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Probability Propagation Nets and Duality

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Abstract. The paper deals with a specific introduction into probability propagation nets. Starting from dependency nets (which in a way can be considered the maximum information which follows from the directed graph structure of Bayesian networks), the probability propagation nets are constructed by joining a dependency net and (a slightly adapted version of) its dual net.

Probability propagation nets are the Petri net version of Bayesian networks. In contrast to Bayesian networks, Petri nets are transparent and easy to operate. The high degree of transparency is due to the fact that every state in a process is visible as a marking of the Petri net. The convenient operability consists in the fact that there is no algorithm apart from the firing rule of Petri net transitions.

Besides the structural importance of the Petri net duality there is a semantic matter; common sense in the form of probabilities and evidence-based likelihoods are dual to each other.

1 Introduction

With this paper, we aim to demonstrate the close relationships between *probability propagation nets (PPNs)* an the *Petri net duality*. PPNs were introduced ([1–5]) to make the processes in Bayesian networks (BNs) more transparent and understandable. This is achieved because PPNs are simple to deal with and all possible system states can easily be followed step by step.

The main reason for that is that the firing rule consists of multiplying the tuples (probability or likelihood tuples) on the input places and subsequently multiplying the result vector by a matrix that is attached to the transition. Finally, the result tuple is put on the (only) output place. The tuple multiplication is either the cross product or a components product for two tuples of equal length (where the result vector contains the products of the respective components of equal arity).

All that expresses the important fact that besides the understandable net structures the algorithms are reduced to most simple vector and matrix operations.

In improving BNs by replacing them by PPNs, we start by representing the dependencies between the random variables by *dependency Petri nets (DNs)*. DNs are "overlays" of net representations of t-invariants which are also Horn nets. They are cycle-free, have a transition boundary, have places with exactly one input arc, and have transitions with at most one output arc. The net representations of t-invariants, which are Horn nets as well, are in a one-to-one relationship with the initializing processes; i.e. every process to calculate the non-prior probabilities takes place in a Horn net, and identifying the Horn nets in a DN will be managed by calculating t-invariants. The concept of DNs was inspired by [6].

The concept of a dual marked net ([7, 8]) leads to nets with two sorts of tokens. Apart from the usual tokens on places (p-tokens) there exist tokens on transitions (t-tokens).

A t-token on a transition t prevents t from firing, even if t is fully enabled by p-tokens. Similar to the p-token flow caused by transitions there exists a t-token flow caused by places which are enabled by t-tokens. The regular direction of t-token flow is against the arc direction. If, unexpectedly, a transition does not fire one can mark it by a t-token and find the reason by observing the backwards directed flow of that t-token. In forward direction the t-tokens indicate the consequences of the non-firing of a transition.

Whereas the flow of probabilities (initializations) in BNs corresponds to the processes in DNs, we will see that the evidence- or observation-driven flow of likelihoods corresponds to the processes in the nets dual to the DNs. This seems to be a deep insight into the inner relationships of "common sense" (probabilities) and evidence (likelihoods), which of course is also valid for BNs. A technicality should be mentioned. The dual net $\mathcal{N}^d$ to a DN $\mathcal{N}$ has also to be provided with a transition boundary (now named $\mathcal{N}^{d*}$). Then $\mathcal{N}$ and $\mathcal{N}^{d*}$ are of the same type (an overlay of Horn nets) and can now be unified to a PPN.

The paper is organized as follows. In section 2 the basic concepts of place/transition nets are introduced to lay the foundation for defining the concept of duality. In the very short section 3 one finds the definition of Horn nets. Section 4 is the introduction into DNs and their expansion to PPNs by joining DNs and their duals. To demonstrate the working PPNs, we use two popular examples. Section 5 is the conclusion.

2 The Duality of Place/Transition Nets

In this section some basics of place/transition nets are introduced. After that the concept of a dual place/transition net is presented as the fundamental concept for defining flows of evidence based on probability flows.

2.1 Place/Transition Nets

Definition 1. *1. A* place/transition net (p/t-net) *is a quadruple* $\mathcal{N} = (P, T, F, W)$ *where*

(a) P and T are finite, non empty, and disjoint sets. P is the set of places *(in the figures represented by circles). T is the set of* transitions *(in the figures represented by squares).*

(b) $F \subseteq (P \times T) \cup (T \times P)$ *is the set of directed* arcs.
(c) $W : F \to \mathbb{N}_0 \backslash \{0\}$ *assigns a* weight *to every arc.*
In case of $W : F \to \{1\}$*, we will write* $\mathcal{N} = (P, T, F)$ *as an abridgement.*

2. *The* preset (postset) of a node $x \in P \cup T$ *is defined as* ${}^\bullet x = \{y \in P \cup T | (y, x) \in F\}$ ($x^\bullet = \{y \in P \cup T | (x, y) \in F\}$).
The preset (postset) of a set $H \subseteq P \cup T$ *is* ${}^\bullet H = \bigcup_{x \in H} {}^\bullet x$ ($H^\bullet = \bigcup_{x \in H} x^\bullet$).
For all $x \in P \cup T$ *it is assumed that* $|{}^\bullet x| + |x^\bullet| \geqq 1$ *holds; i.e. there are* no isolated nodes.
3. *A place p (transition t) is* shared *iff* $|{}^\bullet p| \geqq 2$ or $|p^\bullet| \geqq 2$ ($|{}^\bullet t| \geqq 2$ or $|t^\bullet| \geqq 2$).
4. *A place p is an* input (output) boundary place *iff* ${}^\bullet p = \emptyset$ ($p^\bullet = \emptyset$).
5. *A transition t is an* input (output) boundary transition *iff* ${}^\bullet t = \emptyset$ ($t^\bullet = \emptyset$).

□

Definition 2. *Let* $\mathcal{N} = (P, T, F, W)$ *be a p/t-net.*

1. *A* marking *of* $\mathcal{N}$ *is a mapping* $M : P \to \mathbb{N}_0$. $M(p)$ *indicates the number of* tokens *on p under M.* $p \in P$ *is* marked *by M iff* $M(p) \geqq 1$. $H \subseteq P$ *is* marked *by M iff at least one place* $p \in H$ *is marked by M. Otherwise p and H are* unmarked*, respectively.*
2. *A transition* $t \in T$ *is* enabled *by M, in symbols* $M[t\rangle$*, iff*
$$\forall p \in {}^\bullet t : M(p) \geqq W((p, t)).$$
3. *If* $M[t\rangle$*, the transition t may* fire *or* occur*, thus leading to a new marking* M'*, in symbols* $M[t\rangle M'$*, with*
$$M'(p) := \begin{cases} M(p) - W((p,t)) & \text{if } p \in {}^\bullet t \backslash t^\bullet \\ M(p) + W((t,p)) & \text{if } p \in t^\bullet \backslash {}^\bullet t \\ M(p) - W((p,t)) + W((t,p)) & \text{if } p \in {}^\bullet t \cap t^\bullet \\ M(p) & \text{otherwise} \end{cases}$$
for all $p \in P$.
4. *The set of all markings* reachable *from a marking* M_0*, in symbols* $[M_0\rangle$*, is the smallest set such that*
$$\begin{array}{l} M_0 \in [M_0\rangle \\ M \ \in [M_0\rangle \wedge M[t\rangle M' \Longrightarrow M' \in [M_0\rangle. \end{array}$$
$[M_0\rangle$ *is also called the set of* follower markings *of* M_0.
5. $\sigma = t_1 \dots t_n$ *is a* firing sequence *or* occurence sequence *for transitions* $t_1, \dots, t_n \in T$ *iff there exist markings* $M_0, M_1, \dots, M_n$ *such that*
$$M_0[t_1\rangle M_1[t_2\rangle \dots [t_n\rangle M_n \text{ holds;}$$
in short $M_0[\sigma\rangle M_n$. $M_0[\sigma\rangle$ *denotes that* σ starts from M_0. *The* firing count $\overline{\sigma}(t)$ *of t in* σ *indicates how often t occurs in* σ*. The (column) vector of firing counts is denoted by* $\overline{\sigma}$.
6. *The pair* $(\mathcal{N}, M_0)$ *for some marking* M_0 *of* $\mathcal{N}$ *is a p/t-system or a* marked *p/t-net.* M_0 *is the* initial marking.

7. *A marking* $M \in [M_0\rangle$ *is* reproducible *iff there exists a marking* $M' \in [M\rangle$, $M' \neq M$ *s.t.* $M \in [M'\rangle$. □

Definition 3. *Let* $\mathcal{N} = (P, T, F, W)$ *be a p/t-net and* M_0 *a marking of* $\mathcal{N}$.

