Catastrophe, Chaos, and Self-Organization in Social Systems

Invited Papers of a Seminar Series on Catastrophic Phenomena in Soviet Society and Self-Organized Behaviour of Social Systems
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Edited by Klaus G. Troitzsch

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Preface

From September 4 to 11, 1992, a first meeting between Ukrainian and German scientists interested in mathematical and computer modeling of social processes was held at Vorzel' near Kiev. The meeting had been planned for nearly three years by Igor V. Chernenko and Mikhail V. Kuz'min, then members of the research group on mathematical modeling in sociology at the Institute of Sociology of the Academy of Science of the Ukrainian Republic, and had to be postponed twice due to the political development in the former Soviet Union, but thanks to the organizers' perseverance (and in spite of a strike of the airport personell at Kiev Borispol Airport on the eve of the conference) the conference could at last be realized.

The main purpose of the conference was to discuss a synergetic interpretation of large-scale destructive social processes as catastrophic phenomena in self-organized systems.

Collaboration between German and Ukrainian scientists in the field of mathematical simulation of socio-economic processes has a good tradition that gave a set of original nonlinear models of creative self-organization as well as a new catastrophic approach to examine social evolution. The conditional forecasts of social cataclysms obtained in the papers presented at the conference were corroborated by the subsequent events in the former Soviet Republics.

Social catastrophe means that the self-organized driver of system growth is transmuted into a factor of system decay. Nonlinear models of such crucial metamorphoses were the most exciting subjects of discussions, because these models provide an opportunity to see into the contradictory nature of living systems and to study hidden functional links between micro and macro levels of self-reproductive systems and conditions of their transformations.

From the beginning of the conference preparations it had been clear that the "chaotic" and sometimes "catastrophic" processes in state and society of the successors to the Soviet republics should be in the center of interest of the participants. One of the crucial questions was whether mathematical analysis and computer simulation can make a useful contribution to the understanding of the processes of rapid social and economic change in the Community of Independent States, and whether reasonable use can be made of the precise mathematical concepts of "chaos" and "catastrophe" to describe these processes — since the everyday meanings of these words are so often used for such a description.

These proceedings try to answer this question, starting from four papers which are intended to give some orientation in the field of modeling complex systems: Kisil gives a somewhat pessimistic view on sociological measurement as a fundamental for sociological modeling and simulation, while Weidlich and Troitzsch present the modeling approaches they have been using for some time to show that even in systems of small complexity processes of self-organization may arise. Shatikhin finishes this first part with a paper concentrating on the macroscopic view of systems.

The second part of these proceedings presents some modeling tools and algorithms. We start with two papers on the MIMOSE modeling and simulation system which was developed by Möhring's and Troitzsch's working group at Koblenz. Borodyanski's and Burgin's papers open a new perspective on non-algorithmic procedures. The second part is finished by Polumiyenko's paper on a very complex game theoretical model of an ecosystem which serves as a means to show the possible contribution of game theory as a tool for understanding complex systems.

The third part contains a number of special models of self-organizing processes, of catastrophic and of chaotic phenomena. Flache and his coauthors discuss a model of cooperation in a group rewarded team, thus focussing the microscopic perspective. The papers by Tsvetkov, Dubrovskiy, Chernenko, Chernyshenko, Kuzmin, and Platon describe several variations of a model of evolution in a society consisting of several cooperating and competing subpopulations (a model which is also used to illustrate the facilities of the MIMOSE simulation system in Möhring's and Strotmann's paper). While this model and the variations described in these three papers is macroscopic, Troitzsch's paper investigates the catastrophic jump from a microscopic perspective. Chesnokov and Chernenko in their paper try to reconcile concepts of Eastern mystic and modern mathematics to understand self-organization processes. Niyazov gives a semi-empirical analysis of large cycles in US economy during the last two centuries and proposes a model which could make these cycles understandable.

The proceedings end in a paper presented by Erdmann and discussed for hours by the conference participants, in which he models economic aspects of institutional change in Eastern Europe, proposing some political recommendations how to improve the economical situation in the countries of Eastern Europe.

Acknowledgements

It is a pleasure to express thanks to the organizers of the conference who welcomed their German guests with an overwhelming hospitality after they had taken lots of trouble in inviting their German guests, finding financial support for the conference and accomodation for the guests, coordinating the Ukrainian speakers, replanning the conference after it had to be cancelled first for lack of money, then for the consequences of the Five Days in August. It is due to Igor V. Chernenko's and Mikhail V. Kuz'min's patience and perseverance that the conference participants eventually assembled safely at Vorzel'.

The preparation of the conference would not have been successful without Dr. Wolfgang Döke, Vice Consul for Education and Science at the German Embassy at Kiev, who often served as a mail office when ordinary mail would not have been fast and secure enough to transport letters, papers, and floppy disks between Kiev and Koblenz. In the same way the support of Gerhard Kalab, my brother-in-law who works in Moscow for an important German firm, must be appreciated who also acted as a mail and telecommunications relais station between Kiev and Koblenz. Many a message would have got lost without the patient readiness of both of them to forward them by mail, telefax, and telephone.

Our thanks go to the Association of Ukrainian Composers, whose recreation center in Vorzel' housed the conference, and especially to Mrs. Elena I. Kuts, the director of this center, and her personell, who hosted the German guests for five days.

Another whole-hearted thanks go to Galina S. Nechiporenko who prepared many of the texts of the Ukrainian authors for TEX, and to Olga Sinitsa who served the conference as a translator. She, of course, had to bear the greatest burden during the conference days since not every Ukrainian participant was fluent in English (and none of the German guests understood Russian or Ukrainian).

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Part I

Catastrophes and Self-Organization: Some Fundamentals

Chapter 1

Vladimir V. Kisil, Odessa: Some Subjective Notes about Mathematical Simulation of Social Systems

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Abstract

The purpose of this paper[1] is to give some brief notes about today usage of mathematical models in social science. By the author's opinion, up-to-date mathematical simulation in social science is based on some analogies between social and physical systems. We shall investigate the conditions of applicability of such physical analogies on two examples: from statistics and quantum physics.

1.1 Introduction

All time man constructs long walls and short bridges.

For many years scholars have been divided into two large classes whether the processes they studied were natural or social. We could not say that this division satisfied them. On the contrary, from ancient days to our very specialized century the most outstanding scientists working in one field were often interested in the other one. Moreover, in the last fifty years exchanges (of both peoples and ideas) between these two main scientific areas were as active as never before. But I must say that the boundary wall between natural and social is as firm now as in the past. Maybe, this hopeless firmness compelled R. Descartes to ignore this division completely. Otherwise I cannot explain so trivial a Cartesian opinion that any animal is only an automaton.

[1] I am grateful to Dr. Boris A. Veytsman for discussion of this paper.

On the contrary, ancient scientists found will (and other characteristics of “living” creatures) in matters so simple from modern view as water, flame and air. But as we learned to describe such matters through partial differential equations their “soul” abandoned them. In our times, the same equations are often used for describing social systems and, I believe, with the same results.

Living and unliving, social and natural coexist and interplay in the world, but separated by a “Berlin wall” in scientific theories. Why?

1.2 Mathematical Approaches to Social Study: Simulation of Simulation

> The knowledge of the fellow creature is always restricted to the acquaintance with their deficiencies.

Simulation of social processes using mathematical methods is now a common disease. There are no more branches of pure mathematics from graph theory to nonlinear equations and catastrophe theory not been used for this goal. A variety of these approaches does not witness the power of mathematical methods. I think, it shows that we have no tools corresponding to tasks of social investigation.

Practically all up-to-date mathematical methods were developed to describe the world of physical things, i. e. natural processes. So under the attractive label “Mathematical Simulation” one usually finds a group of people treated either as molecules in a vessel or as a viscous liquid under pressure. Some authors, for example Peter Gould , deduced mathematical methods are completely unsuitable for description of human world:

> If we take the mathematical structures devised to describe the worlds of celestial mechanics, statistical mechanics, quantum mechanics, continuum mechanics . . . and just plain mechanics, and then map the human world unthinkingly onto such structures, is it possible that the human world so described can look anything but mechanical? In brief, does the mathematical “language” chosen allow the description, and allow our thinking, to appear as anything but mechanistic?

This argument has a weak spot: mathematics is not some calculation rules, but rather a self-growing tool for reflecting the outer world. The physical origin of up-to-date mathematics is obvious to everyone. It is obvious too, that the mechanical and deterministic character of mathematical theories is dominant now, but losing its leading role. Neurocomputers are coming instead of Turing’s machine and fuzzy set theory is coming instead of Boolean logic. I am sure that in the future mathematical language will be dominant in the description of social processes. And I am sure that it will be a completely different mathematics no more similar to the modern one than the latter is similar to the Babylonian.

But what should we do with the present-day mathematical models of social processes, where Navier-Stokes partial differential equations are used for describing people's opinion during a pre-election campaign? I think, it is very natural for people (and for monkeys, too) to use the nearby tool. You may like it or not, but in the near future we shall not have any other methods in mathematical simulation and, consequently, any other results.

Anyway, it is very tempting to use pick-and-ready models of physical processes for the dawning social simulation. The author does not consider such borrowing blameworthy provided that the conditions of applicability of the model used are clearly defined. These conditions, as a rule, are limited to qualitative analogies between social and physical systems. It is almost hopeless to expand them further, e.g. up to quantitative results, at least at present. So, we do not have mathematical simulation of social processes now, but only simulation of this simulation.

We shall discuss below two such models, which, seem to be adequate enough. Using them as examples we shall investigate conditions of applicability of physical analogies. Then we shall discuss the status of such models in the framework of the general picture of modeling.

1.3 Mathematical Approaches to Social Study: We All Live in a Vessel under Pressure, between Us, Molecules, Speaking

It is really nice, that there is equal pressure anywhere in our world!

Yes, if you do not count that high strata press on low ones.

Talks between molecules.

The first analogy to be discussed is the analogy between social and molecular groups. 150 years ago a problem of describing the behavior of gas in a vessel appeared to be more complex than a problem of describing the society. Society was seen to be completely guided by laws and decrees, and its constituting individuals were seen to be led by virtue and justice. The apparent imperfectness of social life could be explained by some trifle, e. g. by the lack of people's enlightenment or by private property on means of production. In any case it was doubtless that the better and just society lay in the near future and could be achieved either by royal decree or by decision of Street Committee.

On the contrary, the laws governing the behavior of gas were completely unclear in the 19th century. It was understood that the motion of a single molecule is purely deterministic and governed only by Newtonian laws, but there was no understanding of the ways to study the behavior of 10^{24} molecules. Nevertheless Boltzmann solved this problem, which served as the beginning of a new branch of statistical physics. The results appear to be amazing: to describe macroscopic behavior of gas it is not necessary to know all the parameters of the constituent molecules. It is sufficient to know the laws of molecular motion rather than the values of coordinates and velocities of molecules. The averaged quantities such as mean quadratic velocity could be calculated even not knowing any velocity, but using the macroscopic temperature.

Be it coincidence or not, social sciences of that time were intrinsically statistical, i. e. their subject was the averaged person, Mr. Jack Statistics, rather than individual people.

To further investigate this analogy between social group and molecular statistical ensemble and to explore its limitation let us discuss several basic assumptions, which are implicit in this way of thinking:

1. From the sociology viewpoint people are as similar as the molecules of same kind. This assumption is so important for statistical sociology, that to provide its validity various kinds of technique are used. E.g. if in the given group deviations from average values are too great, the group is divided into clusters using such sociological parameters as “profession”, “age”, “education level”, etc. The individuals inside such groups are more homogeneous, especially with respect to the indicators correlated to the parameters of clusterization. This technique is sufficient for a number of problems and the simpler problems are discussed the more adequate is it. It is easy to forestall results of a poll among Russian workers with the question “Which Pharaoh do you back up: Ramses XII or Tutankhamon?” I think that it is fully compatible with the scientific reasoning to study not the individual respondents but rather an averaged individuum responding “Do not know”.

   But the more attributes essential and relevant to the reference group are investigated, the more important are individual deviations from averaged values. The general polarization is still more important. To understand events in France in the 18th and 19th centuries, in Germany in 1930–1940, or in Russia in 1905–1919, one should reject the averaged picture of peoples.

   Nevertheless the assumption of social equivalence is a pragmatic rather than a methodological one.

   Up to the moment of developing statistical mechanics, the theory of individual motion based on Newtonian laws was well understood. The situation in sociology is quite different. At present and in the near future, individual psychology cannot provide an adequate mathematical model of the individuum suited to be a base for socio-mathematical simulation. Therefore, for sociology the individuum is a black box and will remain so in the near future. The assumption of the equivalence of black boxes seems to be sufficiently sound.

2. The collective variables of a social group are taken to be sums of the respective individual variables. This is analogous to the temperature of gas, which can be obtained as sum of the molecules’ energies. The sum here is an arithmetical one or a little bit more complex. E.g., if at the sociological poll or referendum the number of votes “aye” is known, the result is taken to be “vox populi” from the viewpoints of both law and science.

The problem of measurement appears to be well understood. The common opinion is that sociological information usually can be analyzed quantitatively. The main problem in these cases is whether it is possible to use metric scales or one should use order scales only. A metric scale allows one to forget completely about the complexity of the object under investigation and to apply their arithmetical abilities. Results of such studies are usually formulated as statements like "Social tension grows by 57 %", "The government popularity rating dropped 1,37 times after introducing a new tax", etc. To enable one to use such material it is important to perform the first step: to declare uncomparable objects equivalent. E.g. one adds up all "Nay" answers to a question "Do you support the government policy", without further inquiries, whether this "Nay" is the opinion of a housewife glued to her TV set, a student on a demonstration, or a general planning a coup d'etat.

If metric scales cannot be used even after such assumptions, one uses order scales. In such works there are fewer figures and formulas, but the conclusions are straightforward, too: "Comparing to last year, the degree of popularity of Government grows."

Nevertheless the question still remains, just how adequate is the application of arithmetic to such a fine matter as "Public Conscience"? It should be taken in mind that mankind and all of us in our green years learned arithmetic while calculating apples, stones, etc. All such things are identical for the purpose of calculation. If this identity is not taken a priori, it is inevitably introduced in the calculation process. It is noteworthy that one of the most essential elements of army service — formal drill — begins with the arithmetical command "Count!". Therefore the scaling itself is based on the assumption of equivalence of people discussed in the previous section.

Compared to metric scales, order scales seem to be more flexible, and very near a generalization of the concept of number. It seems to be the last trench of arithmetic. But this is not the case. The relationships "more" and "less" have sense only in one-dimensional systems, such as real numbers. Complex numbers are "arithmetical" objects but as two-dimensional ones they have no order relationships. They can be "more" or "less" with respect to modulus, argument or any other one-dimensional parameter. This parameter can be introduced in any convenient way. In the social sciences the choice of method of introducing one-dimensional space is often caused by aims different from the objective study.

Thus the foundation of mathematical simulation in social sciences (introducing of reference points and scales) is based on two fragile hypotheses. The first is the hypothesis of the equivalence of people constituting a social group. The second is the hypothesis of the possibility of contraction of a space (of indefinite dimension) into a set of one-dimensional scales. But it can be shown that the problem of measurement is even more profound.

1.4 Mathematical Approaches to Social Study: Getting Uncertainty

Do you know where you are going?
Yes, of course!
That is a pity, because then you do not know where you are located!

Talks between electrons.

Measurement is not only the first step in mathematical simulation. In the process of measurement the object under investigation is mapped into the set of the given abstractions. These abstractions are numbers in the case of metric scales or the elements of an ordered structure for order scales. Thus even on this step the analogy between objects under investigation is postulated. In other words this step is also a simulation. The investigation is performed in the following manner:

1. "measurement" — it is postulated that the model parameters can be simulated by numbers or elements of an ordered set;

2. "simulation" — the results of the previous step are linked by methods which give the investigated objects the properties obtained from the manipulation with numbers.

So even at the first step we faced with all the problems of simulation. I believe that one of the most essential problems is the following. When we state an analogy between the investigated object and its model, we seek to understand properties of the object via known properties of the model. The negative consequences of the analogy are that not only a given number of investigated properties is ascribed to the object, but also a number of other properties, for which the analogy is undesirable. These undesirable properties are not listed explicitly, to further worsening of the situation.

Speaking of measurement means that to the sociological objects there are ascribed not only such properties as "arithmeticity" and "order", but also a property of existing independently in our mind.

Let us discuss this property in detail. The paradigm existed in physics before the 20th century included the assumption that such physical quantities as velocity, pressure or electrical field exist objectively, independently on experimentation and their devices. The physical quantities were real numbers, and every decimal figure existed objectively and could be measured provided sufficient time and money. The quantum mechanical crisis in physics changed these assumptions. Maybe all gnoseological consequences of the fact that momentum and coordinate cannot be measured simultaneously are to be understood yet. The problem is not that they are represented by noncommutative operators. The objective reality does not concern either commutative operators or real numbers. The real world does not have to be contained in the smallness of our minds. Apparently nobody will argue with this notion, but it is very difficult psychologically to understand that it is valid also with respect to numbers. The farewell to Euclidean geometry was very difficult to physics. But our farewell to numbers will be even more difficult than to Euclid. The ideal line or triangle we have never seen, but the ideal (as we think) number 2 we can see in any pair of apples or boots.

In natural sciences the existence of objective laws independent on the scientist's mind is not denied. It would be senseless to perform experiments if the phenomena investigated were governed not by their own logic but rather by publications in scientific journals. But even in physics the concept of objective laws does not deny the problems of measurement. There are thousands of papers where the authors do not assume that reality cannot be comprehensively exhausted by such fiction as "real" number.

In the social sciences the situation is even more complex. It is unlikely to divide objective and subjective matters in the society consisting of interacting subjects. A "scientific" theory born in a scientist's mind can change lives of hundreds of millions people and finally become an "objective" factor of social reality. In any case there are many examples in history when some scientific theory used to change the reality (but always not for long time). And there are no examples of scientific theory that explained the world as it is, in a way that Newton mechanics explained celestial motion. If the proof of a scientific theory is the social practice then the most "scientific" results are obtained by those "scientists" who have the power to change that practice.

The principle of measurability in the social sciences has some additional sources of existence, purely pragmatic ones. It is very important for any political force to be able to pretend on expressing "true" public opinion. So there should be no doubt about the existence of public opinion and the possibility of its measurement and its comparison with other interests. All politics is based on the simple algorithm: a political force first forms public interests and then defends them as an objective reality. It is analogous to the advertising practice of developed countries: first the mass-media forms a public need, and then a big branch of industry works to satisfy it, not necessarily for the public benefit.

The second possible analogy between sociology and physics is connected with quantum mechanics. We'll discuss the sociological "uncertainty principle" only. Philosophers have written many works speculating on this principle in physics. This principle states essentially the following: there are pairs of physical quantities (such as coordinate and momentum) which cannot be measured simultaneously with an accuracy exceeding some limit. The formal explanation of than principle is that the corresponding operators do not commute. But such explanation is not much useful for the gnosiology. For a gnosiologist the following extreme explanations are possible.

1. These quantities do not exist objectively, they belong to our model of reality, not to reality itself. So it is not surprising that they cannot be measured simultaneously. It is surprising that these fictions can be measured separately.

2. The uncertainty is the result of the influence of the experimentator on the object. In this framework it is difficult not to assume that the electron has an independent will and interacts with the experimentators as an equal. The most essential part of any textbook on quantum mechanics is the attempt to keep the electron inanimate. This is done at the cost of abandoning the common concept of trajectory, velocity, coordinate.

While discussing the electron's diffraction on two apertures we cannot assume that the electron can "know" how many apertures are open and "decide" how to move. It is more simple to us to abandon our concept and to assume that the electron moves through both apertures simultaneously.

In sociological polls there also exists an "uncertainty principle". By proposing questions, their wording and order the researcher can influence if not determine the respondent's answers. And the more questions are proposed and the more connections are studied, the more possibilities are open to influence on the respondent. Of course, it is possible to develop methods reducing this interference to the possible minimum, but sterility of experiment is not aim of the sociology. The fact that the respondent has free will is doubtless and interaction of this will with other wills (such as that of the researcher or of a political organization) is probably the everlasting condition of sociological research. In any case, there is a quantum mechanical analogy to the sociological object.

The main point of this paper is the unacceptability of the common mathematical physical models for sociology. An adequate description of the socium requires other methods and will give birth to a new mathematics. The earlier the need for this mathematics will be recognized, the earlier we shall have a new science as powerful as beautiful.

Chapter 2

Wolfgang Weidlich: Modelling Concepts of Synergetics with Application to Transitions between Totalitarian and Liberal Political Ideologies

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2.1 Introduction

Synergetics is a new branch of science dealing with the universal laws of the dynamic macro-structures which are generated in multi-component systems through the interactions between their elements.

The human society is such a multi-component system with a manifold of material and mental interactions between its elements, the individuals. Therefore, synergetics should also be applicable to the society, that means to the modelling of social processes!

In the following such a modelling framework is presented. It consists in the combination of concepts taking into account the special nature of social systems with concepts taken from statistical physics and synergetics, which are universally applicable to stochastic multi-component systems (A more comprehensive presentation of the general modelling concepts is given in [WH83] and [Wei91]. Applications to migration processes of human populations including empirical evaluations are presented in [WH88].)

The purpose of this modelling procedure is the development of an integrated concept of theory construction for the quantitative description of collective evolutions in the society. The following partial purposes are included:

1. The formulation of the interrelation between the microlevel of individual decisions and the macrolevel of dynamical collective processes in the society.

2. The derivation of a probabilistic description of the macro-process including stochastic fluctuations, and the derivation of a quasi-deterministic description, in which the fluctuations are neglected.

3. The investigation of model solutions by analytical methods, e.g. the exact or approximate solution of master equations or meanvalue equations, or by numerical simulation of characteristic scenarios.

4. The evaluation of empirical systems, including field inquiries on the microlevel, as well as regression analysis on the macrolevel for the determination of model parameters, and forecasting of future evolutions by model simulation.

The potential domains of application belong to different sectors of social science, namely to

- *Sociology* (for instance socio-political opinion formation, the example treated below in this article)
- *Demography* (for instance migration of populations)
- *Regional Science* (for instance formation of settlements and urban dynamics)
- *Economics* (for instance nonlinear models for business cycles and market instabilities)

We finish this introduction with a scheme of the general conceptual framework for the quantitative modelling of socio-dynamics in synergetics. The blocks constituting this scheme are explained in more detail in the next two sections.

2.2 Characterization of the Social System

2.2.1 The state of the Society

a) Microvariables: The Social Role of the Individual

The individual possesses an *"attitude vector"*, that means a multiple

$$\mathbf{i} = (i_1, i_2, \ldots, i_a, \ldots, i_A) \tag{2.1}$$

of publicly exhibited *"external" attitudes* (e.g. opinions, activities, behavioral modes) with respect to A different aspects $a = 1, 2, \ldots, A$. These attitudes are conditioned by internal inclinations and by external social constraints.

Furthermore, the individual may possess an *internal propensity* with respect to its own publicly exhibited attitude i. This propensity is described by the *trendparameter* ϑ_i assuming integer values only for simplicity. The "internal" propensity may be in agreement with or in opposition to the "external" attitude i.

$$\begin{array}{rcl} \text{Values } \vartheta_i > 0 \text{ describe} & \textit{internal affirmation} & \text{of attitude } i \\ \text{The value } \vartheta_i = 0 \text{ describes} & \textit{internal neutrality} & \text{to attitude } i \\ \text{Values } \vartheta_i < 0 \text{ describes} & \textit{internal opposition} & \text{to attitude } i \end{array} \tag{2.2}$$

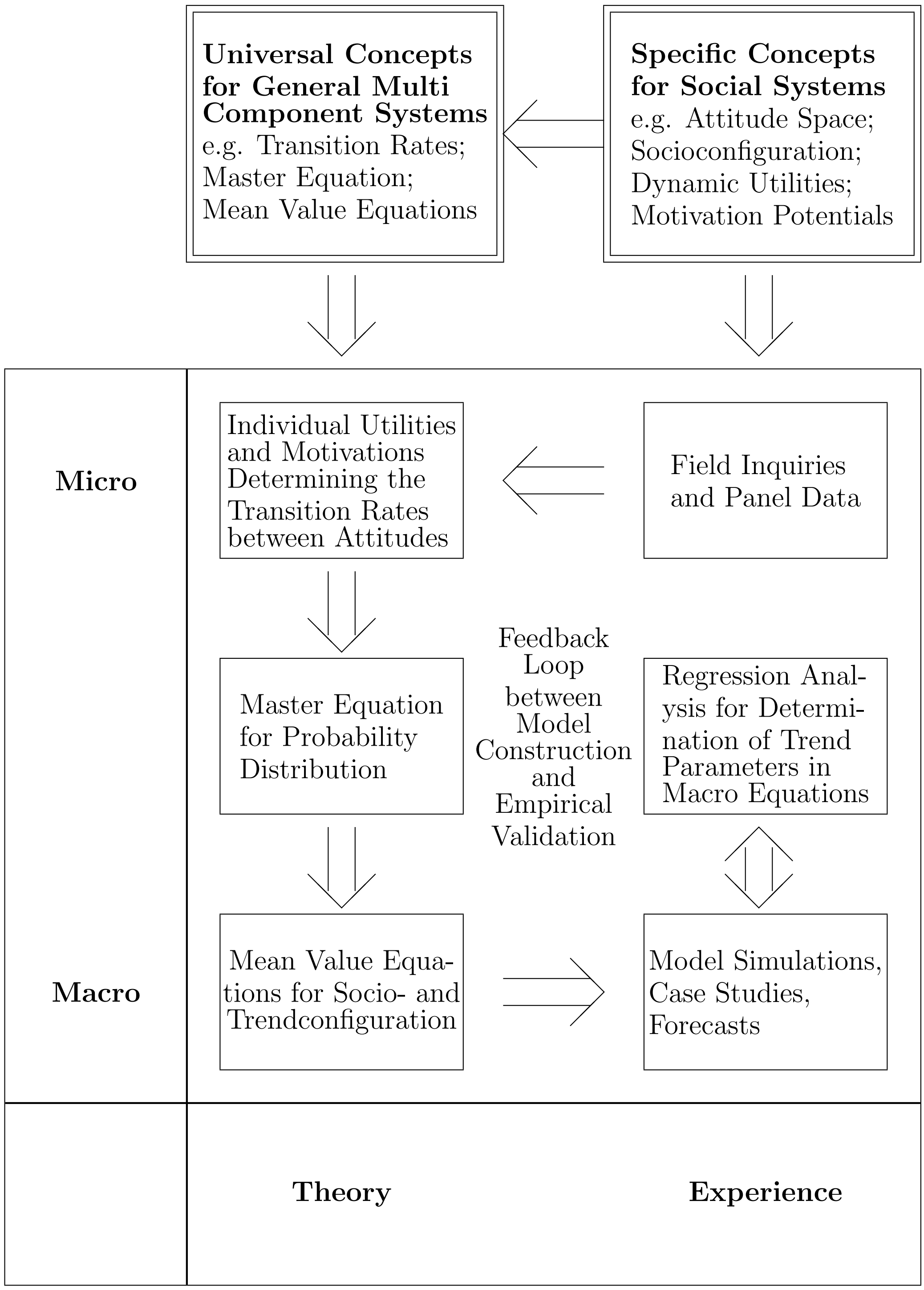

Figure 2.1: The general conceptual framework for the quantitative modelling of socio-dynamics

b) The Macrovariables of the Society

There exist *subpopulations* $\mathcal{P}_\alpha, \alpha = 1, 2, \ldots, P$, each consisting of individuals of the same social background (e.g. social “classes”). A *coarse-grained* model uses one or few subpopulations $\mathcal{P}_\alpha$ only, but a *fine-grained* model uses many differentiated subpopulations $\mathcal{P}_\alpha$.

A central macrovariable is the *socioconfiguration*. It describes the distribution of attitudes (opinions, activities, behaviours) among the subpopulations $\mathcal{P}_\alpha$ of the society and thus characterizes the macrostate of the society. The *socioconfiguration* consists of a multiple of integers

$$\mathbf{n} = \{n_1^1, \ldots, n_i^\alpha, \ldots, n_C^P\} \tag{2.3}$$

where n_i^α is the number of individuals of subpopulation $\mathcal{P}_\alpha$ having the attitude i.

If *trendparameters* ϑ_i^α are assigned to the members of $\mathcal{P}_\alpha$ who have the attitude i, then also the *trendconfiguration*

$$\boldsymbol{\vartheta} = \{\vartheta_1^1, \ldots, \vartheta_i^\alpha, \ldots, \vartheta_C^P\} \tag{2.4}$$

can be introduced as a further set of macrovariables in addition to the socioconfiguration. It is assumed that the integer value ϑ_i^α varies between a minimal amplitude $-\Theta$ and a maximal amplitude $+\Theta$.

2.2.2 Elements of Socio-Dynamics

a) Utilities and Motivation Potentials

The *utility* u_i^α is defined as a measure of the usefulness of the adoption of the external attitude i (namely the publicly exhibited opinion, activity, behaviour) for a member of subpopulation $\mathcal{P}_\alpha$. The utility u_i^α is a real number: $-\infty < u_i^\alpha < +\infty$. It may be a function of the socioconfiguration $\mathbf{n}$ and the trendconfiguration $\boldsymbol{\vartheta}$ as follows

$$\begin{aligned} u_i^\alpha(\mathbf{n}, \boldsymbol{\vartheta}) &= g_i^\alpha(\mathbf{n}) + h_i^\alpha(\mathbf{n})\vartheta_i^\alpha \\ &\text{with} \;\; h_i^\alpha(\mathbf{n}) \geq 0 \end{aligned} \tag{2.5}$$

Formula (2.5) indicates, that the utility u_i^α of attitude i will in general depend on the collective *external* social situation by the term $g_i^\alpha(\mathbf{n})$. On the other hand this external influence will be modified by the term $h_i^\alpha(\mathbf{n})\vartheta_i^\alpha$ in (2.5) containing the *internal* trend ϑ_i^α of a member of population $\mathcal{P}_\alpha$ with attitude i: An affirmative trend $\vartheta_i^\alpha > 0$ will enhance the utility u_i^α, whereas an opposing trend $\vartheta_i^\alpha < 0$ will diminish the utility u_i^α.

The *motivation potential* v_i^α is defined as a measure of the internal psychological satisfaction of a member of $\mathcal{P}_\alpha$ being in attitude i, with his own internal trend ϑ_i^α. Putting

$$v_i^\alpha = b_i^\alpha(\mathbf{n})\vartheta_i^\alpha, \text{ where } b_i^\alpha(\mathbf{n}) \gtrless 0 \tag{2.6}$$

the satisfaction is increasing for growing trend ϑ_i^α, if $b_i^\alpha(\mathbf{n}) > 0$, but decreasing for growing ϑ_i^α, if $b_i^\alpha(\mathbf{n}) < 0$. Hence, positive b_i^α describe a high psychological satisfaction with positive affirmative values of ϑ_i^α, whereas negative b_i^α describe a high internal satisfaction with negative, dissident values of ϑ_i^α.

b) Utility- and Motivation-Guided Individual and Configurational Probability Transition Rates

Our application of the utility concept differs from that utilized in conventional economics in two respects: In classical economics the utility concept is applied in *static* situations to the *deterministic* behaviour of economic individuals. Instead, we make use of utilities and motivation potentials in order to describe the *dynamic* evolution of situations by modelling the *probabilistic* behaviour of homogenous ensembles (subpopulations) of individuals.