1. *A transition* $t \in T$ *is* live under M_0 *or in* $(\mathcal{N}, M_0)$ *iff* $\forall M \in [M_0\rangle\ \exists M' \in [M\rangle : M'[t\rangle$.
2. *A transition* t *is* dead *in* $(\mathcal{N}, M_0)$ *iff* $\forall M \in [M_0\rangle : t$ *is not enabled.* $(\mathcal{N}, M_0)$ *or* M_0 *is* dead *iff* $\nexists t \in T : M_0[t\rangle$.
3. $(\mathcal{N}, M_0)$ *or* M_0 *is* weakly live *(deadlock-free) iff* $\forall M \in [M_0\rangle\ \exists t \in T : M[t\rangle$.
4. $(\mathcal{N}, M_0)$ *or* M_0 *is* live *iff* $\forall t \in T : t$ *is live under* M_0.
5. *A place* $p \in P$ *is* bounded under M_0 *iff* $\exists k \in \mathbb{N}_0\ \forall M \in [M_0\rangle : M(p) \leqq k$. $(\mathcal{N}, M_0)$ *or* M_0 *is* bounded *iff* $\forall p \in P : p$ *is bounded under* M_0.
6. *A place* p *is* markable *in* $(\mathcal{N}, M_0)$ *iff* $\exists M \in [M_0\rangle : M(p) > 0$.
   A set $A \subseteq P$ *is* markable *in* $(\mathcal{N}, M_0)$ *iff* $\exists p \in A : p$ *is markable in* $(\mathcal{N}, M_0)$. □

2.2 Place Vectors and Transition Vectors

Definition 4. *Let* $\mathcal{N} = (P, T, F, W)$ *be a p/t-net.*

1. $\mathcal{N}$ *is* pure *iff* $\nexists (x, y) \in (P \times T) \cup (T \times P) : (x, y) \in F \wedge (y, x) \in F$.
2. *A* place vector *($|P|$-vector) is a column vector* $v : P \longrightarrow \mathbb{Z}$ *indexed by* P.
3. *A* transition vector *($|T|$-vector) is a column vector* $w : T \longrightarrow \mathbb{Z}$ *indexed by* T.
4. *The* incidence matrix *of* $\mathcal{N}$ *is a matrix* $[\mathcal{N}] : P \times T \longrightarrow \mathbb{Z}$ *indexed by* P *and* T *such that*

$$[\mathcal{N}](p, t) := \begin{cases} -W((p, t)) & \text{if } p \in {}^\bullet t \setminus t^\bullet \\ W((t, p)) & \text{if } p \in t^\bullet \setminus {}^\bullet t \\ -W((p, t)) + W((t, p)) & \text{if } p \in {}^\bullet t \cap t^\bullet \\ 0 & \text{otherwise} \end{cases}$$

Column vectors whose entries are all 0 (1) are denoted by $\mathbf{0}$ $(\mathbf{1})$. v^t *and* A^t *are the* transposes *of a vector* v *and a matrix* A*, respectively. The columns of* $[\mathcal{N}]$ *are* $|P|$*-vectors, the rows of* $[\mathcal{N}]$ *are transposes of* $|T|$*-vectors. Markings are representable as* $|P|$*-vectors, firing count vectors as* $|T|$*-vectors. The* $|P|$*-vector* $\mathbf{0}$ *denotes the empty marking* $\emptyset$. □

Definition 5. *Let* i *be a place vector and* j *a transition vector of* $\mathcal{N} = (P, T, F, W)$.

1. i *is a* place invariant (p-invariant) *iff* $i \neq \mathbf{0}$ *and* $i^t \cdot [\mathcal{N}] = \mathbf{0}^t$
2. j *is a* transition invariant (t-invariant) *iff* $j \neq \mathbf{0}$ *and* $[\mathcal{N}] \cdot j = \mathbf{0}$
3. $\|i\| = \{p \in P | i(p) \neq 0\}$ *and* $\|j\| = \{t \in T | j(t) \neq 0\}$ *are the* supports *of* i *and* j*, respectively.*
4. *A p-invariant* i *(t-invariant* j*) is*

– non-negative *iff* $\forall p \in P : i(p) \geqq 0$ ($\forall t \in T : j(t) \geqq 0$)
– positive *iff* $\forall p \in P : i(p) > 0$ ($\forall t \in T : j(t) > 0$)
– minimal *iff* i (j) *is non-negative*
and $\nexists$ *p-invariant* i': $\|i'\| \subsetneqq \|i\|$ ($\nexists$ *t-invariant* j': $\|j'\| \subsetneqq \|j\|$)
and the greatest common divisor of all entries of i (j) *is 1*

5. *The* net representation $\mathcal{N}_i = (P_i, T_i, F_i, W_i)$ of a p-invariant i *is defined by*

$$\begin{aligned} &P_i := \|i\| \\ &T_i := {}^\bullet P_i \cup P_i^\bullet \\ &F_i := F \cap ((P_i \times T_i) \cup (T_i \times P_i)) \\ &W_i \textit{ is the restriction of } W \textit{ to } F_i. \end{aligned}$$

The net representation $\mathcal{N}_j = (P_j, T_j, F_j, W_j)$ of a t-invariant j *is defined by*

$$\begin{aligned} &T_j := \|j\| \\ &P_j := {}^\bullet T_j \cup T_j^\bullet \\ &F_j := F \cap ((P_j \times T_j) \cup (T_j \times P_j)) \\ &W_j \textit{ is the restriction of } W \textit{ to } F_j. \end{aligned}$$

6. $\mathcal{N}$ *is* covered *by a p-invariant i (t-invariant j) iff* $\forall p \in P : i(p) \neq 0$ ($\forall t \in T : j(t) \neq 0$) □

Proposition 1. *Let* $(\mathcal{N}, M_0)$ *be a p/t-system, i a p-invariant; then*

$$\forall M \in [M_0\rangle : i^t \cdot M = i^t \cdot M_0.$$

□

Proposition 2. *Let* $(\mathcal{N}, M_0)$ *be a p/t-system,* $M_1 \in [M_0\rangle$ *a follower marking of* M_0*, and* σ *a firing sequence that reproduces* $M_1 : M_1[\sigma\rangle M_1$*; then the firing count vector* $\overline{\sigma}$ *of* σ *is a t-invariant.* □

Definition 6. *Let* $\mathcal{N} = (P, T, F, W)$ *be a p/t-net,* M_0 *a marking of* $\mathcal{N}$*, and* $r \geqq \mathbf{0}$ *a* $|T|$*-vector;* r *is* realizable in $(\mathcal{N}, M_0)$ *iff there exists a firing sequence* σ *with* $M_0[\sigma\rangle$ *and* $\overline{\sigma} = r$. □

Proposition 3. *Let* $\mathcal{N} = (P, T, F, W)$ *be a p/t-net,* M_1 *and* M_2 *markings of* $\mathcal{N}$*, and* σ *a firing sequence s.t.* $M_1[\sigma\rangle M_2$*; then the linear relation*

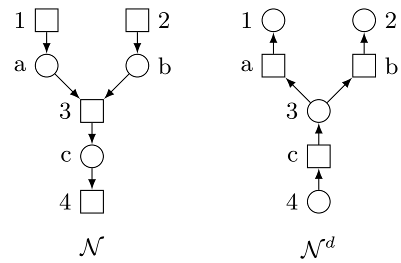
$$M_1 + [\mathcal{N}]\overline{\sigma} = M_2 \text{ holds.}$$

□

2.3 Dualizing the Structure

Definition 7. (**Dual p/t-net**) *Let* $\mathcal{N} = (P, T, F, W)$ *be a p/t-net with*

– $P \neq \emptyset$ *(set of places)*
– $T \neq \emptyset$ *(set of transitions)*
– $P \cap T = \emptyset$
– $F \subseteq (P \times T) \cup (T \times P)$ *(Flow relation, set of arcs)*
– $W : F \to \mathbb{N}_0 \backslash \{0\}$ *(arc weight function)*;



$[\mathcal{N}]$	1	2	3	4
a	1		−1	
b		1	−1	
c			1	−1

$[\mathcal{N}^d]$	a	b	c
1	1		
2		1	
3	−1	−1	1
4			−1

Fig. 1. P/t-nets $\mathcal{N}$ and $\mathcal{N}^d$ and incidence matrices $[\mathcal{N}]$ and $[\mathcal{N}^d]$

the p/t-net $\mathcal{N}^d = (P^d, T^d, F^d, W^d)$ *is the* dual net *of* $\mathcal{N}$ *iff*

- $P^d = T$
- $T^d = P$
- $F^d = F^{-1} = \{(y, x) | (x, y) \in F\}$
- $W^d((y, x)) = W((x, y))$ *for all* $(x, y) \in F$

□

Roughly speaking, the dual net $\mathcal{N}^d$ of a p/t-net $\mathcal{N}$ is developed by transposing the incidence matrix $[\mathcal{N}]$ of $\mathcal{N}$. By that, places and transitions are exchanged and the direction of all arcs is changed. If $\mathcal{N}$ is marked, the tokens remain on their places and become transition tokens that way.

Proposition 4. (trivial)

(a) $[\mathcal{N}^d] = [\mathcal{N}]^t$
(b) p-invariants (t-invariants) of $\mathcal{N}^d$ *are*
t-invariants (p-invariants) of $\mathcal{N}$ □

Example 1. Figure 1 shows a p/t-net $\mathcal{N}$ and the dual net $\mathcal{N}^d$ as well as the corresponding incidence matrices $[\mathcal{N}]$ and $[\mathcal{N}^d]$. □

2.4 Dualizing the Behavior

For dualizing the behavior, one needs an extension to nets with markings. The most obvious extension is to leave the tokens (place tokens, p-tokens) on their places. When the places are converted into transitions, the p-tokens are converted into transition tokens (t-tokens).

Remark 1. When marked nets are dualized, a second sort of tokens arises, namely t-tokens *as markings of transitions.*

Before defining all that formally, an introducing example might be advisable. In the figures, p-tokens are drawn as small circles (as usual) and t-tokens as small squares.