The following key-concepts are introduced:

a) The changes of publicly exhibited attitudes $i \Rightarrow j$ and of internal trends $\vartheta_i^\alpha \Rightarrow \vartheta_i^\alpha \pm 1$ of individuals of population $\mathcal{P}_\alpha$ are the *elementary dynamic processes* on the *microlevel* of the society. They induce corresponding changes

$$\begin{aligned} \mathbf{n} &= \{n_1^1 \dots n_j^\alpha, \dots, n_i^\alpha \dots n_C^P\} \\ \Rightarrow \mathbf{n}_{(ji)}^\alpha &= \{n_1^1 \dots (n_j^\alpha + 1), \dots, (n_i^\alpha - 1), \dots n_C^P\} \end{aligned} \tag{2.7}$$

and

$$\begin{aligned} \boldsymbol{\vartheta} &= \{\vartheta_1^1 \dots \vartheta_i^\alpha, \dots, \vartheta_C^P\} \\ \Rightarrow \boldsymbol{\vartheta}_\pm^\alpha &= \{\vartheta_1^1, \dots, (\vartheta_i^\alpha \pm 1), \dots, \vartheta_C^P\} \end{aligned} \tag{2.8}$$

of the socioconfiguration and trendconfiguration, that means of the macrovariables.

b) It is assumed, that the difference between the utility or motivation potential of an origin state and a destination state plays the role of a "driving force" for a transition of an individual between these states.

Therefore, the functional form of the individual and configurational *probability transition rates from origin to destination state* is constructed in terms of these driving forces. The following exponential ansatz proves convenient and plausible as well, because it guarantees the positive-definiteness and simultaneously the factorization into a push and a pull term of the transition rates. Hence we put

Individual rate from attitude i to attitude j for a member of $\mathcal{P}_\alpha$:

$$p_{ji}^\alpha(\mathbf{n}, \boldsymbol{\vartheta}) = \nu \exp[u_j^\alpha(\mathbf{n}_{(ji)}^\alpha, \boldsymbol{\vartheta}) - u_i^\alpha(\mathbf{n}, \boldsymbol{\vartheta})] \tag{2.9}$$

Configurational rate for the transition (2.7):

$$w_{ji}^\alpha(\mathbf{n}, \boldsymbol{\vartheta}) = p_{ji}^\alpha(\mathbf{n}, \boldsymbol{\vartheta}) n_i^\alpha \tag{2.10}$$

Rate for the transition $\vartheta_i^\alpha \Rightarrow (\vartheta_i^\alpha + 1)$ for members of $\mathcal{P}_\alpha$:

$$\begin{aligned} r_{i\uparrow}^\alpha(\mathbf{n}, \boldsymbol{\vartheta}) &= \mu(\Theta - \vartheta_i^\alpha) \exp[v_i^\alpha(\mathbf{n}, \vartheta_i^\alpha + 1) - v_i^\alpha(\mathbf{n}, \vartheta_i^\alpha)] \\ &\equiv \mu(\Theta - \vartheta_i^\alpha) \exp[b_i^\alpha(\mathbf{n})] \end{aligned} \tag{2.11}$$

Rate for the transition $\vartheta_i^\alpha \Rightarrow (\vartheta_i^\alpha - 1)$ for members of $\mathcal{P}_\alpha$:

$$\begin{aligned} r_{i\downarrow}^\alpha(\mathbf{n}, \boldsymbol{\vartheta}) &= \mu(\Theta + \vartheta_i^\alpha) \exp[v_i^\alpha(\mathbf{n}, \vartheta_i^\alpha - 1) - v_i^\alpha(\mathbf{n}, \vartheta_i^\alpha)] \\ &\equiv \mu(\Theta + \vartheta_i^\alpha) \exp[-b_i^\alpha(\mathbf{n})] \end{aligned} \tag{2.12}$$

2.3 Equations of Motion for the Socio- Dynamics

2.3.1 The Stochastic Level of Description

We consider the probability $P(\mathbf{n}, \boldsymbol{\vartheta}; t)$ to find at time t the socioconfiguration $\mathbf{n}$ and the trendconfiguration $\boldsymbol{\vartheta}$ in the society. This probability distribution obeys the following fundamental *master equation*;

$$\begin{aligned}\frac{dP(\mathbf{n},\boldsymbol{\vartheta};t)}{dt} &= \left[\sum_{j,i,\alpha} w_{ji}^{\alpha}(\mathbf{n}_{ij}^{\alpha},\boldsymbol{\vartheta})P(\mathbf{n}_{(ij)}^{\alpha},\boldsymbol{\vartheta};t) - \sum_{j,i,\alpha} w_{ji}^{\alpha}(\mathbf{n},\boldsymbol{\vartheta})P(\mathbf{n},\boldsymbol{\vartheta};t)\right] \\ &+ \left[\sum_{i,\alpha} r_{i\uparrow}^{\alpha}(\mathbf{n},\boldsymbol{\vartheta}_{i-}^{\alpha})P(\mathbf{n},\boldsymbol{\vartheta}_{i-}^{\alpha};t) - \sum_{i,\alpha} r_{i\uparrow}^{\alpha}(\mathbf{n},\boldsymbol{\vartheta})P(\mathbf{n},\boldsymbol{\vartheta};t)\right] \\ &+ \left[\sum_{i,\alpha} r_{i\downarrow}^{\alpha}(\mathbf{n},\boldsymbol{\vartheta}_{i+}^{\alpha})P(\mathbf{n},\boldsymbol{\vartheta}_{i+}^{\alpha};t) - \sum_{i,\alpha} r_{i\downarrow}^{\alpha}(\mathbf{n},\boldsymbol{\vartheta})P(\mathbf{n},\boldsymbol{\vartheta};t)\right]\end{aligned} \tag{2.13}$$

The solution $P(\mathbf{n}, \boldsymbol{\vartheta}; t)$ of eq. (2.13) describes not only the mean trajectory of a society in the configuration space, but also the probability of stochastic deviations from this mean behaviour.

The right hand side of the probability evolution equation (2.13) consists of three terms. The first term describes the change of the probability of the configuration $(\mathbf{n}, \boldsymbol{\vartheta})$ by probability inflows from neighbouring states $(\mathbf{n}_{(ij)}^{\alpha}, \boldsymbol{\vartheta})$ and probability outflows from state $(\mathbf{n}, \boldsymbol{\vartheta})$. Similarly, the second and third term describe changes of the probability of $(\mathbf{n}, \boldsymbol{\vartheta})$ by enhancement or diminution processes of the internal trend, respectively.

2.3.2 The Quasi-Deterministic Level of Description

A description of the macro-evolution of the society in terms of expectation values (meanvalue) is indicated, if one is only interested in the mean behaviour and not in probabilistic fluctuations.

The expectation values of the components of the socio- and trend-configuration are defined as follows

$$n_k^{\beta}(t) = \sum_{\mathbf{n},\boldsymbol{\vartheta}} n_k^{\beta} P(\mathbf{n},\boldsymbol{\vartheta};t) \tag{2.14}$$

$$\vartheta_k^{\beta}(t) = \sum_{\mathbf{n},\boldsymbol{\vartheta}} \vartheta_k^{\beta} P(\mathbf{n},\boldsymbol{\vartheta};t) \tag{2.15}$$

In the case of unimodal (or appropriately truncated) probability distributions the following approximate equations of motion can be derived for $n_k^{\beta}(t)$ and $\vartheta_k^{\beta}(t)$, making use of the master equation (2.13):

$$\frac{dn_k^{\beta}(t)}{dt} = \sum_i w_{ki}^{\beta}(\mathbf{n}(t),\boldsymbol{\vartheta}(t)) - \sum_j w_{jk}^{\beta}(\mathbf{n}(t),\boldsymbol{\vartheta}(t)) \tag{2.16}$$

$$\frac{d\vartheta_k^\beta(t)}{dt} = r_{k\uparrow}^\beta(\mathbf{n}(t), \boldsymbol{\vartheta}(t)) - r_{k\downarrow}^\beta(\mathbf{n}(t), \boldsymbol{\vartheta}(t)) \tag{2.17}$$

The equations (2.16) and (2.17) form a set of coupled selfcontained, autonomous, in general nonlinear differential equations for $n_k^\beta(t)$ and $\vartheta_k^\beta(t)$, with $k = 1, 2, \ldots, C$ and $\beta = 1, 2, \ldots, P$. The standard methods of nonlinear analysis can be applied to this dynamical system.

2.4 Example: A Dynamical Model of Collective Political Opinion Formation

2.4.1 The Components of the Model

We consider the simplest version of such a model by assuming only two competing political opinions $i = +$ and $-$ (that means two parties or two ideologies) and only one homogenous population (this means, that the index $\alpha = 1$ can be skipped). The *socioconfiguration* now consists of

$$\mathbf{n} = \{n_+; n_-\}; \tag{2.18}$$

with the total population number

$$2N = n_+ + n_- \tag{2.19}$$

and

$$n_+ = N + n; \quad n_- = N - n \tag{2.20}$$

after introducing the *majority variable*

$$n = \frac{1}{2}(n_+ - n_-); \quad -N \leq n \leq +N \tag{2.21}$$

The *trendconfiguration* is given by

$$\boldsymbol{\vartheta} = \{\vartheta_+, \vartheta_-\}; \quad -\Theta \leq \vartheta_\pm \leq +\Theta \tag{2.22}$$

The form of the *utility functions* is a special case of (2.5):

$$\begin{aligned} u_+(n_+, \vartheta_+) &= g(n_+) + h(n_+)\vartheta_+ \\ u_-(n_-, \vartheta_-) &= g(n_-) + h(n_-)\vartheta_- \end{aligned} \tag{2.23}$$

with the simplest nontrivial ansatz for g and h:

$$g(n_\pm) = \frac{1}{2}\kappa n_\pm; \quad h(n_\pm) = \frac{1}{2}\gamma \tag{2.24}$$

Similarly the *motivation potentials* are given by (see (2.6)):

$$\begin{aligned} v_+(n_+, \vartheta_+) &= b_+(n_+)\vartheta_+ \\ v_-(n_-, \vartheta_-) &= b_-(n_-)\vartheta_- \end{aligned} \tag{2.25}$$

with the simplest nontrivial form of $b_\pm(n_\pm)$:

$$b_\pm(n_\pm) = \beta(n_\pm - N) = \pm\beta n \tag{2.26}$$

The meaning of (2.23) ... (2.26) becomes evident, if the *transition rates* are constructed according to the general rules (2.9) ... (2.12), with the result:

$$\begin{aligned} \text{a) } w_{+-}(n,\vartheta) &= \nu(N-n)\exp[\kappa n + \gamma\vartheta] \\ \text{b) } w_{-+}(n,\vartheta) &= \nu(N+n)\exp[-(\kappa n + \gamma\vartheta)] \\ \text{c) } r_\uparrow(n,\vartheta) &= \mu(\Theta-\vartheta)\exp[\beta n] \\ \text{d) } r_\downarrow(n,\vartheta) &= \mu(\Theta+\vartheta)\exp[-\beta n] \end{aligned} \tag{2.27}$$

It is plausible here to put

$$\vartheta_+ = -\vartheta_- \equiv \vartheta \tag{2.28}$$

and to characterize the socio- and trendconfiguration (2.18) and (2.22) by the two variables $\{n,\vartheta\}$ only. The transition rates (2.27) then induce the following next neighbour transitions:

$$\begin{aligned} \text{a) } \{n\vartheta\} &\Rightarrow \{n+1,\vartheta\} \\ \text{b) } \{n\vartheta\} &\Rightarrow \{n-1,\vartheta\} \\ \text{c) } \{n\vartheta\} &\Rightarrow \{n,\vartheta+1\} \\ \text{d) } \{n\vartheta\} &\Rightarrow \{n,\vartheta-1\} \end{aligned} \tag{2.29}$$

The name and the interpretation of the paramters $\nu, \mu, \Theta, \kappa, \gamma, \beta$ follows as a consequence of the form of the transition rates in terms of the macrovariables $\{n,\vartheta\}$ of the society with respect to its external opinion (n) and internal trend (ϑ) state.

$$\begin{aligned} \nu &= \text{opinion evolution speed parameter} \\ \mu &= \text{trend evolution speed paramter} \\ \Theta &= \text{maximal trend amplitude} \\ \kappa &= \text{opinion pressure parameter} \\ \gamma &= \text{trend influence parameter} \\ \beta > 0\,(\text{or } \beta < 0) &= \text{propensity parameter} \end{aligned}$$

for developing an affirmative (or a dissident) internal trend in relation to the external majority opinion.

2.4.2 The Equations of Motion of the Model

The master equation for the probability $P(n,\vartheta;t)$ to find the configuration $\{n,\vartheta\}$ at time t is a special case of eq. (2.13). It reads:

$$\frac{dP(n,\vartheta;t)}{dt} =$$

$$\begin{aligned}
[w_{+-}(n-1,\vartheta)P(n-1,\vartheta;t) &+ w_{-+}(n+1,\vartheta)P(n+1,\vartheta;t) \\
-w_{+-}(n,\vartheta)P(n,\vartheta;t) &- w_{-+}(n,\vartheta)P(n,\vartheta;t)] \\
+[r_{\uparrow}(n,\vartheta-1)P(n,\vartheta-1;t) &- r_{\uparrow}(n,\vartheta)P(n,\vartheta;t)] \\
+[r_{\downarrow}(n,\vartheta+1)P(n,\vartheta+1;t) &- r_{\downarrow}(n,\vartheta)P(n,\vartheta;t)] \qquad (2.30)
\end{aligned}$$

Of course, the explicit form (2.27) of the transition rates must be inserted.

The equations of motion for the expectation values $n(t)$ and $\vartheta(t)$ follow from the general form (2.16), (2.17) and read

$$\begin{aligned}
\frac{dn}{dt} &= w_{+-}(n,\vartheta) - w_{-+}(n,\vartheta) \\
&= 2\nu\{N\sinh(\kappa n+\gamma\vartheta) - n\cosh(\kappa n+\gamma\vartheta)\} \qquad (2.31)
\end{aligned}$$

$$\begin{aligned}
\frac{d\vartheta}{dt} &= r_{\uparrow}(n,\vartheta) - r_{\downarrow}(n,\vartheta) \\
&= \mu\{(\Theta-\vartheta)\exp(\beta n) - (\Theta+\vartheta)\exp(-\beta n)\} \qquad (2.32)
\end{aligned}$$

Introducing the scaled variables

$$\begin{aligned}
x = \frac{\vartheta}{\Theta};\ -1 \le x \le +1; \quad y = \frac{n}{N};\ -1 \le y \le +1 \\
\tau = 2\nu t;\ \tilde{\mu} = \frac{\mu}{\nu};\ \tilde{\kappa} = N\kappa;\ \tilde{\gamma} = \Theta\gamma;\ \tilde{\beta} = N\beta \qquad (2.33)
\end{aligned}$$

one obtains the scaled form of the meanvalue equations:

$$\frac{dy}{d\tau} = \{\sinh(\tilde{\kappa}y+\tilde{\gamma}x) - y\cosh(\tilde{\kappa}y+\tilde{\gamma}x)\} \qquad (2.34)$$

$$\frac{dx}{d\tau} = \tilde{\mu}\{\sinh(\tilde{\beta}y) - x\cosh(\tilde{\beta}y)\} \qquad (2.35)$$

2.4.3 Simulation of Characteristic Scenarios and their Interpretation

The model equations are now solved numerically for concrete parameter sets, which correspond to characteristic scenarios of political behaviour. In all cases we exhibit the fluxlines of the meanvalue equations (2.34), (2.35) and the corresponding stationary solution of the master equation (2.30). Throughout all simulations we choose $\tilde{\mu} = 2$ and $2N = 20$. The very small value of $2N$ has been chosen for illustrative purposes, because it yields broad probability distributions, whereas the scaled meanvalue equations do not explicitely depend on N.

We distinguish two groups of scenarios: the group A with affirmative trenddynamics (i.e. with $\beta > 0$) and the group D with dissident trenddynamics (i.e. with $\beta < 0$). In both groups we vary the value of the parameter $\tilde{\beta}$ (i.e. the strength of the propensity to go in case A to affirmative or in case D to opposing internal trends). Furthermore we vary the opinion pressure parameter $\tilde{\kappa}$. A small (large) $\tilde{\kappa}$ means a liberal (totalitarian) society with a small (high) pressure on the individual to adapt his external opinion to the majority opinion.

Group A: Affirmative Trend Dynamics ($\tilde{\beta} > 0$)

Case A.1 corresponds to a liberal society without opinion pressure and with weak affirmation propensity. The origin $(\hat{y}, \hat{x}) = (0, 0)$, which corresponds to a balanced opinion and trend situation, is a stable fixed point, (see Fig. 2.2 left) around which probabilistic fluctuations occur (see Fig. 2.2 right).

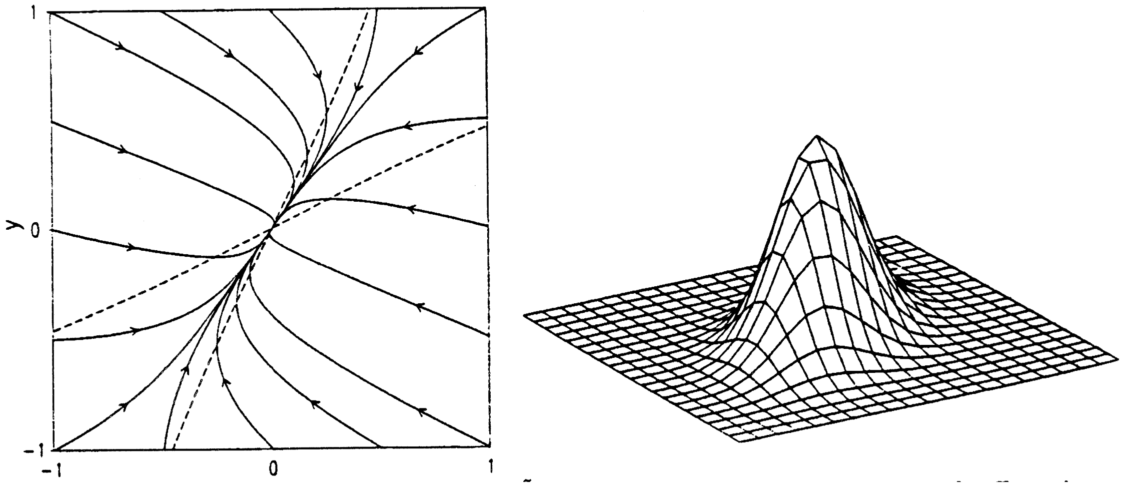

Figure 2.2: Case A.1 Parameters: $\tilde{\kappa} = 0$; $\tilde{\beta} = \tilde{\gamma} = 0.5$. No opinion pressure; weak affirmation strength. *Left:* The flux-lines approach the stable balanced opinion situation $(\hat{y}, \hat{x}) = (0, 0)$. *Right:* Unimodal stationary probability distribution peaked around the origin $(0, 0)$.

Case A.2 corresponds to a liberal society without opinion pressure but with a strong affirmation propensity. The balanced opinion / trend situation is now unstable, because the strong inclination for affirmation leads to the self-stabilization of stable majority opinions $\hat{n} > 0$ or $\hat{n} < 0$ (see Fig. 2.3 left) and to fluctuations around these stable situations (see Fig. 2.3 right).

Case A.3 corresponds to a society with considerable opinion pressure, that means with totalitarian tendencies, and with intermediate affirmation propensity. We find again stable opinion majorities (see Fig. 2.4 left) with fluctuations around them (see Fig. 2.4 right). However, the opinion majority is now partially stabilized by an affirmative internal trend, and partially by the effect of opinion pressure.

Group D: Dissident Trend Dynamics ($\tilde{\beta} < 0$)

Case D.1 corresponds to a liberal society without opinion pressure but a strong propensity to develop an opposing internal trend. Therefore, any existing external opinion majorities are soon removed by opposing trends. The flux-lines spiral into the balanced opinion / trend situation $(0, 0)$ (see Fig. 2.5 left), around which fluctuations occur (see Fig. 2.5 right).

Case D.2 corresponds to a society with totalitarian tendencies (considerable opinion pressure) and weak inclination to develop opposing trends. The balanced opinion / trend situation is still stable, but opinion majorities ($n > 0$ or $n < 0$), once established, have a long lifetime as demonstrated by the broad probability distribution of Fig. 2.6 right.

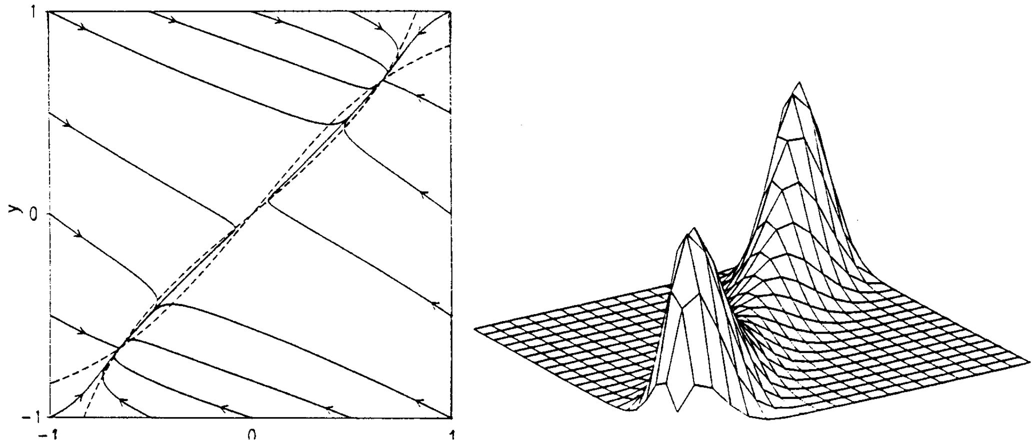

Figure 2.3: Case A.2 Parameters: $\tilde{\kappa} = 0$; $\tilde{\beta} = \tilde{\gamma} = 1.2$. No opinion pressure; strong affirmation strength. *Left:* Unstable balanced opinion situation $(0, 0)$. The flux-lines approach unbalanced opinion majority situations stabilized by affirmation. *Right:* Bimodal stationary probability distribution peaked around the stable opinion majority situations.

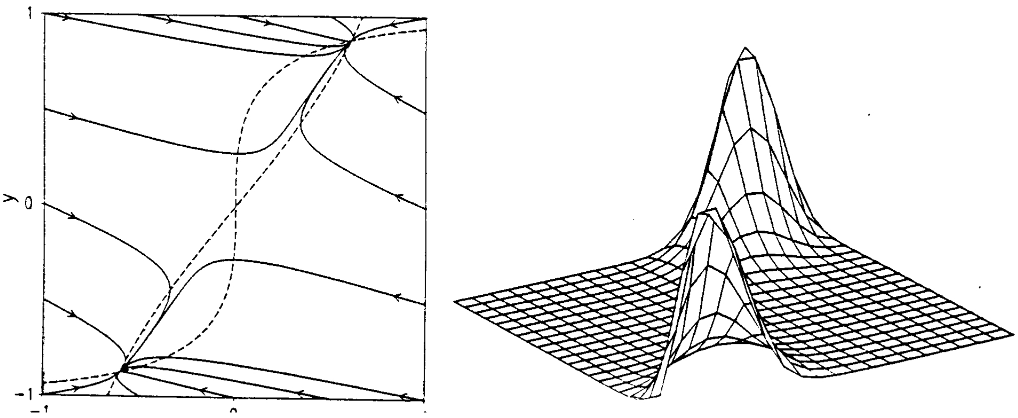

Figure 2.4: Case A.3 Parameters: $\tilde{\kappa} = 1$; $\tilde{\beta} = \tilde{\gamma} = 0.8$. Considerable opinion pressure; moderate affirmation strength. *Left:* Unstable balanced opinion situation $(0, 0)$. The flux-lines approach unbalanced opinion majority situations stabilized by affirmation and opinion pressure. *Right:* Bimodal stationary probability distribution peaked around the stable opinion majority situations.

Case D.3 describes a totalitarian society with strongly developed opinion pressure and weak propensity to develop dissident trends. The society stabilizes in a state of high external opinion majority induced by opinion pressure, and of simultaneous weak opposing internal trend (see Fig. 2.7 left). The bimodal probability distribution (see Fig. 2.7 right) describes the probabilistic fluctuations around the stable fixed points.

This case corresponds to a totalitarian society with fully developed opinion pressure, but equally strong propensity to develop a dissident internal trend. A dramatic revolutionary dynamics arises from the antagonistic competition between the effects of opinion pressure and dissidence strength. The evolution approaches a limit cycle (see Fig. 2.8 left) whose phases can be interpreted as follows.

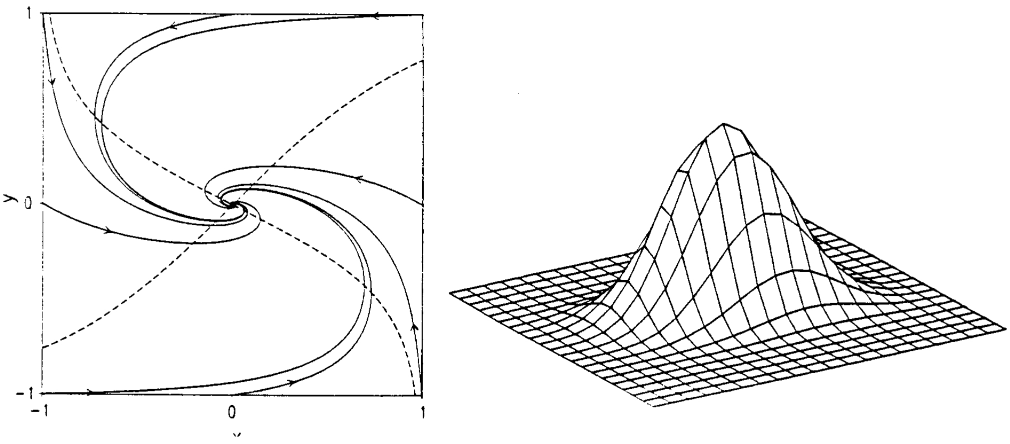

Figure 2.5: Case D.1 Parameters: $\tilde{\kappa} = 0$; $\tilde{\beta} = -2$; $\tilde{\gamma} = 1$. No opinion pressure; strong dissidence strength. *Left:* Stable balanced opinion situation $(0, 0)$. The flux-lines spiral into the origin $(0, 0)$. *Right:* Unimodal stationary probability distribution peaked around the balanced opinion situation $(0, 0)$.

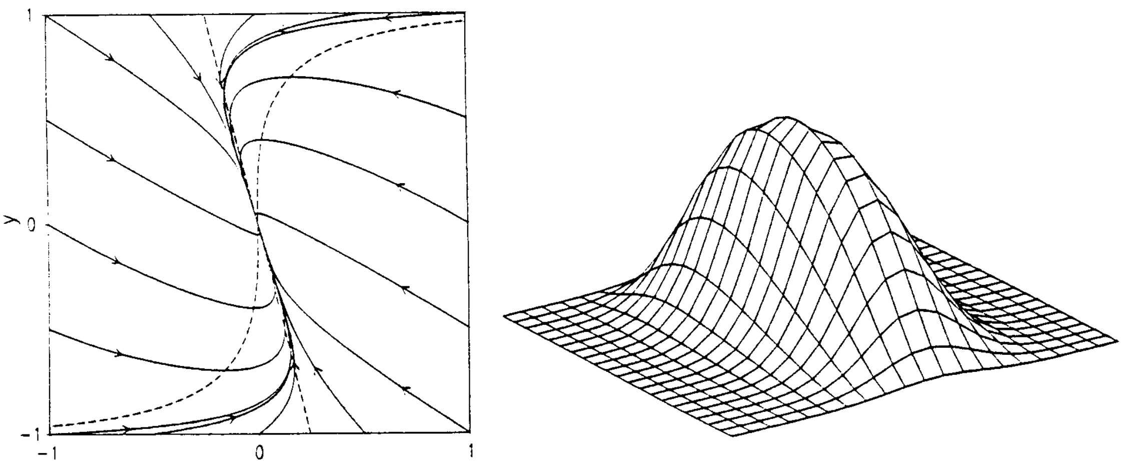

Figure 2.6: Case D.2 Parameters: $\tilde{\kappa} = 1$; $\tilde{\beta} = -\frac{1}{4}$; $\tilde{\gamma} = 1$. Considerable opinion pressure; weak dissidence strength. *Left:* Still (marginally) stable balanced opinion situation $(0, 0)$. The flux-lines approach the origin via strongly unbalanced opinion situations. *Right:* Broad but still unimodal stationary probability distribution with large variance around balanced opinion situation $(0, 0)$.

Starting somewhere in the first quadrant, transient states with affirmative trend are quickly traversed, until in the second quadrant a long-living metastable state builds up. Its lifetime is proportional to the stationary probability (see Fig. 2.8 right). In this state there exists a strong antagonism between the external opinion majority $(n \lesssim N)$ sustained by opinion pressure, and the simultaneous opposing internal trend $(\vartheta \gtrsim -\Theta)$. The metastable state finally breaks down by the "victory" of the opposing trend. Thereupon intermediate transient states are traversed in the third quadrant until there stabilizes in the fourth quadrant another metastable state with the opposite opinion majority $(n \gtrsim -N)$ and a trend $(\vartheta \lesssim +\Theta)$ again in dissidence to this new opinion

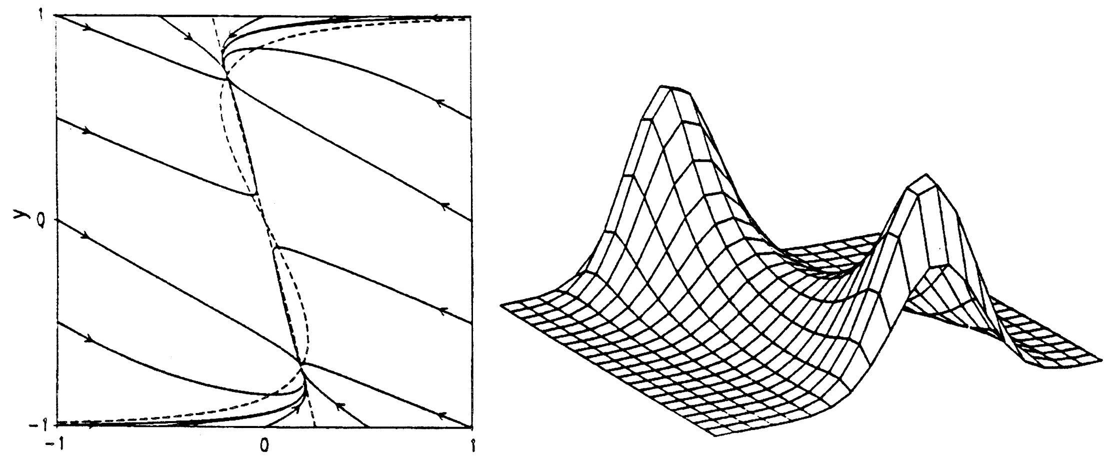

Figure 2.7: Case D.3 Parameters: $\tilde{\kappa} = 1.5$; $\tilde{\beta} = -\frac{1}{4}$; $\tilde{\gamma} = 1$. Strong "totalitarian" opinion pressure; weak dissidence strength
Left: Unstable balanced opinion situation $(0, 0)$. The flux-lines approach stable unbalanced opinion majority situations, in which the opinion pressure outweighs the dissidence strength. *Right:* Bimodal stationary probability distribution peaked around the stable opinion majority situations.

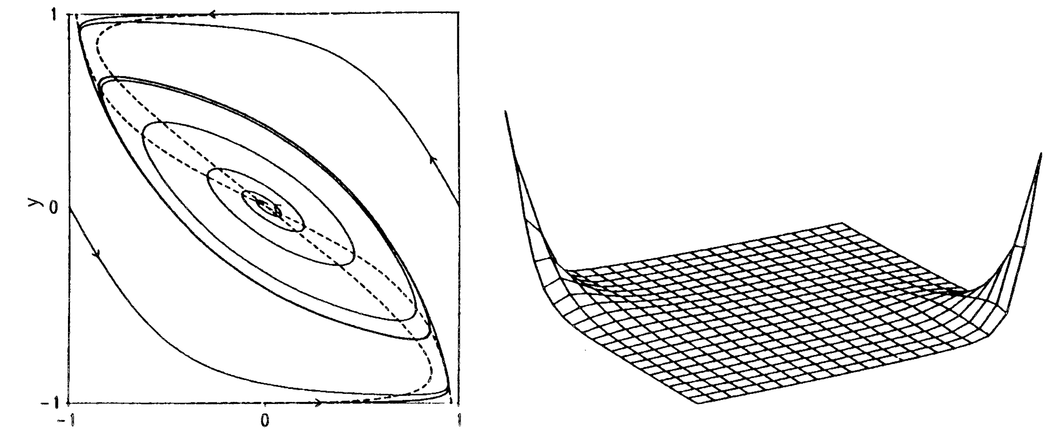

Figure 2.8: Case D.4 Parameters: $\tilde{\kappa} = 3.5$; $\tilde{\beta} = -2$; $\tilde{\gamma} = 2$. Very strong "totalitarian" opinion pressure and strong dissidence strength. No stable stationary situations. The flux-lines approach a limit cycle comprising metastable pronounced opinion majority situations sustained by very strong opinion pressure but finally destabilized by the strong dissidence trend.

majority. This metastable state has again a high perseverance probability (see Fig. 2.8 right), but will finally break down, too.