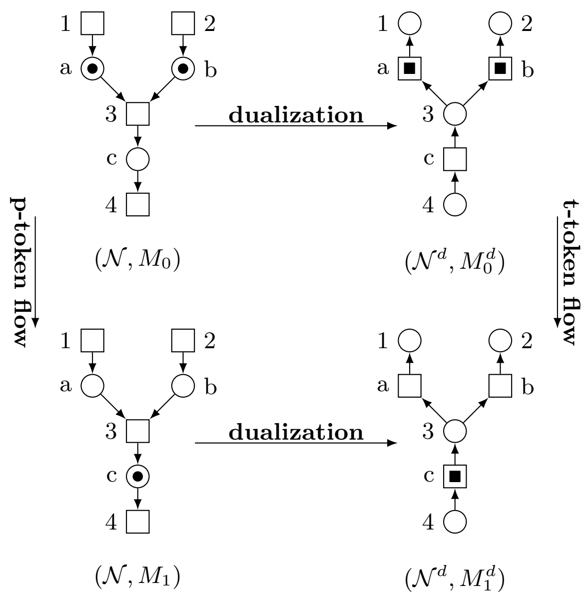


Fig. 2. Token flow in $\mathcal{N}$ and $\mathcal{N}^d$

Example 2. Figure 2 shows four marked p/t-nets (cf. Fig. 1). In $\mathcal{N}$ $M_0[3\rangle M_1$ holds, i.e. M_1 follows from M_0 by firing transition 3. Now, we *demand* $M_0^d[3\rangle M_1^d$ also in $\mathcal{N}^d$, i.e. M_1^d follows from M_0^d by firing place 3. So places *fire backwards (against the arc direction).* □

Remark 2. Dualizing marked p/t-nets induces the firing of enabled places. A place is enabled *if its output transitions are sufficiently marked by t-tokens.*

Of course, now the question of the meaning of t-tokens arises.

Example 3. Transition 4 of the first net of the first row in Fig. 3 is crossed out, what is assumed to mean that this transition was not able or not allowed to fire. The reason for it is that before (shown in the second net of the first row) transition 3 was not able or not allowed to fire. Here the reason is that transition 1 or 2 was not able or not allowed to fire. Comparing the first two rows shows that the crosses and the t-tokens behave without any difference because of the firing rule for t-tokens. □

Now an important question arises: What can be gained by duality? T-tokens and firing places yield only a *new interpretation* of the traditional net dynamics and nothing else because of $(\mathcal{N}^d, M_0^d)^d = (\mathcal{N}, M_0)$. But the dual should enrich the original net. That is to be achieved by permitting *nets with both sorts of tokens.*

Example 4. This is a modification of Example 3. In all nets of Fig. 4, node B is marked by one suitable token. In row one it is no longer sensible to assume

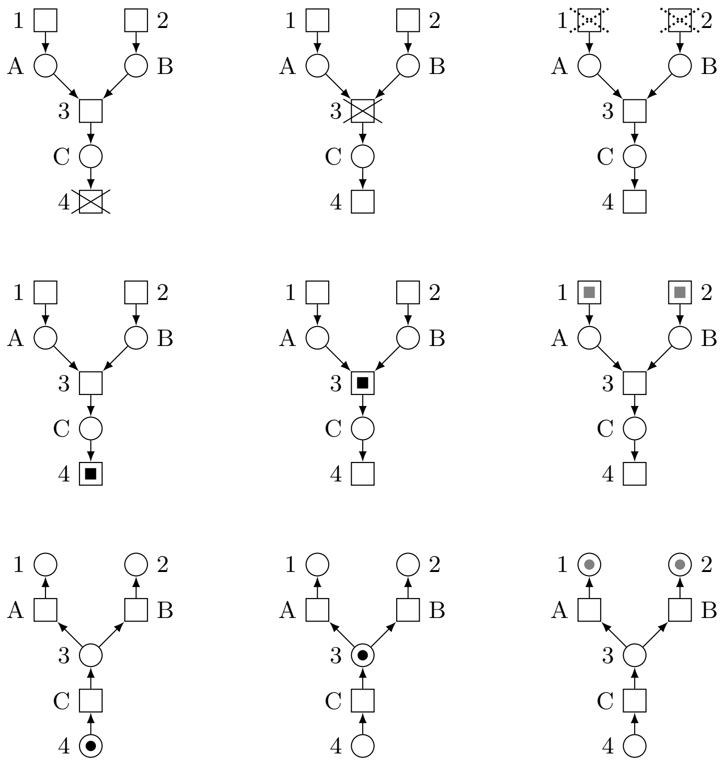


Fig. 3. Interpretation of t-tokens (1)

that transition 2 was not able or not allowed to fire because the p-token on B might be the result of a firing of transition 2. Now we assume that a *marked* node (place and transition) cannot be enabled, regardless of the node being "enabled" in the usual way. Consequently, the t-tokens in the second row behave like the crosses. □

Remark 3. p- and t-tokens block each other.

Definition 8. **(p/t-marking)** *Let* $\mathcal{N} = (P, T, F, W)$ *be a p/t-net;*
M *is* a place/transition marking (p/t-marking) *iff* $M : P \cup T \to \mathbb{N}_0$*;*

$p \in P$ is p-marked (marked) *iff* $M(p) \geq 1$*,*
$t \in T$ is t-marked (marked) *iff* $M(t) \geq 1$*;*

the tokens on places are p-tokens*;*
the tokens on transitions are t-tokens*;*

$p \in P$ is enabled for M *iff* $M(p) = 0 \wedge \forall y \in p^\bullet : M(y) \geq W((p, y))$.
$t \in T$ is enabled for M *iff* $M(t) = 0 \wedge \forall x \in {}^\bullet t : M(x) \geq W((x, t))$.
So, marked nodes cannot be enabled.

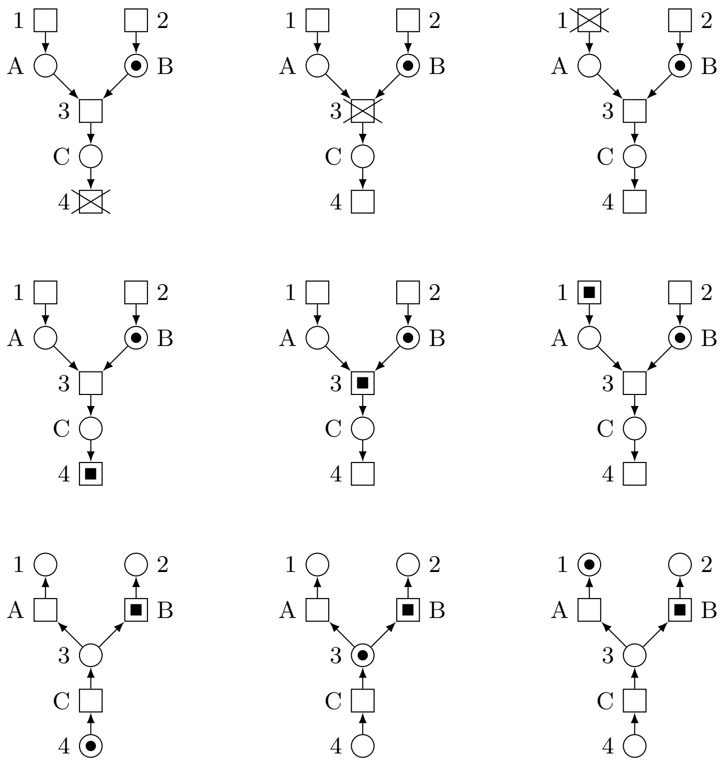


Fig. 4. Interpretation of t-tokens (2)

Let $p \in P$ *be enabled for M;*
the follower marking M' of M after one firing of p *is given by*

$$M'(y) := \begin{cases} M(y) - W((p,y)) & \text{if } y \in p^\bullet \setminus {}^\bullet p \\ M(y) + W((y,p)) & \text{if } y \in {}^\bullet p \setminus p^\bullet \\ M(y) - W((p,y)) + W((y,p)) & \text{if } y \in {}^\bullet p \cap p^\bullet \\ M(y) & \text{if } y \notin {}^\bullet p \cup p^\bullet \end{cases}$$

$$\text{for all } y \in T$$

$$M'(x) := \quad M(x) \qquad \text{for all } x \in P;$$

let $t \in T$ *be enabled for M;*
the follower marking M'' of M after one firing of t *is given by*

$$M''(x) := \begin{cases} M(x) - W((x,t)) & \text{if } x \in {}^\bullet t \setminus t^\bullet \\ M(x) + W((t,x)) & \text{if } x \in t^\bullet \setminus {}^\bullet t \\ M(x) - W((x,t)) + W((t,x)) & \text{if } x \in {}^\bullet t \cap t^\bullet \\ M(x) & \text{if } x \notin {}^\bullet t \cup t^\bullet \end{cases}$$

$$\text{for all } x \in P$$

$$M''(y) := \quad M(y) \qquad \text{for all } y \in T; \qquad \square$$

Definition 9. (**dual marking**) *Let $\mathcal{N} = (P, T, F, W)$ be a p/t-net and $\mathcal{N}^d = (P^d, T^d, F^d, W^d)$ its dual net, such that $P^d = T$, $T^d = P$; let $M : P \cup T \to \mathbb{N}_0$ be a p/t-marking of $\mathcal{N}$. $M(P)$ is a $|P|$-vector, $M(T)$ is a $|T|$-vector.*

$M^d = P^d \cup T^d \to \mathbb{N}_0$ is the dual marking *of M iff $M^d(P^d) = M(T)$ and $M^d(T^d) = M(P)$.* □

Example 5. In the second net of the second row of Fig. 3, the places A and B are in a conflict (so, they are enabled!). In the corresponding net of Fig. 4, only place A is enabled. In the second row of both figures, transition 4 is only disabled in the first net. The corresponding statements hold for the dual nets in the third row. □

Although the concept of duality for marked nets was already introduced in [7], even for a class of higher level nets, it took quite a long time to ultimately get convinced that marked transitions and firing places might yet be useful concepts and no "net-theoretical sacrilege".