This result is a consequence of the oversimplified model assumption, that the parameters $\tilde{\kappa}, \tilde{\beta}, \tilde{\gamma}$ of political psychology remain constant during the evolution. It is however highly plausible (but not yet included in the model equations) that the revolutionary breakdown of the metastable state also leads to the evolution of other psychological trend parameters $\tilde{\kappa}, \tilde{\beta}, \tilde{\gamma}$. If, for instance, the opinion pressure $\tilde{\kappa}$ and the dissidence strength $\tilde{\beta}$ would decrease after the breakdown of the metastable totalitarian state, the further evolution could approach the balanced opinion / trend situation instead of the opposite totalitarian metastable state.

Summarizing, it seems that the model, in particular its scenarios D, can provide semiquantiative insights into the dynamics of recent historical events, even in spite of the assumed oversimplifications.

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Chapter 3

Klaus G. Troitzsch, Koblenz: Methodological Problems of Computer Simulation in Social Science

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Abstract

The paper discusses the state of the art in mathematical modelling and computer simulation of social processes. It describes the steps taken in any scientific modelling activity — the identification of some part of reality as a "real system", the detection and/or reconstruction of the laws governing the part of reality to be modelled, the combination of our notions of these laws into a more or less formal model, and (in the case of computer simulation) the running of a computer language version of the model.

Models — and computer simulation as well — may be classified for their dynamicity, stochasticity, linearity, for the level (micro vs. macro) and for the structure of their state and time spaces (discrete vs. continuous).

In a second section the paper discusses uses and problems of some well known simulation approaches used in the social sciences — DYNAMO and microanalytical simulation models — and mentions some recent achievements in the development of microsimulation tools.

A third section of the paper is devoted to general problems of computer simulation in the social sciences as compared to mathematical approaches: wherever inferences from multiple hypotheses cannot be drawn by logical or mathematical deduction, simulation at least allows us to draw inferences from fixed initial conditions and combinations of parameters, yet it is no substitute for any logical and mathematical analysis. Whoever makes use of simulation to contribute to the solution of socially or politically relevant questions should be aware and make his audience aware that simulation is never more than the solution of a formal model for a given parameter vector and a given set of initial conditions — which both have to be justified —, and that stochastic simulation is even less: one single realization of a stochastic process. Simulation tools should not only make this awareness possible, they should promote and — even better — enforce it.

3.1 The formal analysis of social processes

The formal analysis of social processes has a tradition of at least four decades. The leading article [Sim57, pp. 99–114] of the second part of Herbert A. Simon's "Models of Man" — the famous formalization of Homans' theory of interaction in social groups [Hom50] —, an article which originally appeared in April, 1953, is concerned with a mathematical macro model of the process ongoing in a social group (taken as a whole), four attributes of which (intensity of interaction, level of friendliness, amount of endogenous activity, and amount of activity imposed by the external environment) are taken as time-dependent functions in a system of linear differential equations. A (mathematically) similar approach was taken in Richardson's famous arms race model the first version of which was published in 1948 [Ric48]. The late fifties and early sixties saw the first computer simulation models, e.g. of election campaigns (cf. e.g. [SPA62]), while at the end of the sixties and in the seventies global modelling was en vogue[1] (cf. e.g. [For80, For71, $M^{+}74$]). At about that time, mathematical modelling and computer simulation were introduced to Germany, too (cf. e.g. [May67, Zie72]).

Considering these early efforts in simulation, Erwin K. Scheuch states:

> Very early in the use of computers in social research, there were high expectations that simulation could become a major form of application. . . . Subsequently ⟨i.e. after Sola Pool's *Simulmatics* project [SPA62]⟩ there was little use made of simulation. If and when modelling becomes more popular in empirical research, it is likely that there will be a revival of computer simulation.

— a paragraph of some ten lines on simulation in a paper of 18 pages on "The Impact of Computers on Quantitative Social Research" [Sch90a, p. 16] which makes sufficiently clear that formal modelling of social processes, whether mathematically or by computer simulation, has never become part of mainstream sociology.

The main purpose of this paper will be to consider

- why formal modelling — including computer simulation — did not succeed in making a substantial contribution to the development of social science, a realm of science dealing with systems of high complexity undergoing changes in time or, to formulate it the other way round, dealing with processes going on in complex systems,
- what we need to add to the common practice of formal modelling to make it successful in contributing to the progress of social science.

We attempt, initially, to answer the first question thus (and this answer is derived from Scheuch's last remark in the quotation above): Most formal models of processes failed to be linked to empirical research.

Either the model is simple enough to be linked to the empirical data that are at hand — then it fails to be appropriately complex to explain reality. This is the

[1] Perhaps, global modelling will go through a renaissance after Meadows's new book [MMR92].

typical case we have in curve fitting procedures that model a (univariate) time series by a simple linear or polynomial regression against time, or in univariate or multivariate ARIMA models where the dynamics of a single variable or of very few variables is modelled (multivariate time series are analysed seldom enough).

Or the model is so complex that any empirical analysis must fail due to the enormous number of parameters that have to be estimated from empirical data. This is the case in both global modelling and microanalytical simulation models — the case discussed in section 3.3 of this paper.

While the first drawback originates in the attempt to design process models according to the data analysis procedures at hand, the latter results from the attempt to formally describe notions about a complex object that can and will never be tested empirically. This attempt, then, is made without regard to any data analysis procedures.

The way out of this dilemma seems to be somewhere between the simplicity demanded by data analysis and the complexity demanded by reality. But this is not the whole story. Mainstream data analysis methodology is linear, and its preferred probability distribution is the Gaussian distribution. Even the most sophisticated linear model, however, will never fit a reality that is inherently nonlinear. It is, of course, true, that a large LISREL model may be fitted against a bulk of empirical data, taken from a multi-wave panel study to analyse the processual nature of reality, but as good as the fit might be numerically, the conceptual fit will be bad: the linear model will always predict the process to stabilize at the only equilibrium point possible while everyday experiences teaches us that sufficiently complex systems are systems far from equilibrium [PV79]. We shall go into further details in section 3.4 of this paper.

Moreover, mainstream data analysis techniques are statistical in a sense that reflects much of the etymological root of "statistics", namely "static". Statistics is used to cope with measurement and sampling errors (the distribution of which may easily and appropriately be modelled with the help of the Gaussian distribution due to the central limit theorem), whereas the inherent stochasticity of reality is not catered for, the only exceptions being survival analysis and time series analysis of the ARIMA type. Any deeper analysis of stochastic processes is still impossible with the general purpose statistical packages (and statistical package manufacturers will not supply any procedures for analysing nonlinear multivariate stochastic processes before there is a demand from the field of social sciences).

Disregard of nonlinearity and of inherent stochasticity corroborate each other in a vicious cycle: Linearity masks the difference between measurement and sampling error on the one hand and inherent stochasticity on the other hand, since linear processes have expected values which are their maximum likelihood values at the same time because their random variables are always normally distributed. Only nonlinear stochastic processes can consist of random variables with bi- or multimodal distributions whose expected values may even be — as it were — minimum likelihood values, as has been shown for example for one of the simplest nonlinear multilevel processes in [WH83]. We shall discuss this, too, in section 3.4 of this paper.

Multilevel modelling is another realm of social science that has been treated shabbily by mainstream social science. Only this kind of modelling, which is obviously more than "microanalytical simulation modelling" (which only aggregates the higher level from the lower level, neglecting the impact of the higher level on the lower level) is able to handle the problem of interactions between the individual and the group level, i.e. of mutual dependencies between attributes or properties of a group and attributes and properties of the individuals within the group. In addition to the above, we must also include the case of more than two levels, for example between individuals, households, and markets, or between party members, party organizations (on different organizational levels!), and the political system as a whole. Multilevel modelling is a prerequisite for detecting "some sort of order ⟨that⟩ arises as a result of individual action but without being designed by any individual ... ⟨ — ⟩ a problem ... which demands a theoretical explanation" [Hay44, p. 288], since only multilevel modelling supplies the means for describing both individual action *and* order (which can only be detected at a level above the individual level).

Thus, we might come to a first conclusion (which needs further elaboration in the sections to follow):

- Formal modelling did not succeed in making a substantial contribution to the development of social science because
  - it did not supply an appropriate stock of data analysis procedures for
    - nonlinear
    - stochastic

    processes in complex systems consisting of interacting elements and subsystems on many levels, but
  - it tried to present results which at a first glance looked so much like reality that any empirical test seemed unnecessary.
- To overcome the gap between modelling and data analysis,
  - complexity was reduced to the highest level (global models), model parameters and initial values of the numerous model variables were finely tuned to make the model yield results for the past that looked much like the historical data used for tuning, thus making the audience believe that the future of reality would look much like the results predicted by the model, or
  - complexity was reduced to the lowest level (microanalytical simulation models and its predecessors like Simulmatics), model parameters and initial properties of the numerous model elements (simulated individuals) were finely tuned to achieve analogous aims as in the case of the global models,

  and all this happened with little comprehension of the possible trajectories of the processes modelled.

Thus, what has to be taken away from the common practice of formal modelling to make it successful in contributing to the progress of social science is — at least for a while —

- the belief that our theories founding our predictive models are ready for use, and consequently
- the practice of taking models as if they were well tested and able to predict the future development of social systems.

Instead, we should behave a little more modestly and begin with the formulation of mathematical and computer simulation models of rather short range, completely renouncing curve fitting prediction, but first seeking an explanation of macro phenomena that is grounded on micro hypotheses since this is what non-formalized theories can barely offer. A formal model — be it a mathematical or a computer simulation model — proves that the theory behind it is consistent in so far as it shows that the asserted consequences indeed follow on from the theoretical assumptions. Whether the theory is empirically "correct" or not or whether it has empirical content, is a problem posed to theories of any kind (cf. [Sch90b]).

In section 3.2 we shall first discuss the modelling process, then continue with a critique of some well known simulation approaches in section 3.3, and finally address the general problem of modelling nonlinear stochastic systems in section 3.4.

3.2 The use of computer simulation in the social sciences

Modelling — which includes computer simulation — is a scientific activity that has the aim of creating a replication of some real system in such a way that this replication will react to inputs in a manner that resembles the reaction of the real system to the same inputs. The first step in this modelling activity is thus the identification of some part of reality as a "real system" consisting of elements, of relations defined on these elements, and of relations defined on the elements of the system and its environment [Bun79]. We believe that the properties of the system, of its elements, and of its environment will change because of to some deterministic or stochastic laws, and we make our (mathematical or computer simulation) model follow the same laws that we believe will control that part of reality which is to be modelled. Thus the second step of any modelling activity will be the detection — or rather the reconstruction — of the laws governing that part of reality we are about to model. Kreutzer [Kre86, p. 2] calls this step "system representation". In a third step we shall try to combine our notions of the laws governing reality into a model which may be real by itself (as is the case in animal experiments used to detect primary and secondary effects of drugs), iconic, verbal, or formal, i.e. written down in a formal language like the language of mathematics or like a computer programming language. In each of the four cases we try to draw our inferences about expected model behaviour from the premises incorporated into the model. In the cases of the informal models — real, iconic, verbal — these inferences are weak because real, iconic, and verbal models abound in ("natural") properties of their own that are not shared by the part of reality to be modelled (the "abundancy class" of Stachowiak [Sta73, 155–157]), whereas in the case of formal models — mathematical and computer simulation — the inferences are just as certain as our assumptions about the laws governing reality are completely incorporated into our model. In most instances, of course, they are not completely incorporated. There are rather a lot of properties of the real system that we do not know or that we cannot measure or that we cannot or do not want to formalize (the "preterition class" of Stachowiak).

Simulation is thus the last of four steps of computer assisted modelling or — as Hanneman [Han88] puts it — of "computer assisted theory building", and its results will be no better than the outputs of the first three steps (and, of course, they will not be better than the simulation tools we make use of).

Normally, computer simulation models will be

- dynamic process models, in which changes of properties are due to other properties,
- discrete in time, since digital computers work in discrete time steps, though they can approximate processes in continuous time,
- discrete or approximately continuous in state space, i.e. qualitative or quantitative,
- deterministic or pseudo-stochastic,
- linear or nonlinear,
- on the micro or on the macro level,
- based more or less on concepts and more or less on data.

For our purpose, and for discussing the problems of computer simulation, it will suffice to talk further about only the last four alternatives (though the third alternative — qualitative vs. quantitative — is by no means trivial, and tools which allow us to combine qualitative and quantitative approaches are rather rare).

3.3 Uses and problems of some well known simulation approaches

There are various approaches and traditions of social science computer simulation. None of them come close to pursuing a unified methodology and have thus, as yet, made little progress. Unified methodology cannot be created by majority rule, but can be expected to emerge when the isolation, in which most social science simulation studies are at present carried out, is overcome. Simulation will support interdisciplinarity, and the emergence of a unified social science simulation methodology will be supported by interdisciplinarity.

Each of the social science simulation traditions and approaches has its merits and deficiencies which will now be exemplified for two well known cases.

Let us begin with the Systems Dynamics approach, a computer simulation approach which delivers models that are dynamic, discrete in time and continuous in state space, usually deterministic and nonlinear, on a macro level and concept and/or data based.

As for the label “discrete in time”, it has to be added that for Forrester [For80, 5-8–5-9], the founder of Systems Dynamics, DYNAMO (the Systems Dynamics programming language) is to be used for integrating systems of differential equations in a manner that “the solution interval ... ⟨is a⟩ quantity ... that does not have significance in the real system that the model represents”, whereas Zwicker [Zwi81, 30–31] defends DYNAMO as a tool for discrete time modelling with various arguments such as that many economic decisions are made at nearly equidistant points of time, or that parameter estimation is much easier in systems of difference equations than in systems of differential equations — the premise that changes occur only at equidistant points of time is not always true: children are born, and infectious diseases are spread, and people die almost every second, not only at midnight on New Year’s Eve. On the other hand, DYNAMO is a coarse tool for the integration of differential equations.

Usually, DYNAMO models will be continuous in state space, because discontinuous processes can only be modelled in a rather clumsy manner; the same is true for stochastic models.

Systems Dynamics only allows us to understand the whole world (or the part of reality which is to be modelled) as one and indivisible, no properties can be attributed to elements of a system.

Whereas DYNAMO can be used to write down small models, for example, of a predator-prey population interactions on the macro level — concept based, and suitable for didactic purposes —, most applications of DYNAMO abound in parameters (which makes those models data based). WORLD2, for example, has far more than one hundred parameters in its TABLE equations, which must have been found out (or rather: estimated) empirically by curve fitting.

In spite of its deficiencies, DYNAMO is called “a semiformal language for continuous state continuous time dynamics” [Han88, 14], but at the end of his book the same author states that “the particular problems of mixed continuous and discrete state models, mixed continuous and discrete time dynamics, and network relation models ... seem most in need of development of language” [Han88, p. 326] — to which nothing need be added.

To cope with the shortcomings of the Systems Dynamics approach and other “one-of-a-kind entities” models [Orc86, 11], so-called ‘microanalytic simulation models’ have been created which — according to one of the founders of this methodology — were “originally devised to facilitate effective use of the output of microanalytical research” [Orc86, 13]. They aim at predicting effects of (and thereby supporting) social and financial policy — take for example the tax simulation models [Hab86], [Lie86], the microsimulation model for the German Federal Training Assistance Act (BAFöG; this law “gives categorically eligible students a legal claim to individual training assistance of his or her preference, qualifications and performance, if the funds needed for training expenses and subsistence cannot be provided by the student him(her)self, the spouse or the parents”) [BQ86, esp. p. 172] or the microsimulation model of the Sfb3 for the analysis of programs such as pension reform or shortening of work hours [GW86], [Sfb83].

“... in microanalytical modelling, operating characteristics can be used at their appropriate level of aggregation with needed aggregate values of variables being obtained by aggregating microentity variables generated by microentity operating characteristics" [Orc86, p. 14]. The main advantage of this kind of procedure is that "available understanding about the behaviour of entities met in everyday experience can be used ... to generate univariate and multivariate distributions" (ibid.). The main difference between Systems Dynamics and microanalytic simulation models is the same as the difference between deterministic macro models and stochastic micro models — which, it is well known, make quantitatively and qualitatively different predictions ([Bar73, p. 307], [Rap84, p. 22], [Tro89], see also [WH83]). In Henize's [Hen84, p. 571] classification, microanalytic simulation models show up as static — which seems to be due to the fact that there is usually no feedback from the reactions of microentities on the macro policies.

Since microanalytic simulation models usually use "available detailed information about the initial state of microunits such as persons and families", they are not only extremely data based but also very demanding, as far as the necessary computing and data storage capacity is concerned. These very high demands cannot be met (at least not within the computer technology of today), because storage capacity is proportional to the number of microunits represented in the model, and computing time may be proportional even to the square of this number. These special requirements, however, are not only due to the peculiarities of the micro level, but also to the manner in which (and the tools with which) microanalytic simulation models are designed and implemented. "There is no universal general-purpose microsimulation software available ..., most MSMs are developed in a conventional way from scratch" [Klö86, pp. 485–486], i.e. they have to be programmed in a general-purpose language like FORTRAN or PL/1, and they have to be run on large mainframes.

Small microanalytical simulation models, however, like the ones facilitated by IPMOS [FS90] may be developed and run on any modern desktop micro computer with the same ease as DYNAMO offers to its users. These IPMOS models, of course, allow for feedback between micro and macro level in both directions. As an experimental tool (and as a tool designed to implement Weidlich's and Haag's stochastic models [WH83]) IPMOS is suitable only for models with discrete state space on the micro and quasi continuous state space on the macro level, and there are only two levels of aggregation.

Another tool — MIMOSE [Möh90] — is being designed to meet the requirements of social science computer simulation far more thoroughly: MIMOSE enables the specification of stochastic, dynamic, nonlinear, mixed quantitative and qualitative, multi-level models, and at the same time it enables the simulation of such models — and thus helps to solve some of the special problems of computer simulation in the social sciences. Moreover, MIMOSE is a language in which a wide range of computer simulations may be formulated in such a way that they can be easily understood and compared to each other — an advantage that other languages do not have either because they are restricted to a narrow range of problems (like DYNAMO) or because they must incorporate a large bulk of technical details (like simulation models written in general purpose languages like FORTRAN or PASCAL). Easy comprehension, however, is a prerequisite for any critical evaluation of computer simulation models (cf. [Sch90b, p. 124]).

A rather new trend in social science simulation is the multi-agent simulation approach [DFC92] which makes use of artificial intelligence methods to model individual agents' behaviour in a rule based setting. Whereas in classical simulation approaches subsequent states of objects are generated determistically from differential equations or stochastically in Markov processes, multi-agent simulation in the strict sense of this term defines agents as entities "defined by their ability to perceive specific kinds of communication, by their skills, i.e. their ability in performing various actions, their deliberation model if any, and their control structures, i.e. their ability to relate perception to action." [DFC92, p. 52] The main difference between the so-called multi-agent simulation approach and the approach represented by our MIMOSE tool lies in the use of artificial intelligence methods by the former; what they have in common is their ability to model emergent phenomena in multi-level processes.

As far as can be judged up to now, there is a strong trend towards dynamic multi-level models, but they come in two types: concept based and data based. Concept based models are useful for their heuristic value since they allow inferences from multiple hypotheses, whereas data based models might be useful for their predictive value — but only provided that they have been tested for internal inconsistencies and against empirical data.

3.4 General problems of computer simulation in the social sciences

Simulation may be understood as experimenting with models: wherever inferences from multiple hypotheses cannot be drawn by logical or mathematical deduction, simulation at least allows us to draw inferences from fixed initial conditions and combinations of parameters. Simulation as experimenting with models, however, is a kind of experimentation completely different from experimenting in physics since it gives us only answers concerning the implications of our thoughts but no answers concerning reality. Thus the truth value of a simulation result is just the same as the truth value of the hypotheses combined into the simulation model.

Computer simulation allows us to vary initial conditions and parameter vectors abundantly, and thus to get a first glance at the diversity of possible consequences of initial conditions, but it is no substitute for any logical or mathematical analysis. Simulation, however, allows us refinement of analytical tools. Once computer simulation has shown that for example a certain nonlinear deterministic system displays unpredictable "chaotic" motion, because it depends sensitively on its initial conditions and because a minute change of the initial conditions leads to completely changed consequences after a very short span of time, we can go to work and mathematically analyse the conditions in which unpredictability of deterministic systems originates — as has been done in physics, meteorology, chemistry, and biology for the last 30 years, since Lorenz's [Lor63] well known paper on "Deterministic nonperiodic flow" appeared.

Social systems are always essentially nonlinear, and they are always so complex that they can display chaotic behaviour. This is why the application of methods for the analysis of complex nonlinear systems that are being developed in other sciences will be especially fruitful in the social sciences — though not for the purpose of quantitative prediction of social processes, because this approach, which was pursued a bit too euphorically in the early seventies (system dynamics, "The Limits to Growth" [MMJM72]) and maybe again recently when Meadows stated that his WORLD3 had to be modified only slightly to retrodict the world's development in the seventies and eighties [MMR92], is condemned to failure owing to our present knowledge of nonlinear systems. On the other hand, nonlinear modelling allows for multiple equilibrium, non-equilibrium, and nonperiodic solutions — which is best suited to the purposes of social science because in its realm (single) equilibrium and periodic solutions are almost never found. Meanwhile economists and social scientists, too, have learnt from simulations of models like the ones described in Troitzsch's and Klee's contribution to this proceedings (see section 6.2) that qualitative predictions are possible even in complex systems, when they analyse whether a certain system is susceptible to chaotic behaviour or to catastrophes, which parameters are responsible for the outbreak of chaos or for a catastrophe, and by which measures chaos or catastrophe might be prevented. Empirical processes of, for example, stock exchanges that seem to be erratic or random, may be analysed more precisely with these new methods which had to be developed with the help of numerical simulation of concept based theoretical models. Very slowly, we are acquiring a new understanding of processes that were until now believed to be random processes.

Simulation, however, only allows us the refinement of analytical tools; it does not facilitate data analysis, neither does it give a complete view of the relationship between initial conditions and parameter vectors on the one hand and the outcomes of simulation experiments on the other. Simulation allows us to search a continuous multidimensional parameter space, but only point for point. At the most, it may be possible to detect boundaries in parameter space which separate regions of stable equilibrium from regions in which the parameter vector results in limit cycles or chaotic attractors, but those boundaries in parameter space may at the most be imagined in models of minor complexity.

To be only moderately realistic the dimension of parameter space will be so high as to prevent the detection of boundaries between regions of qualitatively different system behaviour. In these cases, simulation will only allow us the prediction whether a given parameter combination and a given set of initial conditions will make the system approach a stable equilibrium, a stable orbit, or a chaotic attractor. This is no doubt useful, albeit only — at huge expense — to find the initial condition and parameter vector displaying the desired behaviour by trial and error.

Social science computer simulation, however, must not only be experimentation with formal models. We have to stress that simulation outcomes always have to be systematically tested against empirical data. Here social science data gathering and analysis is confronted with a wide area of tasks, and simulationists should try to create a common data base which allows for empirically testing their formal models. There are only a few fields of social science — demography, electoral and attitude research — in which there are data sets for many equidistant points of time on hand which could be analysed with the new tools appropriate to nonlinear deterministic and stochastic processes.

About twenty years ago, Hayward R. Alker [AJ74] put the question whether computer simulation is inelegant mathematics and even worse social science. He answered that indeed "an open simulation is bad mathematics even if it is a good social system representation." [AJ74, p. 153] The ideal he aimed at — unique solvability of a system of mathematical equations allowing for deterministic prediction — is too far fetched considering the complexity and nonlinearity of systems of differential equations, or of inhomogeneous Markov processes with which we are concerned when we are modelling social processes (but the mathematics of chaotic systems or of stochastic processes is by no means "bad mathematics"). Whether — on the other hand — a good representation of the social system is possible, be it by mathematical models, or by computer simulation, has to be decided by theory based empirical analysis.

Whoever makes use of simulation to contribute to the solution of socially or politically relevant questions should be aware, and make his audience aware, that simulation is never more than the solution of a formal model for a given parameter vector and a given set of initial conditions — which both have to be justified —, and that stochastic simulation is even less: one single realization of a stochastic process. Simulation tools should not only make this awareness possible, they should promote and — even better — enforce it.

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Chapter 4

Leonid G. Shatikhin: Some Thoughts on General Principles of Predicting Extreme Situations

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4.1 Introduction: The Problem

Extreme situations as initial phase of serious events like havaries and catastrophes may occur in various areas of life and human activities. In any single case they occur in different manners. But there is the question, whether the conditions of their emergence do not have something in common, so that they might be predicted early and that advice might be worked out so that they might be prevented right from the beginning in systems of arbitrary nature.

If general indicators of an approaching extreme situation cannot be found, mankind will always be in a position of being threatened by unexpected momentous events.

But if we succeed in finding such general indicators, then it will be possible to live more or less in peace, even in completely novel situations, for which there is no statistics reporting possible deviations from the established course of processes.

We shall try to get on with these questions.

4.2 The Concept of System: System as Closed Path

One of the many definitions of a system reads: “A system is a process going on”. It stems from S. Optner[Opt69] and — in spite of its shortness — describes the essence of an arbitrary dynamical system exhaustingly.

[0]Editor's note: The help of H. Neugebauer in preparing a German version of Professor Shatikhin's original russian text to support the English translation is greatly appreciated.

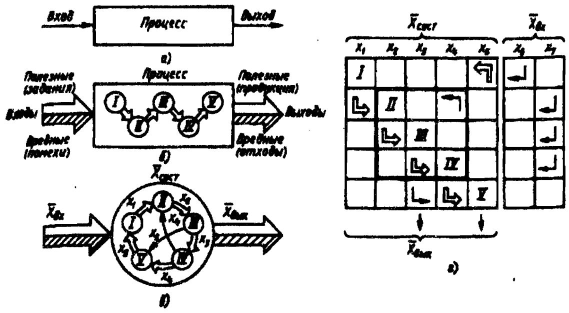

Figure 4.1: Succesive representations of a system (taken from [Shat91, p. 153, fig 91]) — *а:* elementary original representation (input–process–output), *б:* first detailed representation of three principal components of a system (inputs/utilities (tasks)/detriments (disturbances)–process–outputs/utilities (production)/detriments (losses)), *в:* second detailed representation of the system kernel, *г:* matrix representation of a system

It is obvious that we judge any system as a means to achieve an *output* which is of interest to us, an output which is the result of a given system's functioning. But any concrete output is given as the result of a process (or of processes) *within the system.*

This inner part of a system is called "system kernel". If we represent processes going on within the system as a directed graph (see fig. 4.1*в*) we will always find at least one closed path within the system kernel. For stable systems this path will have a negative sign, i.e. it will consist of an odd number of negative edges and an arbitrary number of positive edges. In the end, the desired output of a system — with the given technology of redesigning the system kernel — can only be achieved if there is an input of completely fixed factors as resource. *Input, kernel, and output* are the necessary and fundamental parts of any dynamical system.

We must remark, though, that within the kernel several processes may go on at the same time. This means that within our system there are several *subsystems.* Each of them in turn consists of input, kernel, and output. Beside, any input and output may have both favourable and deleterious components for the process under consideration. In the end, we have to make a last remark concerning the concept of complex and simple systems: Any system which is described by *only one* closed path is *simple*, independent of the number of the elements belonging to the path. We regard systems as complex, however, which have inner paths with crossing inner connections.

After having defined the concepts concerning the essence of a system we can switch over to the consideration of the conditions of their functioning which is inseparably connected with the concept of *situation.*

4.3 The Concept of Situation: Situation as the Totality of External Conditions

According to dictionary definitions, a situation is "a connection of conditions and circumstances which create a certain position". Or: "a reasonable, rule-bound multitude of common states of objects". Anyway, we must count the concept and object of situation in the field of *external influences* on the system kernel which work upon its input and belong — in a certain measure — to the concept of *environment* within which a given systems functions.

In a lot of cases a situation is created and defined by a random totality of external conditions and circumstances. Often a situation will be brought about by an *external system* (which the system under consideration is part of), and it will be brought about in a *goal-directed* manner to create a special complex of conditions for a certain inner system which further or hinder the fulfilment of its functions.

In any of these (and other!) cases our system — which must care to perform its obligations (albeit to satisfy the needs of its parts) — will have to analyze the situation being created around it, searching it for changes and influencing it in the desired direction. Sometimes — e.g. in parliamentary practice — this is done by lobbies. In other cases it is done by direct or concealed bribing of higher rank officials in charge of related decisions. Anyway, we must consider that a situation is created by a certain layer of elements of higher order structures in which processes go on more rapidly than the fundamental processes in the system we defend.

Completely analogous conditions regarding the influence of external impacts also exist in the field of technical systems where external conditions may also change suddenly or stepwise and may complicate (or sometimes even facilitate) the functioning of various machines, aggregates or whole enterprises.

4.4 The Concept of an Extreme Situation and the Possibilities of Its Prediction

A first concept of extreme situations and of the necessity to predict them was given in the introduction. Let us now regard these questions at large. We shall suppose that *extreme situations* occur under conditions which disturb the stable functioning of a dynamical system. But a dynamical system is stable as long as it is described by a closed path with negative sign. If the path is broken off, the system fails, an *extraordinary situation* is brought about — a havary or catastrophe.

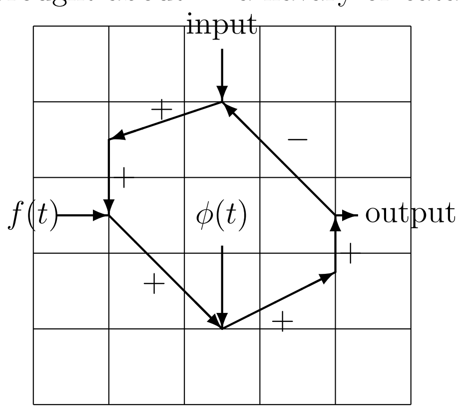

Figure 4.2: An illustrative example of a directed graph of a system

The initial phase of an extraordinary situation is the extreme situation, if the conditions for an imminent breakdown of the path are just *coming into being.* These conditions must necessarily be *predicted*. For this prediction the emerging situation around the system must be analyzed.

Thus to predict extreme situations around a (served) system, an uninterrupted, systematic monitoring of all processes entering the given space is necessary, just as atmospheric changes are watched for drawing synaptic maps and for weather prediction. At the same time all paths existing in a system must be monitored, carefully and in a qualified manner judging the following parameters:

1. size of the forces influencing the path from inside and from outside,
2. constancy of connections within the system,
3. tendency of sign change within the path under consideration,
4. relations between positive and negative paths in the system as a whole.

For the analysis of situations in systems with multiple parameters, a representation by *structure matrices* is better suited. Let us now look at their properties and possibilities.

4.5 Structure Matrices as a Method Suited For Representing Complex Systems of Arbitrary Nature

Structure matrices were recommended as an apparatus for representing and analyzing complex systems by the author[Shat91, p. 254]. They allow for creating a correct base for working with structures of complex dynamical systems of an arbitrary physical nature without additional graphical representations.