3 P/T-Net Representation of Propositional Formulae

In this section, we define a p/t-net representation for propositional formulae in conjunctive normal form (CNF). If in the net representation of a Horn formula the empty marking is reproducible, the formula is contradictory. So, indirect proofs can be given by reproducing the empty marking.

Definition 10. (**p/t-net representation of a formula**) *Let α be a propositional CNF-formula and $\mathcal{N}_\alpha = (P_\alpha, T_\alpha, F_\alpha, W_\alpha)$ a p/t-net with $W_\alpha : F_\alpha \to \{1\}$; $\mathcal{N}_\alpha$ is the* net representation *of α iff*

$P_\alpha = \mathbb{A}(\alpha)$ *(the* set of atoms *of α)*

and $T_\alpha = \mathbb{C}(\alpha)$ *(the* set of clauses *of α)*

and $\forall(\tau = \neg a_1 \vee \ldots \vee \neg a_m \vee b_1 \vee \ldots \vee b_n \in \mathbb{C}(\alpha)$ *with* $\{a_1, \ldots, a_m, b_1, \ldots, b_n\} \subseteq \mathbb{A}(\alpha))$:

$^\bullet\tau = \{a_1, \ldots, a_m\}$ *and* $\tau^\bullet = \{b_1, \ldots, b_n\}$;

i.e.: the negated literals are input places, the non-negated literals are output places of the transition representing τ. In [9] this net representation is called "canonical". □

Definition 11. (**Horn clause, Horn formula, Horn net**) *Let α be a propositional CNF-formula;*

a clause κ of α is a Horn clause *iff it contains at most one non-negated literal;*

α is a Horn formula *iff all its clauses are Horn clauses;*

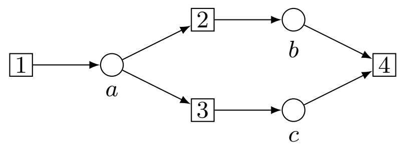
the net representation $\mathcal{N}_\alpha$ of a Horn formula α is a Horn net. □

Theorem 1. (**see [9]**) *A Horn formula α is contradictory iff in $\mathcal{N}_\alpha$ the empty marking is reproducible.* □

Example 6.

$$\alpha = \underset{(1)}{a} \wedge \underset{(2)}{(\neg a \vee b)} \wedge \underset{(3)}{(\neg a \vee c)} \wedge \underset{(4)}{(\neg b \vee \neg c)}$$

is a Horn formula; the net representation $\mathcal{N}_\alpha$ is:



The empty marking $\mathbf{0}$ is reproduced by the firing sequence $\sigma = (1\ 2\ 1\ 3\ 4)$, i.e. $\mathbf{0}[\sigma\rangle\mathbf{0}$. So, α is contradictory or, what is the same, $\mathbf{0}[\sigma\rangle\mathbf{0}$ is an indirect proof of the fact that $b \wedge c$ is a logical consequence of clauses 1,2, and 3. □

4 Dependency Petri Nets and Probability Propagation Nets

The dependency nets introduced in this section play a double role. First, they are to replace the graph structure of Bayesian networks as the means for describing dependencies between random variables. Second, a dependency net joined with its dualization represents by far the largest part of a probability propagation net ([1–5]). Thus, each dependency net is the backbone of a simple construction principle for PPNs. Within the PPN the dependency net governs the probability propagation and its dual net governs the propagation of new evidences (the likelihoods).

Definition 12. (structural dependency Petri net) *Let* $\mathcal{N} = (P, T, F, W)$ *be a p/t-net;* $\mathcal{N}_S$ *is a* structural dependency net *iff*

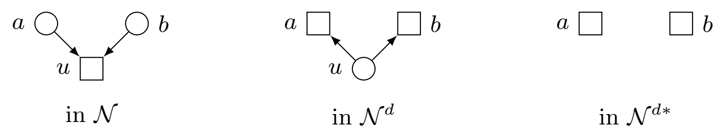
$W : F \to \{1\}$
and $\mathcal{N}_S$ *has a transition boundary*
and $\mathcal{N}_S$ *is connected and cycle-free*
and $\forall k \in P \cup T : (|{}^\bullet k| \geqq 2 \Rightarrow k \in T) \wedge (|k^\bullet| \geqq 2 \Rightarrow k \in P)$
and $\forall t \in T : t^\bullet = \emptyset \Rightarrow |{}^\bullet t| = 1$.

This concept (see [10]) was inspired by [6]. □

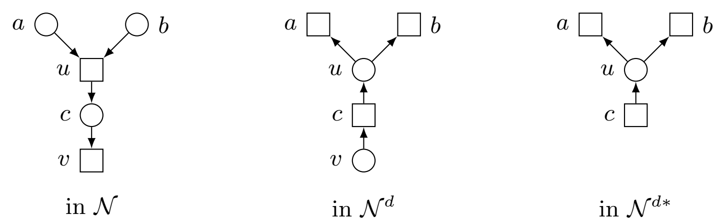
The transition boundary of Horn nets and net representations of t-invariants ensure important properties which have to do with reproducing the empty marking. In DNs, which are "overlays" of Horn nets, the reproductions of the empty marking are the initializing flows of probabilities and vice versa. Unfortunately, the dual nets of DNs have a place boundary. So, they are unable to reproduce the empty marking. Fortunately, this shortcoming can be corrected.

Definition 13. (modified dual nets) *Let* $\mathcal{N}_S$ *be a structural DN and* $\mathcal{N}_S^d$ *the dual net;* $\mathcal{N}_S^{d*}$ *is the* modification *of* $\mathcal{N}_S^d$ *where all input boundary places are omitted and all output boundary places get an additional individual (unshared) output transition.* □

The last condition of Definition 12 is due to the joining of a DN $\mathcal{N}$ and a modification of $\mathcal{N}^d$. If an output transition u of $\mathcal{N}$ has more than one input place, the dual net $\mathcal{N}^d$ has an input place u with more than one output transition.



When constructing $\mathcal{N}^{d*}$ the input place u will be omitted which is defective, because a and b are no longer output transitions of one place. A dummy place and transition resolve the problem. Now u meets the last condition of Definition 12.



Example 7. Figure 5(a) shows a structural DN $\mathcal{N}_S$. $\mathcal{N}_S$ is covered by two Horn nets with the respective sets of nodes

$$\{P, Q, a, b, U, d, V, g, X\} \text{ and } \{P, Q, a, b, U, d, W, h, Y\}.$$

In addition, both Horn nets are net representations of t-invariants. The corresponding t-invariants permit to reproduce the empty marking which proves that g and h can be inferred from

$$a \wedge b \wedge (\neg a \vee \neg b \vee d) \wedge (\neg d \vee g) \quad \text{and}$$
$$a \wedge b \wedge (\neg a \vee \neg b \vee d) \wedge (\neg d \vee h) \text{ , respectively.}$$

Figure 5(b) shows the DN $\mathcal{N}$ corresponding to $\mathcal{N}_S$ with standard inscriptions for arcs and transitions. Each π-value as arc inscription denotes a probability tuple. The p-values at the transitions denote the respective conditional probability matrices.

P and Q (as transitions without input places) are permanently enabled. When firing they put prior probability tuples $\pi(a)$ and $\pi(b)$ on a and b, respectively. Then U is enabled and takes $\pi(a)$ and $\pi(b)$ from a and b and puts $(\pi(a) \times \pi(b)) \cdot p(d|ab)$ on place d. Now V and W are enabled, but in a conflict situation. If V fires, $\pi(d)$ is taken from d and $\pi(g) = \pi(d) \cdot p(g|d)$ is put on place g. Similarly, if W fires, $\pi(d)$ is taken from d and $\pi(h) = \pi(d) \cdot p(h|d)$ is put on place h. The transitions X and Y are to clear the net, thus completing the reproduction of the empty marking. So, instead of inferring g and h in $\mathcal{N}_S$, in $\mathcal{N}$ $\pi(g)$ and $\pi(h)$ are calculated.

The outcomes of reproducing the empty marking in $\mathcal{N}$ and of the initializing phase in the Bayesian network in Fig. 6 are identical, except that the cause of action in $\mathcal{N}$ is observable and comprehensible. □

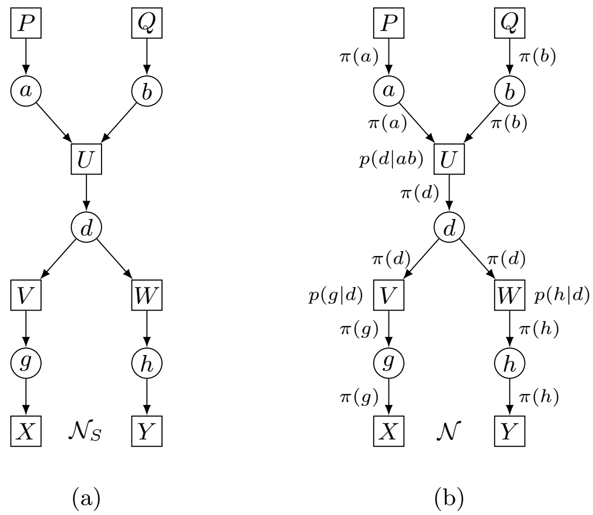


Fig. 5. Structural DN $\mathcal{N}_S$ and the corresponding DN $\mathcal{N}$

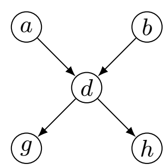


Fig. 6. Bayesian network

Originally, the construction of PPNs was a translation of algorithms from the books by Pearl [11] and Neapolitan [12] (see [1, 4]). In this paper, we will construct a PPN by dualizing a DN $\mathcal{N}$ and then joining $\mathcal{N}$ and a slight modification $\mathcal{N}^{d*}$ of the dual net $\mathcal{N}^d$.

The appeal of this approach is due to an inherent localization principle. First, when dualizing the DN $\mathcal{N}$, this can be done node by node and arc by arc in any order. Next, in joining $\mathcal{N}$ and $\mathcal{N}^{d*}$, only the shared nodes are affected and can also be chosen in any order.

Definition 14. (shared node) *Let $\mathcal{N} = (P, T, F, W)$ be a DN; a node $k \in P \cup T$ is* shared *iff*

$$|{}^\bullet k| \geqq 2 \text{ or } |k^\bullet| \geqq 2 \text{ holds.}$$

□

That means in particular that only nodes have to be taken into account and no paths (see [10]), which would make all calculations more complex.

We now come to depict the joining algorithm that expands a DN to a PPN. Building the structure of the PPN is quite simple. It is a bit complicated to describe the names of nodes and arcs of the new structure. Therefore we will split the description of the algorithm into two parts and start with DNs which have shared transitions but no shared places.

Joining Algorithm (1st part)

Let $\mathcal{N} = (P, T, F, W)$ be a DN with unshared places, i.e. $\forall p \in P : |p^\bullet| = 1$; let $\mathcal{N}_S$ be the corresponding structural DN and $\mathcal{N}_S^{d*} = (P^{d*}, T^{d*}, F^{d*}, W^{d*})$ its dualization; let $f = f_A(A_1, \ldots, A_n)$ be a shared transition with ${}^\bullet f = \{A_1, \ldots, A_n\}$ and $f^\bullet = \{A\}$, where $f_A(A_1, \ldots, A_n)$ is the name of the transition and of the matrix attached to it with $f_A(A_1, \ldots, A_n) := p(A|A_1 \ldots A_n)$.

When joining $\mathcal{N}_S$ and $\mathcal{N}_S^{d*}$, in $\mathcal{N}_S^{d*}$ every output transition $A_i \in f^\bullet$ of $f \in P^{d*}$ gets $n-1$ additional input places $(\{A_1, A_2, \ldots, A_n\} \cap P) \setminus \{A_i\}$, $1 \leqq i \leqq n$, from $\mathcal{N}_S$. In Fig. 7 the respective arcs are dashed.

Now some names have to be changed because the usual allocation of names in dual nets and in PPNs is different. $\mathcal{N}$ is given, so the names of its nodes are given. All arc names are of the form $\pi(U)$ if $U \in P$ is the name of the incidenting place. The names of the dashed arcs get the names $\pi(U)$, too, if $U \in P$ is the incidenting place.

The names of the transitions in $\mathcal{N}^{d*}$, which currently have the names $A_1, \ldots,$ A_n, can be uniquely replaced if one takes into account that the corresponding matrices are generalized transposes of $f_A(A_1, \ldots, A_n)$. So, they have the same variables. The transition $A_1 \in T^{d*}$, for example, has the input places $A_2, \ldots, A_n \in P$ and $f \in P^{d*}$. Consequently, $A_1 \in P$ must be an output place and the transition A_1 must be renamed as $f_{A_1}(A, A_2, \ldots, A_n)$. That means that the place with the current name $f \in P^{d*}$ must get the name A. $A_2, \ldots, A_n \in T^{d*}$ are renamed likewise.

All arc names in $\mathcal{N}^{d*}$ are of the form $\lambda(V)$ if $V \in P^{d*}$ is the incidenting place (see Fig. 8). Some final remarks:

(a) If $f \in T$ is not shared, the additional (dashed) arcs are omitted, and the transpose is the usual one.
(b) The arc labels $\pi(\ldots)$, $\lambda(\ldots)$ might be replaced by constant tuples.
(c) The names of the input boundary transitions are arbitrary.

The outcome of this algorithm for the DN $\mathcal{N}$ is the corresponding PPN. □

Example 8. (see [13]) Figure 9 shows a Bayesian network (BN). It is a directed acyclic graph whose nodes are random variables. The prior probabilities of B and C are given as well as the conditional probability matrix of A. It can be seen that A depends on its parent nodes B and C. We are mainly interested in Mr. Holmes' belief in the possbility of being burglarized.

Initially, this belief equals 0.01. But after he got a call with the information that his burglar alarm is sounding, his belief increases to 0.476. Then he hears on the car radio that there was an earthquake in his area, and his belief in a burglary goes down to 0.02.

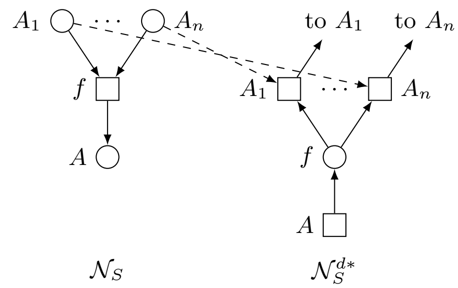


Fig. 7. Joining algorithm, 1st part (1)

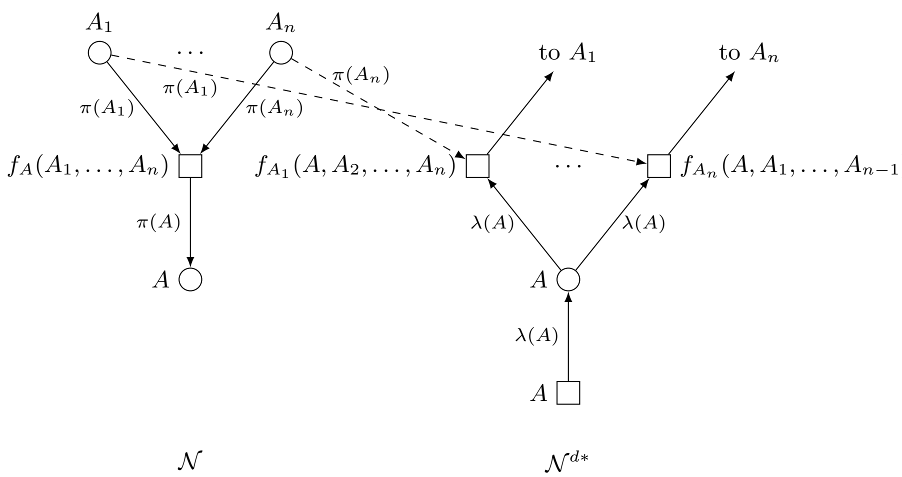


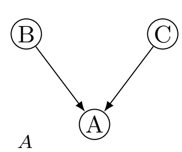
Fig. 8. Joining algorithm, 1st part (2)

$P(B)$:

B	1	0
	0.01	0.99

$P(C)$:

C	1	0
	0.001	0.999

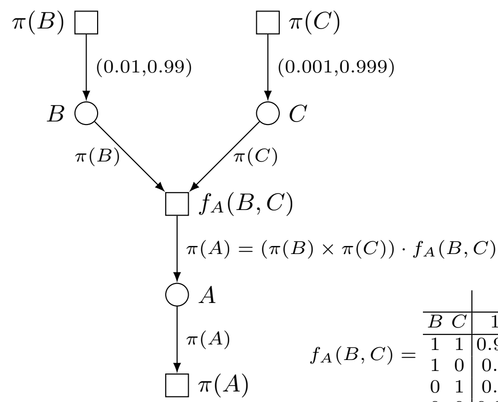


$P(A|BC) =$

B	C	A = 1	A = 0
1	1	0.99	0.01
1	0	0.9	0.1
0	1	0.5	0.5
0	0	0.01	0.99

A_1 = Mr. Holmes' burglar alarm sounds
A_0 = Mr. Holmes' burglar alarm does *not* sound
B_1 = Mr. Holmes' residence is burglarized
B_0 = Mr. Holmes' residence is *not* burglarized
C_1 = there is an earthquake
C_0 = there is *no* earthquake

Fig. 9. A Bayesian network



$f_A(B,C) =$ (table below) $= P(A|BC)$

		A	
B	C	1	0
1	1	0.99	0.01
1	0	0.9	0.1
0	1	0.5	0.5
0	0	0.01	0.99

Fig. 10. The DN corresponding to Fig. 9

This belief revision cannot be seen in the BN. It is "hidden" in the algorithms. We now want to build a PPN by completing the DN shown in Fig. 10 which corresponds to the BN of Fig. 9. The transitions $\pi(B)$ and $\pi(C)$ are input boundary transitions which put the attached tuples on B and C, respectively, when firing. The only non-boundary transition is $f_A(B,C)$ which has the same name as the matrix attached to it. The attached probability tuples are the respective prior probabilities, $f_A(B,C)$ is the conditional probability table $P(A|BC)$. The output boundary transition $\pi(A)$ clears the net, thus completing the reproduction of the empty marking if $\pi(B)$, $\pi(C)$ and $f_A(B,C)$ have fired. The nomenclature is similar to that in the book by Neapolitan [12]. To use the same name for different mathematical terms keeps the number of names manageable. Figure 11 shows the corresponding PPN. Since the calculation of the generalized transpose of the matrix is simple, but quite complicated in the general case, it is sufficient to demonstrate it for the example. The vector form of the matrix