Regarding their structure, structure matrices are ordinary extended matrices of an equation system. The coefficients for the dependent variables are placed on the left, quadratic side, and the coefficients for the independent variables are placed on the right hand side (see fig. 4.1г). The signs of all off-diagonal elements on the right hand side of the system are inverted. Thus, the matrix is completely analogous to the graph if the diagonal elements on the left hand side are regarded as vertices of the graph, and the off-diagonal elements as the edges of the graph.

If we analyze a dynamical system, all coefficients of the matrix must be written in operator form. Then, all diagonal elements of the matrix are regarded as *eigenoperators* or *denominators* of the transfer functions of the respective dynamic terms, and all off-diagonal elements as *connective operators* or *numerators* of the transfer functions.

$$W(p) = \frac{R(p)}{Q(p)} \qquad \text{with} \qquad p = \frac{d}{dt} \tag{4.1}$$

Thus, the diagonal elements reflect the dynamic properties of the respective elements of the system.

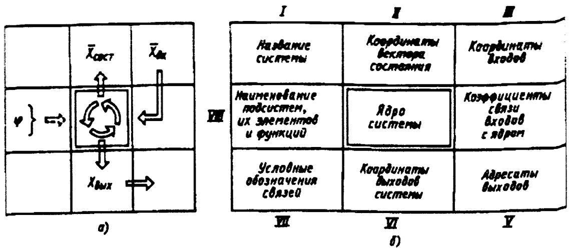

Figure 4.3: Structure matrix in standardized form (taken from [Shat91, p. 16, fig. 15]). *а:* path of passing signals, *б:* functional regulation of the commentary field. Center: system kernel; I: system name: II: coordinates of the state vector; III: coordinates of the input; IV: coefficients connecting the inputs to the kernel; V: adressees of the outputs; VI: coordinates of the outputs from the system; VII: conditional indicators of connections; VIII: name of subsystems, their elements and functions

To make work with the matrix even more comfortable, we do not write all the coordinates of the system on the right hand side, as column vectors, but above, as row vector. Besides, around the coefficient matrix comment fields are placed in which the necessary information on the system analyzed is to be found (names of the subsystems, their elements, functions of these elements, and other necessary information). Such a form of a matrix is called *standardized form* (see fig. 4.3*б*). It allows to avoid any additional textual description of the system during the evaluation of its structure.

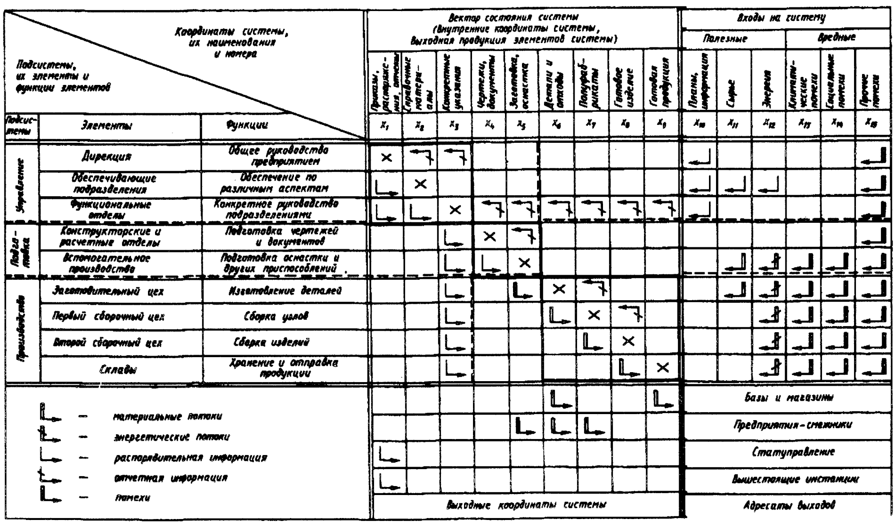

Figure 4.4: Structure matrix of a productive enterprise in first detailed representation (taken from [Shat91, p. 224, fig. 131]). The arrows have the following meaning (translation of the commentaries in the left bottom corner of the drawing, from top to bottom): material flows, energy flows, instructive information flows, report information flows, preventions.

During the initial phase of the evaluation of the structure, the elements of the matrix are represented as icons. Thus, e.g., the diagonal elements are written as oblique crosses, and the off-diagonal elements as right angle arrows from the source element to the target element. All elements in the matrix are organized according to the *block matrix* principle, where the related hierarchy in the system is cared for. During further evaluation, all blocks are detailed in the necessary measure. All closed paths belonging to the system may be followed in the direction of the connective arrows in the quadratric part of the matrix (see fig. 4.4).

The suggested form of representation by matrices guarantees an essentially more disciplined thinking on the side of the researcher in analysis and synthesis of complex dynamical systems of arbitrary nature, and is well suited as a language of interdisciplinary communication.

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Part II

Modeling and Simulation in Social Science: Methods and Tools

Chapter 5

Michael Möhring and Volker Strotmann, Koblenz: Modelling and Simulation of Multilevel Models in the Social Sciences with MIMOSE

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Abstract

The aim of this paper[1] is to present the most important features of the modelling and simulation system MIMOSE. It starts with an overview of the modelling language concept, which considers special demands of modelling in social science, expecially the description of non-linear, quantitative and qualitative relations, stochastic influences, the dynamic change of structure, and micro and multilevel models. At the same time, describing models in MIMOSE does not burden the modeler with a lot of programming and implementation details. By using ideas from general systems theory and influenced by the paradigms of functional programming l nguages, the modelling technique of MIMOSE supports the development of structured, homogenous simulation models, which improves the transparence of the "model programming process" and makes model descriptions and even the corresponding simulation results easier to understand.

Introducing a simple attitude change process of individuals in a population, each step of building a MIMOSE model, including the implemented evaluation strategie will be shown. The definition of the dynamic change of model structure, one of the most important features of MIMOSE, will be described in detail by using a macro model of technological evolution. The specification of a macro and a micro version of this model in MIMOSE shows, that the difference between both version is only slight.

Finally, a detailed description of the current state of development of the model definition language and the simulation system will be given.

[1]The most important parts of this paper were also presented at the International Conference on Social Science Methodology which was organized by the Dipartimento di Politica Sociale of the Universitá di Trento and the Research Council 33 (Logic and Metholology) of the International Sociological Association at Trento, June 22–26, 1992.

5.1 The Simulation System MIMOSE

MIMOSE[2] is an abbreviation for *MI*cro- and multilevel *MO*delling *S*oftwar*E*. It stands for a modelling and simulation software system which consists of a model description l issge and an experimental frame for the simulation of the described models. This presentation concentrates mainly on the model description language, because simulation systems are characterized by the model description language they provide, and because the present language design incorporates the most important ideas of the entire MIMOSE project, which means the development of a new modelling technique.

The basic concept of the experimental frame is the partition of the whole modelling and simulation process in single tasks, which can be performed interactively by a window-oriented user interface:

- *model description*
  Usually, the input of MIMOSE models takes place by using a text editor window. As an alternative to this, a graphic editor has been currently developed, which supports the model development process by the input of graphical symbols [Kl91].

- *model initialization*
  The execution of a correct simulation step requires the initialization of model parameters (i.e. external parameters, number of objects, constant object attributes). Several initialization sets can be easily defined for each model descriptionXXX *control of simulation*
  Additionally to the initialization of model parameters, a complete simulation run requires additional informations, like the size of the time interval between two simulation steps (*DT*), a conditional function for the simulation stop (*STOP*), and a conditional function for a programmed simulation interrupt (*BREAK*).

- *execution of simulation runs*
  Beside the start and stop of simulation runs, the simulation window allows an interactive simulation interrupt and resume, including the definition of new initialization valuesXXX *graphic presentation of simulation results*
  MIMOSE allows two dimensional graphic presentation of simulation results, including facilities like the presentation of different curves in one graphic, zooming, splines, etc.

- *file handling facilities*
  This component includes loading and saving of model descriptions, model initialization sets, simulation control sets, and simulation data.

[2]MIMOSE is funded by the Deutsche Forschungsgemeinschaft, grant no. Tr 225/3-1, 225/3-2.

5.2 Model Description in MIMOSE

5.2.1 Basic Ideas

The analysis of models by computer based simulations requires the translation of models (i.e. verbal hypothesis, icon models, differential equation systems) into a formal representation, which is executable by a computer program. To minimize the translation work, a modeler obviously prefers a modelling language which supports his modelling technique as much as possible. This requires the development of model description languages which are adapted as much as possible to the "world view" of the modeler and which are at the same time powerful enough to fulfil all of his modelling demands. Therefore, the purpose of the MIMOSE project was the development of a modelling language, which considers special demands of modelling in social science, especially the description of

- nonlinear, quantitative and qualitative relations,
- stochastc influences,
- dynamic change of structure, and
- micro and multilevel models.

At the same time, describing models in MIMOSE does not burden the modeler with a lot of programming and implementation details. Furthermore, the l issge concept supports the development of structured, homogeneous simulation models, which improves the transparence of the "model programming process" and makes model descriptions and even the corresponding simulation results easier to understand.

To reach these goals MIMOSE is iased on the following:

- To achieve a general and uniform modelling technique, ideas from *general systems theory* were adapted. Especially the formal approach of Bunge [Bunge 1979] [Bunge 1979a] describing the structure of real entities by a set of attributes and their state at a given point of time by state functions which are assigned to each attribute, became very useful.
- The design and development of the concrete language structure was strongly influenced by the paradigms of *functional programming languages* [Ebert 1986] [Reade 1989]. The most important characteristics of functional languages in reference to MIMOSE are outlined below:
  - *declarative, uniform l issge concept*
    Functional languages are based on expressions which denote concrete data objects (i.e. numbers). The construction of new expressions carried out by
    - ∗ function applications (i.e. arithmetic functions, matrix functions etc.)
    - ∗ composition of functions,
    - ∗ conditional functions (i.e. if-then-else),

  * function abstractions, and so called
  * "higher order functions", in which arguments and values can be functions again.

- *single assignment*
  Identifiers are only used as abbreviations of expressions. They do not denote variables, which do not exist in functional languages. This *avoids side effects*, because an identifier must not be used for the abbreviation of different expressions
- *"referential transparency"*
  Within a special scope (i.e. object type definition (see subsection 5.2.5) each use of an identifier or the corresponding expression denotes the same value, which is evaluated from the correspondent expression. This supports an easy understanding of programs.
- *lazy evaluation strategy*
  In contrast to a "busy strategy" in procedural programming languages (i.e. PASCAL, C) "lazy evaluation" means, that an expression is evaluated, exactly when its value is required (i.e. as an argument of another function evaluation). Using this strategy leads to more flexibility in programming, because the described order of evaluation is independent from the input order of the program statements.

5.2.2 A Simple Model Example

The following chapters demonstrate the main characteristics of MIMOSE by the developing of a simple MIMOSE model for the process of *attitude change of individuals in a population* [Weidlich/Haag 1983]. This model can be described as follows:

Each of the individuals has an attitude *pro* (=1) or *contra* (=0) with respect to a certain issue (i.e. death penalty). At a given point of time an individual can therefore be described by its position (0,1) in a one dimensional *attitude space*. The attitude change depends on the attitude of the individual history as well as on a probability for an attitude change:

$$\begin{aligned} 0 \Longrightarrow 1 &: \nu e^{(\pi+\kappa x)} \\ 1 \Longrightarrow 0 &: \nu e^{(-(\pi+\kappa x))} \end{aligned}$$

with

x	:	state of the population (*socio configuration*)
ν	:	general *flexibility* of an attitude change
π	:	*preference* towards an attitude
κ	:	the *sympathy* to adjust the individual attitude to the majority of the population

Generally, the state of the population at a given point of time is characterized by the *socio configuration* (n_1, n_0), which means the number of individuals with the attitude *pro* (n_1) or *contra* (n_0), respectively. Because of

$$n_1 + n_0 = N, \text{ (N = Number of the population members)}$$

the socio configuration space can be reduced to one dimension and x is restricted to the range $[-1, 1]$ by

$$x = 2\frac{n_1}{N} - 1$$

with

x	=	1	:	all individuals have the attitude *pro*
x	=	-1	:	all individuals have the attitude *contra*

Depending on the choice of the parameter values of π and κ the simulation of the model leads to the following main results of the population's behavior (x):

- If the coupling of individuals and population is weak (i.e. $\kappa = 0.2$), the state of the population leads to a state near 0, which means that the individuals do not adjust their attitude to the population's majority.
- A strong coupling between individuals and population (i.e. $\kappa = 1.5$) leads to a state near one of the extreme states $(1, -1)$ with few random changes between these states.

5.2.3 Multilevel Modelling

According to our example the change of attitudes in a population can be described basically from two different points of view:

- Modelling on a *macro level*
  The population is one single unit, whose behavior is characterized by some time dependent variables (i.e. ratio of the number of individuals with attitude *pro* with respect to the number of all population members) connected by equations (i.e. differential equations).
- Modelling on a *micro level*
  Each individual of the population is described explicitly together with rules for his/her attitude change (i.e. individual probability for an attitude change). The state of the population can be derived after each simulation step, by aggregating the attitudes of the individuals.

Because of the fact that analytical solutions can be achieved only for a small class of differential equation systems[3] and that the socio configuration of the population should influence the individual attitudes too, none of the two modelling techniques would be satisfying. Generally, the analysis of effects on a macro level caused by dynamic processes on a micro level (i.e. interacting individuals) as well as the analysis of influences from macro to micro level (feedback effects) require a modelling formalism, which integrates different modelling levels. Especially the analysis of the so-called "collective phenomena" needs such a modelling technique in the social science. One of the main features of MIMOSE is the description of modelled objects on different (aggregation) levels together with the relations between these levels in one model.

[3] For this simple model example there exists an analytical solution which allows the description of the behavior of the population by the differential equation $\dot{x} = \sinh u - x \cosh u$, with $u = \pi + \kappa x$

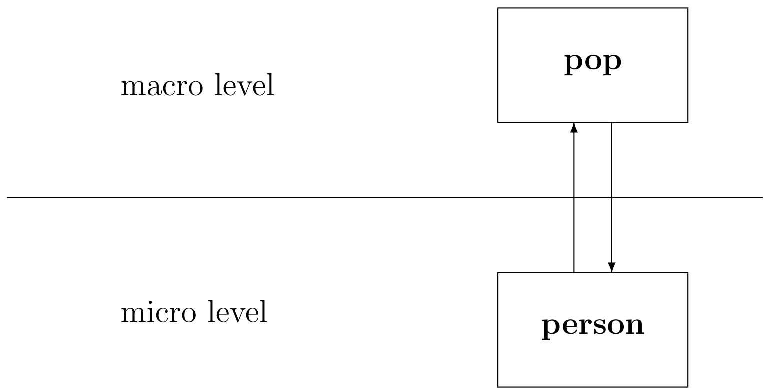

Figure 5.1: Two level modelling of the attitude change model

5.2.4 Model Structure

The basic notions in MIMOSE are *objects*, as formal representations of entities in reality (i.e. individuals, groups, organizations) and *attributes*, as formal representations of real properties of objects (i.e. age, attitude). Starting from these definitions, the general *model structure* in MIMOSE can be described as follows:

- A set of *object* types, of which concrete objects will be created during the model initialization and simulation.
- Description of object types by a set of *attributes*, whose values represent the state of objects at a given time.
- Description of relations between (types of) objects (also by attribute definitions).

Figure 5.1 shows the micro/macro structure of the sample model with the two object types *pop* and *person* together with the relations among them, whereas Figure 5.2 shows the analogous model description in MIMOSE, where each object type is structured by a set of typed attributes, with the following meaning:

x	:	The state of a population (*socio configuration*)
persons	:	The relation of a population to a list of person objects (= members of the population)
att	:	The attitude of a person to a certain issue (i.e. death penalty)
p	:	The relation of a person to a population

5.2.5 Model Behaviour

In addition to the model structure, a simulation model in MIMOSE requires a description of the *model behaviour*, which can be defined as the state change of all model objects over a period of time The structure of state change of an object type can be generally described as follows:

- Assignment of state transition functions to object attributes for the evaluation of the attribute value in each simulation step.

```
 pop
 {
   x        : real := ...;
   persons  : list of person
 };

person
{
   att      : inti:= ...;
   p        : list of pop
};
```

Figure 5.2: Structure of the attitude change model

- Description of the dependencies between object attributes by arguments of the state transition functions.
- Description of external influences and attribute values of the past by arguments of the state transition functions.