$f_A(B,C) =$

		A	
B	C	1	0
1	1	0.99	0.01
1	0	0.9	0.1
0	1	0.5	0.5
0	0	0.01	0.99

is shown in Fig. 12(a). Both columns of $f_A(B,C)$ are unified in one column, where the column A is to enable a unique transformation back. The specific distribution of 1's and 0's characterizes A as output variable; this is also the case for the output variables B and C in Fig. 12(b) and (c), respectively. Also the first and second input variables have specific distributions of 1's and 0's. The most important point is that the tuples in the three vectors are identical. Of course, they are in varying order. The back transformation of the vectors (b)

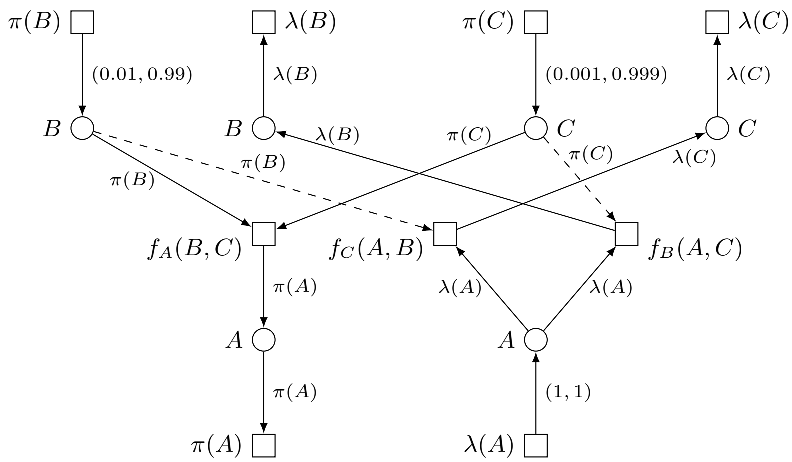


Fig. 11. PPN

A	B	C	$f_A(B,C)$
1	1	1	0.99
1	1	0	0.9
1	0	1	0.5
1	0	0	0.01
0	1	1	0.01
0	1	0	0.1
0	0	1	0.5
0	0	0	0.99

(a)

A	B	C	$f_B(A,C)$
1	1	1	0.99
1	1	0	0.9
0	1	1	0.01
0	1	0	0.1
1	0	1	0.5
1	0	0	0.01
0	0	1	0.5
0	0	0	0.99

(b)

A	B	C	$f_C(A,B)$
1	1	1	0.99
1	0	1	0.5
0	1	1	0.01
0	0	1	0.5
1	1	0	0.9
1	0	0	0.01
0	1	0	0.1
0	0	0	0.99

(c)

Fig. 12. Generalized transposes

and (c) of Fig. 12 yields the derived matrices

$$f_B(A,C) = \begin{array}{cc|cc} & & \multicolumn{2}{c}{B} \\ A & C & 1 & 0 \\ \hline 1 & 1 & 0.99 & 0.5 \\ 1 & 0 & 0.9 & 0.01 \\ 0 & 1 & 0.01 & 0.5 \\ 0 & 0 & 0.1 & 0.99 \end{array} \quad , \qquad f_C(A,B) = \begin{array}{cc|cc} & & \multicolumn{2}{c}{C} \\ A & B & 1 & 0 \\ \hline 1 & 1 & 0.99 & 0.9 \\ 1 & 0 & 0.5 & 0.01 \\ 0 & 1 & 0.01 & 0.1 \\ 0 & 0 & 0.5 & 0.99 \end{array} \quad .$$

As shown in Fig. 10, $\pi(A) = (\pi(B) \times \pi(C)) \cdot f_A(B,C)$. That means when transition $f_A(B,C)$ fires, the tuples $\pi(B)$ and $\pi(C)$ are taken from their input places B and C, and the value $\pi(A) = (\pi(B) \times \pi(C)) \cdot f_A(B,C)$ is put on A. Similarly, when $f_B(A,C)$ fires the tuples $\lambda(A)$ and $\pi(C)$ are taken from places A and C and $\lambda(B) = (\lambda(A) \times \pi(C)) \cdot f_B(A,C)$ is put on B.

Remark 4. The order of the variables in the cross product has to coincide with the order of the input variables in the transition or matrix name. □

Definition 15. (belief, components product, [11]) *The* belief $bel(X)$ *of a random variable is defined as*

$$
\begin{aligned}
bel(X) &:= \alpha \cdot (\pi(X) \circ \lambda(X)) \\
&= \alpha \cdot ((\pi_1, \ldots, \pi_n) \circ (\lambda_1, \ldots, \lambda_n)) \\
&= \alpha \cdot (\pi_1\lambda_1, \ldots, \pi_n\lambda_n) = (b_1, \ldots, b_n) \\
&\textit{where } \sum_{i=1}^{n} b_i = 1.
\end{aligned}
$$

α *is a* normalizing factor.
$\circ$ *is the* components product. □

The functioning of the PPN

$\pi(B)$ and $\pi(C)$ fire and put the constant tuples $(0.01, 0.99)$ and $(0.001, 0.999)$ on places B and C; then $f_A(B, C)$ fires and puts $(0.019, 0.981)$ on place A, which after that is cleared by transition $\pi(A)$.

The current value of $\lambda(A)$ is $(1, 1)$, saying that nothing is known about Mr. Holmes' burglar alarm, neither that it is sounding nor that it is not. So, the belief of A is $bel(A) = ((0.019, 0.981) \times (1, 1)) = (0.019, 0.981)$.

For calculating $bel(B)$, $\lambda(A)$ puts $(1, 1)$ on A and $\pi(C)$ puts $(0.001, 0.999)$ on C; then $f_B(A, C)$ takes these values from A and C and puts $\lambda(B) = ((1, 1) \times (0.001, 0.999)) \cdot f_B(A, C) = (1, 1)$ on B, so $bel(B) = ((0.01, 0.99) \circ (1, 1)) = (0.01, 0.99)$.

Now the alarm is sounding and $(1, 1)$ at the arc $(\lambda(A), A)$ is replaced by $(1, 0)$, which yields $bel(B) = (0.476, 0.524)$ (after normalizing). The fact of the earthquake is set down in replacing $(0.001, 0.999)$ at the arc $(\pi(C), C)$ by $(1, 0)$, which leads to $\lambda(B) = (0.02, 0.98)$ (again after normalizing).

In both cases the transition $\lambda(B)$ clears B, thus completing a reproduction of the empty marking. Similarly, the transition $\lambda(C)$ clears C after calculating $\lambda(C)$, thus also completing a **0**-reproduction.

The important fact is that the three t-invariants, respectively their net representations, which are also Horn nets, and the processes to calculate $\pi(A)$, $\lambda(B)$, and $\lambda(C)$ correspond to each other one-to-one. □

This example shows a remarkable fact, namely the roles of the DN $\mathcal{N}$ and the modified dual net $\mathcal{N}^{d*}$. In $\mathcal{N}$ probabilities are propagated which represent the "common sense", the experience hithertho. $\mathcal{N}^{d*}$ is the net for propagating the new evidences based on recent observations. So the "common sense" and the new evidence are propagated in nets which are dual to each other. Partly, the evidences are communicated from $\mathcal{N}$ into $\mathcal{N}^{d*}$ via the "dashed" arcs. Besides this function the "dashed" arcs are interesting because of a technical point of view. Figures 13 and 14 show $\mathcal{N}$ and $\mathcal{N}^{d*}$ in two different representations (without arc inscriptions). Figures 14 and 15 have in common that a p-token on place $A \in P^{d*}$ (in $\mathcal{N}^{d*}$) cannot enable f_C but does enable f_B; i.e. the dashed arcs are the means to manage the propagation in the dual nets $\mathcal{N}^{d}$ and $\mathcal{N}^{d*}$ without t-tokens (or t-objects like t-tuples in general).

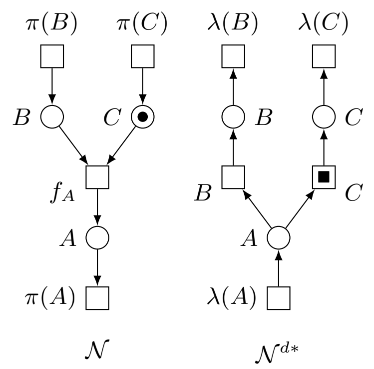


Fig. 13.

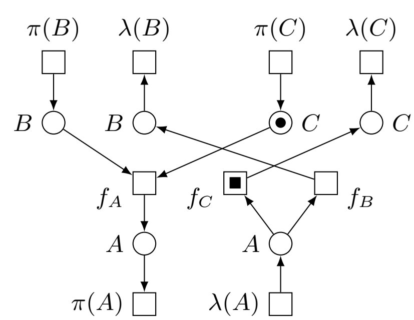


Fig. 14.

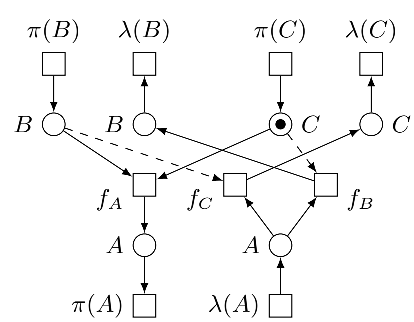


Fig. 15.

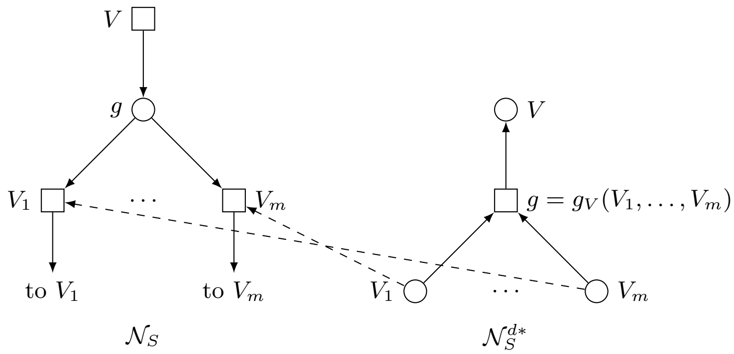


Fig. 16. Joining algorithm, 2nd part (1)

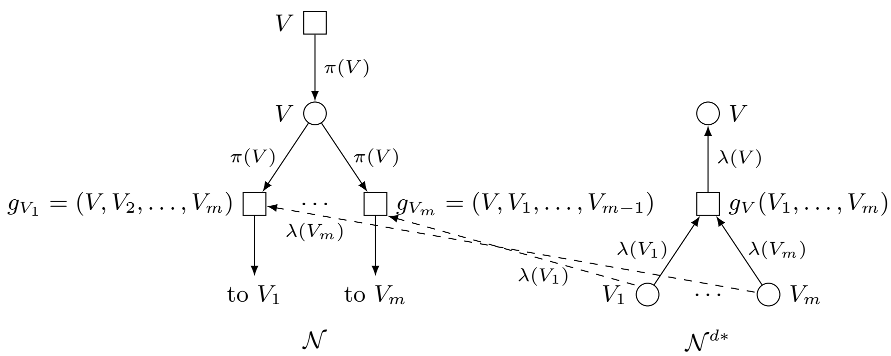


Fig. 17. Joining algorithm, 2nd part (2)

Joining Algorithm (2nd part)

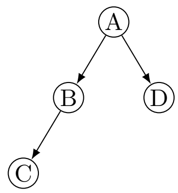
Let $\mathcal{N} = (P, T, F, W)$ be a DN with unshared transitions, i.e. $\forall t \in T : |{}^\bullet t| \leqq 1$; let $\mathcal{N}_S$ be the corresponding structural DN and $\mathcal{N}_S^d = (P^d, T^d, F^d, W^d)$ its dualization; let $\mathcal{N}_S^{d*} = (P^{d*}, T^{d*}, F^{d*}, W^{d*})$ be the modification of $\mathcal{N}_S^d$; let $g = g_V(V_1, \ldots, V_m)$ be a shared place with ${}^\bullet g = \{V\}$ and $g^\bullet = \{V_1, \ldots, V_m\}$.

When joining $\mathcal{N}_S$ and $\mathcal{N}_S^{d*}$, in $\mathcal{N}_S^{d*}$ every output transition $V_j \in g^\bullet$ of $g \in P$ gets $m-1$ additional input places $(\{V_1, \ldots, V_m\} \cap P^{d*}) \setminus \{V_j\}$, $1 \leqq j \leqq m$, from $\mathcal{N}_S^{d*}$. In Fig. 16 the respective arcs are dashed. Again, some names have to be changed because of the different standard allocation of names in dual nets and PPNs.

The transitions $V_1, \ldots, V_m$ of $\mathcal{N}$ are uniquely renamed as $g_{V_1}(V, V_2, \ldots, V_m)$, $\ldots, g_{V_m}(V, V_1, \ldots, V_{m-1})$. The place $g \in \mathcal{N}_S$ must get the name V.

Whereas the arc names in $\mathcal{N}$ are given, all arc names in $\mathcal{N}^{d*}$ are of the form $\lambda(V_i)$ if $V_i \in P^{d*}$ is the incidenting place (see Fig. 17). Again, some formal remarks:

(a) If $g \in P$ is not shared, the additional (dashed) arcs are omitted.



A_1 = spouse is cheating
A_0 = spouse is *not* cheating
B_1 = spouse dines with another
B_0 = spouse does *not* dine with another
C_1 = spouse is reported seen dining with another
C_0 = spouse is *not* reported seen dining with another
D_1 = strange man/lady calls on the phone
D_0 = *no* strange man/lady calls on the phone

$$P(A) = \begin{array}{c|cc} A & 1 & 0 \\ \hline & 0.1 & 0.9 \end{array} \qquad P(B|A) = f_B(A) = \begin{array}{c|cc} & \multicolumn{2}{c}{B} \\ \hline A & 1 & 0 \\ \hline 1 & 0.7 & 0.3 \\ 0 & 0.2 & 0.8 \end{array}$$

$$P(C|B) = f_C(B) = \begin{array}{c|cc} & \multicolumn{2}{c}{C} \\ \hline B & 1 & 0 \\ \hline 1 & 0.4 & 0.6 \\ 0 & 0.001 & 0.999 \end{array} \qquad P(D|A) = f_D(A) = \begin{array}{c|cc} & \multicolumn{2}{c}{D} \\ \hline A & 1 & 0 \\ \hline 1 & 0.8 & 0.2 \\ 0 & 0.4 & 0.6 \end{array}$$

Fig. 18. "Cheating spouse" example

(b) The arc labels $\pi(\ldots)$, $\lambda(\ldots)$ might be replaced by constant tuples.
(c) The names of the input boundary transitions are arbitrary.

The outcome of this algorithm for the DN $\mathcal{N}$ is "nearly" the PPN; the transitions g of $\mathcal{N}^{d*}$ have a simple inner structure which will be demonstrated in the following example. □

Example 9. (see [12]) The people in this scenario are a spouse and a strange man/lady. The random variables, the prior probability of A, the Bayesian network and the conditional probability tables are given in Fig. 18; the corresponding DN $\mathcal{N}$ is shown in Fig. 19. The ground structure of the PPN evolved from DN $\mathcal{N}$ is shown in Fig. 20. So far, the inner structure of the g-transitions is missing. Initially, the prior values are

$\pi(A) = (0.1, 0.9)$ and
$\lambda(C) = \lambda(D) = (1, 1)$ indicating that there is no information about C and D.

Figures 20, 21, 22, 23 show

$$\lambda(B) = \lambda(C) \cdot f_B(C) = \lambda(C) \cdot f_C^t(B)$$
$$\pi(B) = ((\lambda(D) \cdot f_D^t(A)) \circ \pi(A)) \cdot f_B(A)$$
$$\pi(C) = \pi(B) \cdot f_C(B)$$
$$\pi(D) = ((\lambda(B) \cdot f_B^t(A)) \circ \pi(A)) \cdot f_D(A)$$
$$\lambda(A) = (\lambda(B) \cdot f_B^t(A)) \circ (\lambda(D) \cdot f_D^t(A)).$$

Please note that the m-transitions are to calculate the components product $\circ$. The initializing phase yields

$$\pi(B) = (((1,1) \cdot [\begin{smallmatrix} 0.8 & 0.4 \\ 0.2 & 0.6 \end{smallmatrix}]) \circ (0.1, 0.9)) \cdot [\begin{smallmatrix} 0.7 & 0.3 \\ 0.2 & 0.8 \end{smallmatrix}] = (0.1, 0.9) \cdot [\begin{smallmatrix} 0.7 & 0.3 \\ 0.2 & 0.8 \end{smallmatrix}]$$
$$= (0.25, 0.75)$$
$$\lambda(B) = (1,1) \cdot [\begin{smallmatrix} 0.4 & 0.001 \\ 0.6 & 0.999 \end{smallmatrix}] = (1,1)$$

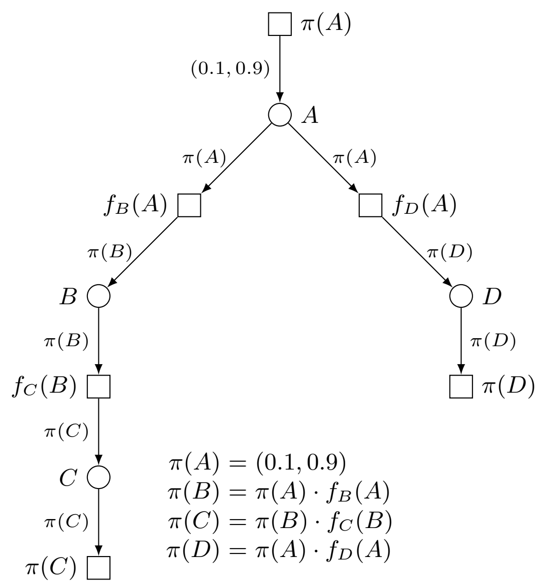


Fig. 19. DN $\mathcal{N}$

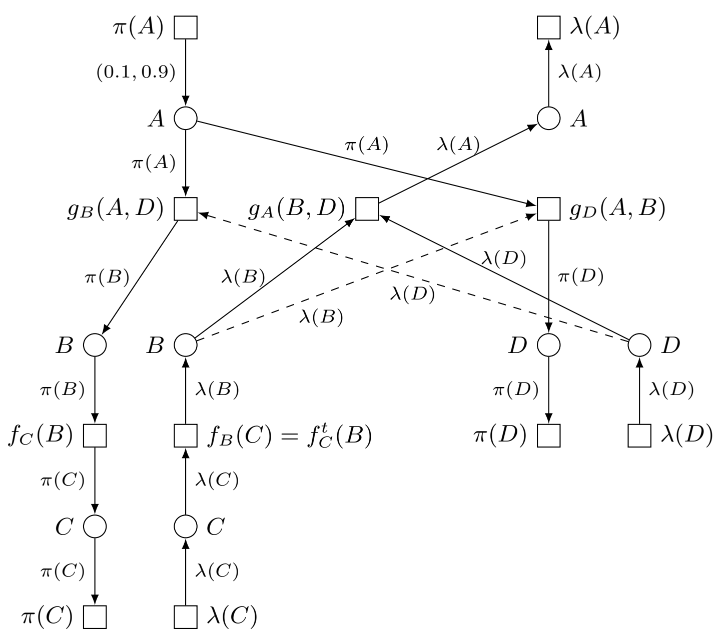


Fig. 20. Ground structure of the PPN

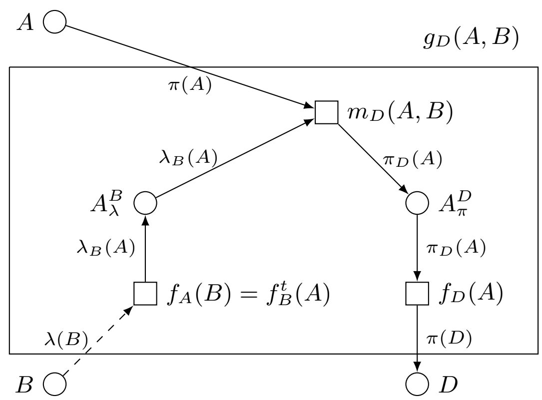


Fig. 21.

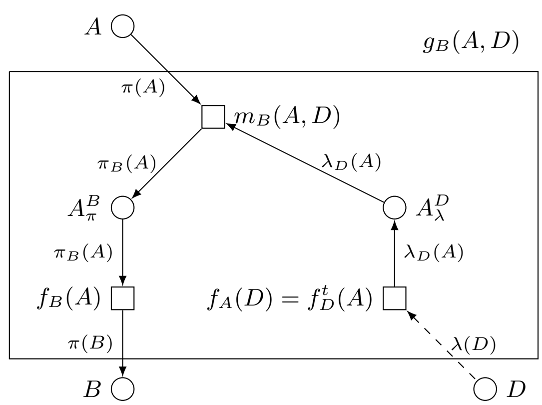


Fig. 22.

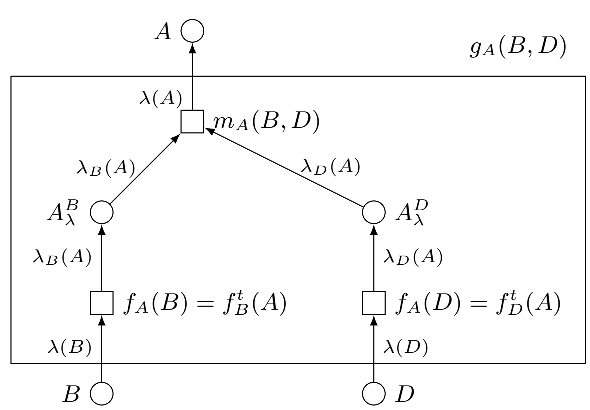


Fig. 23.

$\pi(C) = (0.25, 0.75) \cdot \left[\begin{smallmatrix} 0.4 & 0.6 \\ 0.001 & 0.999 \end{smallmatrix}\right] = (0.10075, 0.89925)$
$\pi(D) = (((1,1) \cdot \left[\begin{smallmatrix} 0.7 & 0.2 \\ 0.3 & 0.8 \end{smallmatrix}\right]) \circ (0.1, 0.9)) \cdot \left[\begin{smallmatrix} 0.8 & 0.2 \\ 0.4 & 0.6 \end{smallmatrix}\right] = (0.1, 0.9) \cdot \left[\begin{smallmatrix} 0.8 & 0.2 \\ 0.4 & 0.6 \end{smallmatrix}\right]$
$\quad = (0.44, 0.56)$
$\lambda(A) = ((1,1) \cdot \left[\begin{smallmatrix} 0.7 & 0.2 \\ 0.3 & 0.8 \end{smallmatrix}\right]) \circ ((1,1) \cdot \left[\begin{smallmatrix} 0.8 & 0.4 \\ 0.2 & 0.6 \end{smallmatrix}\right]) = (1,1)$

So, the initial beliefs are

$bel(A) = \pi(A) \circ \lambda(A) = (0.1, 0.9)$
$bel(B) = \pi(B) \circ \lambda(B) = (0.25, 0.75)$
$bel(C) = \pi(C) \circ \lambda(C) = (0.10075, 0.89925)$
$bel(D) = \pi(D) \circ \lambda(D) = (0.44, 0.56).$

All λ-values equal (1,1), thus indicating that there is no new evidence about A, B, C, D. Consequently, the beliefs are equal to the respective probabilities. From a computational point of view the formulae can be improved by saving intermediate results.

Now let be $\lambda(B) = (1, 0)$, which indicates that spouse dines with another. In detail:

$$\pi(A) = (0.1, 0.9),\ \lambda(C) = \lambda(D) = (1,1),\ \lambda(B) = (1,0)\ .$$

Now the previous value $\pi(B) = (0.25, 0.75)$ is obsolete and has to be replaced by $bel(B) = \pi(B) \circ \lambda(B) = (1, 0)$;

$\pi(B) := (1, 0)$
$\pi(C) = \pi(B) \cdot f_C(B) = (1,0) \cdot \left[\begin{smallmatrix} 0.4 & 0.6 \\ 0.001 & 0.999 \end{smallmatrix}\right] = (0.4, 0.6)$
$\pi(D) = ((\lambda(B) \cdot f_B^t(A)) \circ \pi(A)) \cdot f_D(A)$
$\quad = (((1,0) \cdot \left[\begin{smallmatrix} 0.7 & 0.2 \\ 0.3 & 0.8 \end{smallmatrix}\right]) \circ (0.1, 0.9)) \cdot \left[\begin{smallmatrix} 0.8 & 0.2 \\ 0.4 & 0.6 \end{smallmatrix}\right]$
$\quad = ((0.7, 0.2) \circ (0.1, 0.9)) \cdot \left[\begin{smallmatrix} 0.8 & 0.2 \\ 0.4 & 0.6 \end{smallmatrix}\right]$
$\quad = (0.07, 0.18) \cdot \left[\begin{smallmatrix} 0.8 & 0.2 \\ 0.4 & 0.6 \end{smallmatrix}\right]$
$\quad = \alpha \cdot (0.128, 0.122);$
this tuple is not yet normalized; for $\alpha := \frac{1}{0.128+0.122} = \frac{1}{0.25}$ we get
$\pi(D) = \frac{1}{0.25}(0.128, 0.122) = (0.512, 0.488).$

$\lambda(A) = (\lambda(B) \cdot f_B^t(A)) \circ (\lambda(D) \cdot f_D^t(A))$
$\quad = ((1,0) \cdot \left[\begin{smallmatrix} 0.7 & 0.2 \\ 0.3 & 0.8 \end{smallmatrix}\right]) \circ ((1,1) \cdot \left[\begin{smallmatrix} 0.8 & 0.4 \\ 0.2 & 0.6 \end{smallmatrix}\right])$
$\quad = (0.7, 0.2) \circ (1,1) = (0.7, 0.2)$

$bel(A) = \pi(A) \circ \lambda(A) = (0.1, 0.9) \circ (0.7, 0.2)$
$\quad = (0.07, 0.18)$
$\quad = \alpha \cdot (0.07, 0.18)$
$\quad = \frac{1}{0.25}(0.07, 0.18)$
$\quad = (0.28, 0.72)$
is the new value for $\pi(A)$; so we set $\pi(A) := (0.28, 0.72)$.

The new beliefs are

$bel(A) = (0.28, 0.72)$
$bel(B) = (1, 0)$
$bel(C) = \pi(C) \circ \lambda(C) = (0.4, 0.6) \circ (1,1) = (0.4, 0.6)$
$bel(D) = \pi(D) \circ \lambda(D) = (0.512, 0.488) \circ (1,1) = (0.512, 0.488)$

That means that after spouse was dining with another ($\lambda(B) = (1, 0)$) the belief

- that spouse is cheating (A_1) has increased from 0.1 to 0.28,
- that spouse is reported seen dining with another (C_1) has increased from 0.10075 to 0.4, and
- that strange man calls on the phone (D_1) has increased from 0.44 to 0.512.

□

5 Conclusion

In this paper we showed close relationships between probability propagation nets and the Petri net duality, whose special feature is a duality of structure and behaviour. The flows of common sense (probabilities) and evidence (likelihoods) are dual to each other. But beyond this fact, which is interesting in itself, the simple construction of the dual to some given net leads to a construction principle for probability propagation nets. Starting from dependency nets (describing probabilistic dependencies) it is possible to build probability propagation nets by joining dependency nets and their (slightly adapted) duals.

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