The power of modelling in MIMOSE depends directly on the definition of state transition functions. The expression oriented functional l issge concepts allow the uniform and flexible description of state transition functions, which include the application of base functions (arithmetic, logic, list functions, matrix functions, etc.) and user defined functions, function composition, conditional functions (if-then-else, case), and "higher order functions".

The complete definition of the example model in Figure 5.3 shows the behaviour between the object types *pop* and *person* by the state transition functions for the object attributes x and *att*, directly translated from the described socio configuration and individual attitude change in chapter 2.2:

- The state of a population (x) depends on the attitudes of all members (*persons.att*). The selection of all members with an attribute equal 1 is performed by the application of the function *haselements* by counting all individual attitudes equal 1 in the list *persons.att*. The number of the population members is calculated by the function application *length (person)*.
- The attitude of a person depends on the attitude in the time step before (*att_1*), on the state of the population in the time step before (*elem(p.x_1,1)*[4]), and on the external model parameters ν, π and κ. The new attitude is determined by the application of a transition probability function (i.e. *prob(1,ny * exp (pi + kappa * elem(p.x_1)))*) within a case expression, which realizes the described cases for an individual attitude change.

Apparently, the example shows some general regulations which a modeler must consider when defining multilevel models in MIMOSE.

[4]The basic function *elem* selects an element, whose index is given in the second argument, from a list (=first argument).

```
pop
{
 x       :  real
         := 2 * haselements(persons.att,1) / length(persons) - 1;
 persons :  list of person
};

person
{
 att :  int
     := case att_1 of
        1       : 0 if prob (1,ny * exp(-(pi + kappa * elem(p.x_1,1))))
                    else 1;
        default : 1 if prob (1,ny * exp(  pi + kappa * elem(p.x_1,1)))
                    else 0;
        end;
 p   :  list of pop
};
```

Figure 5.3: Behaviour of the attitude change model

1. Attributes from other object types can only be used as arguments of a state transition function if a relation attribute to the correspondent object type is defined, too (*persons, p*).

2. Attributes from other object types can be obtained by using the access function "." together with the defined relation attribute (*persons.att, p.x_1*).

3. The access to attribute values from the simulation history can be easily performed by adding "*_num*" to the attribute identifier (i.e. *att_1* for the attribute value in the time step before).

4. The definition of relations between object types in *both* directions requires a non-cyclic access of attributes within the actual time step to guarantee a correct evaluation of the state transition functions (*persons.att* $\Longrightarrow$ *p.x_1*) (see also the *lazy evaluation* strategy described below).

In our example the relation attributes *person* and *p* do not receive any state transition functions. This means that the initialized relations remain constant over the entire simulation period. Generally, the definition of relations as object attributes allows state transition functions for relation attributes too. This enables a dynamic variation of "relation values" during a simulation (i.e. the dynamic creation and deletion of objects in birth/death processes).

The model description in Figure 5.3 together with necessary initializations of model parameters (i.e. *pop.x_1, persons.att_1*) allows the execution of a complete simulation step. MIMOSE uses a *lazy evaluation* strategy for the evaluation of models, which means that an attribute is evaluated exactly when its value is required (i.e. as an argument of another function evaluation). The use of this evaluation strategy is demonstrated by the execution of one simulation step of the example model, starting with the evaluation of x:

eval (x)	$\Longrightarrow$	eval (persons)	: evaluated (constant)		
		eval (att)	$\Longrightarrow$	eval (att_1)	: evaluated (history)
			$\Longrightarrow$	eval (pi)	: evaluated (constant)
			$\Longrightarrow$	eval (kappa)	: evaluated (constant)
			$\Longrightarrow$	eval (p)	: evaluated (constant)
			$\Longrightarrow$	eval (x_1)	: evaluated (history)

After evaluating *att*, the evaluation of x is possible. After that, all state functions of the model are evaluated and the simulation step is finished. By using this evaluation

```
pop
{
 x       :  real
         := 2 * haselements(persons.att,1) / length(persons) - 1;
 persons :  list of person;
 outx    :  list of real
         := append(outx_1,x)
};
```

Figure 5.4: Redefinition of *pop*

strategy, the described order of evaluation is independent from the input order in the model description and it realizes the intended level hierarchy of the model: Calculation of a new socio configuration *after* generating the attitudes of all individuals.

If the application of a state transition function is not evaluated by the lazy evaluation strategy, no dependencies to other attributes exists in the model description. Therefore, all of this applications will be evaluated at the end of a simulation step.

The analysis of model behaviour requires all the values of the interestng attributes, which are evaluated during a simulation. In MIMOSE they must be collected explicitly by the modeller, because a model describes only the evaluation of attributes in a single simulation step[5]. Figure 5.4 shows a redefinition of the object type *pop*, in which the values of x are stored in the attribute *outx*: In each simulation step the function *append* adds the actual value of x to the existing list of values in *outx*. The values of all object attributes as well as the values of all external model parameters are accessible at each breakpoint or after the end of a simulation. Therefore MIMOSE needs no special transfer functions from model states to output parameter like in other modelling concepts (see [Zeigler 1976, S. 218ff] [Ropohl 1978, S. 23ff]).

[5]Only the “system variables” *count* and *time* will be collected automatically and stored in *$count* and *$time*.

model parameters		initialization values
pop		
	number of instances	1
	x_1	<< 0 >>
	persons	makeref (pop,person)
	outx_1	<< [] >>
person		
	number of instances	1000
	att_1	makeinst (1000,uniform (1,0,1))
	p	makeref (person,pop)
pi		0.0
kappa		1.2
ny		0.02

Figure 5.5: Change of attitude in a population (initialization)

5.3 Experimentation with Models

5.3.1 Model Initialization

During a simulation step every state function of a model will be evaluated. Therefore, the initialization of a model must guarantee, that an evaluation is possible for the first time This requires the initialization of

- the number of objects, which should be created of each object type,
- all exogenous parameters,
- all object attributes without a state function, and
- all object attributes from the past.

After checking the model description, MIMOSE creates a model initialization window, which provides the user with a list of all the model parameters which must be initialized. The initialization of model objects is done by the initialization of their attributes. According to the proposed number of objects, an attribute gets the value for each object instance by evaluating a special initialization function like

- << 0,2,4,6,8 >> (*explicit initialization*) or
- makeinst (1000, uniform (1,0,1)).

Relation attributes, like *persons* and *p* in our example, gets its values by evaluating the initialization function *makeref (<obj_type1>, <obj_type2>)*, which creates relations from all instances of *obj_type1* to all instances of *obj_type2*. Figure 5.5 shows an initialization of the known attitude change process with one instance of the object type *pop* and 1000 instances of *person*. The attributes *x_1* and *outx_1* of the instance of *pop* are

DT	0.1
$STOP$	$count = 1000$
$BREAK$	$(count \,\%\, 100) = 0$

Figure 5.6: Initialization of a simulation run

explicitly initialized with 0 and an empty list ([]), whereas the instances of *att_1* get its values by evaluating the random function *uniform* a thousand times, creating uniform distributed random numbers between 0 and 1. The attributes x of object *pop* and *att* of object *person* do not need any initialization because they get values by evaluating their state functions during simulation.

5.3.2 Control and Execution of Simulation Runs

After finishing the initialization, a model is prepared for the execution of one simulation stepe A complete simulation run, however, requires additional informations, which the user must define in the provided simulation control window:

DT	:	the size of the time interval between two simulation steps
$STOP$	:	a conditional function for the simulation stop
$BREAK$	:	a conditional function for a programmed simulation interrupt

Modelling and simulation is a complex and highly interactive process, which requires a lot of simulation runs with different model structures and a high variation of model and simulation parameters. Compared to the old-fashioned batch-oriented simulation, the interruption of simulation runs by the modeller provides more flexibility, because the inspection of intermediate results is possible as well as the change of model parameter values, to simulate the impact of asynchron external events.

For the adminstration of simulation time, MIMOSE provides two "system variables", which can be used in model descriptions too:

count : the counter of simulation steps
time : the counter of simulation time
with $time = \sum_{i=0}^{count} DT_i$

Figure 5.6 shows an initialization example, in which the simulation run is interrupted each hundred time steps. After checking the simulation control correctly the experimental frame of MIMOSE provides a simulation window which, beside the *start* and *stop* of simulation runs, additionally allows an *interactive simulation interrupt* and *resume*, including the definition of *new initialization values.*

The model description together with the model initialization and the initialization of the simulation control describes a complete simulation model, which can be started by pressing the start button in the simulation window. Between two simulation steps the "system variables" *count, time, $count* and *$time* must be updated as well as the attribute values. To guarantee a correct evaluation in the next simulation step, each attribute value must be copied to the attribute in the simulation step before, if there exists one. For example, *x_1, outx_1* and *att_1* got the values of *x, outx* and *att*.

5.4 The Evolution of Technologies

5.4.1 The Modeling of Evolution on the Population (Macro) Level

Basic Assumptions

The basic concept of this model was developed by Igor V. Chernenko [Chernenko 1989]. He described a fixed number of three populations using three different kinds of production processes:

- 'acquisitive' hunters' and gathers' production.
- agrarian production,
- industrial production.

The focus of his analysis is which quantities of products are produced in the three sectors and the way the three sectors influence one another. The increment in existing quantities of goods can be written as the difference between the quantities of goods produced (gain) and the losses due to consumption and decay. The quantity of goods produced depends on

- the natural resources,
- the existing quantity of a certain product,
- the support of other products.

The following statements describe the basic assumptions of this model:

- The more natural resources are available, the greater the gains will be.
- The greater the quantity of a certain kind of good already is, the less the gains will be.
- The more auxiliary goods of other kinds are available, the greater are the gains.

The production process can be described as a differential equation:

$$\dot{s}_i = \mathbf{G}_i(\mathbf{s}) - \mathbf{L}_i(\mathbf{s}), \;\; 1 \leq i \leq I \tag{5.1}$$

$$\mathbf{G}_i(\mathbf{s}) = p_i s_i (r_i + \sum_{i \neq j} c_{ij} s_j - c_{ii} s_i) \tag{5.2}$$

Element	Explanation
s_i	(*size*) the quantity of goods produced in population i
$G_i(s)$	(*gain*) quantity of goods produced without losses
$L_i(s)$	(*loss*) the losses due to decay of goods produced in population i.
p_i	(*productivity*) the productivity of population i
r_i	(*resources*) natural resources (taken as regenerating) necessary to produce goods of kind i
c_{ij}	(*coupling*) the effects of other kind j of goods on kind i
c_{ii}	the saturation effect
T	(*totalSize*) the total production of goods

Table 5.1: Explanation of variables and parameters

The losses **L** are proportional to both the quantity of existing goods of this kind and the gains in goods of all kinds. Hence, the loss functions are written as follows:

$$\mathbf{L}_i(\mathbf{s}) = \frac{s_i}{T}\sum_h \mathbf{G}_h(\mathbf{s}) \tag{5.3}$$

The meaning of the variables and parameters of the equations is shown in Table 5.1. The total quantity of produced goods (or as we shall call it later on: the size of the subpopulations producing those goods) is constant. It is easy to see that $\dot{T} = 0$ for all $\mathbf{s}$, since

$$\begin{aligned}
\dot{T} &= \sum_i \dot{s}_i && (5.4)\\
&= \sum_i (\mathbf{G}_i(s) - \frac{s_i}{T}\sum_h \mathbf{G}_h(s)) && (5.5)\\
&= \sum_i \mathbf{G}_i(s) - \frac{\sum_i s_i}{T}\sum_h \mathbf{G}_h(s)) && (5.6)\\
&= 0 && (5.7)
\end{aligned}$$

This means, a population can grow (or expand their production) only at the expense of others.

Chernenko speaks of three stationary states:

1. $s_1 = T$ $\quad s_2 = 0$ $\quad s_3 = 0$
2. $s_1 = \frac{p_2T+(p_1r_1-p_2r_2)}{p_1+p_2}$ $\quad s_2 = \frac{p_1T-(p_1r_1-p_2r_2)}{p_1+p_2}$ $\quad s_3 = 0$
3. $s_1 = r_1$ $\quad s_2 = \frac{r_2}{1-c\beta}$ $\quad s_3 = \frac{\beta r_2}{1-c\beta}$

Depending on the parameters, exactly one of these stationary states is stable[6].

Chernenko seems to suppose T to be a monotonously increasing time-dependent function. Because of this the system shows dynamic behavior. If for the first time

[6] An exact mathematical analysis is shown in [Troitzsch 1992]

T exceeds $r_1 - \frac{p_1}{p_2} r_2$, then state 1 loses its stability, and state 2 becomes stable and ultimately state 3 becomes stable.

The question is how and why does T change? There is no explanation for this by Chernenko, he just supposes that the system changes in the same way the evolution seems to have changed historically.

Including results achieved by Peter M. Allen [Allen 1976] – to whom Chernenko also refers – Klaus G. Troitzsch enlarged the idea of Chernenko's model. He does not only look at three fixed populations, but assumes a variable number of populations.

The simulation starts with one population and one production technology. New populations with new technologies more or even less 'effective' are stochastically created during the simulation process. Furthermore the model contains a mutation and selection process, in which some populations grow (at the expense of others), other populations become extinct (in the macro model this means: they become very small). By doing this, we describe a model with a variable number of differential equations during simulation.

At the population level, a problem arise: On the one hand a new population must have a very small initialization size because it does not make sense to start the evaluation of the differential equation for some population with zero. On the other hand no population become extinct in the macro model because the size of a population is always greater than zero. With a new population we have to increase T and we have to delete a population if the size of the population gets smaller than a fixed limit.

Development of the MIMOSE Model

How can this model be defined using MIMOSE?

In the beginning there is a variable number of quite similar populations. Each population corresponds to the differential equation as shown in equation (5.1). We describe this by a single object type called `pop`. During the model initialization we create the concrete (starting) population. Furthermore we have to create and to delete concrete populations during simulation time.

We assume the birth of a new population as an exogenous influence. Therefore we need another object type called `society`.

In Figure (5.7)[7] the dependence between `pop` and `society` is shown by arrows. The society refers to all populations, the populations all refer to the society and to one another. Now we translate Figure (5.7) into a MIMOSE description (As far as possible we use the labels of equations (5.2) and (5.3) as attribute names):

```
society :=
{ popKey        : inti:= ...
  subpops       : list of pop
                := concat (subpops1,
                           copy(head(subpops1), (time >= nextEvent_1),
                                [ /* initialization */] ));
  subpops1      : list of pop
                := delete( subpops_1, sel(fct $1 to $1.size_1 < 0.05 end,
```

[7] $a \rightarrow b$ means: attributes from a are used by b

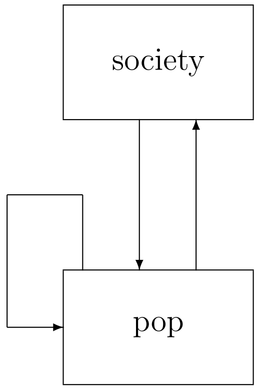

Figure 5.7: object types of the evolution model

```
                                                  subpops_1));
 ...
  T             : real := ... ;
};
pop :=
{ soc           : list of society;
  otherPops     : list of pop := makeref(pop,pop);
  key           : int;
  r             : real;
  p             : real;
  sizeList      : list of real := ... ;
  c             : list of real := ... ;
  G             : real := ... ;
  L             : real := ... ;
  ...
  s             : real := ... ;
}
```

The attributes `subpops` and `subpops1` describe the birth and death process of an concrete population by using the functions `copy` and `delete`. A new population is initialized with stochastical values.

We still have to include state transition functions to complete the model description. The differential equation (5.1) can be written as:

```
s        : real
         := s_1 + DT * (G - L);
```

`G` and `L` are calculated by equation (5.2) and (5.3)

```
G       : real
        := p * s_1 * (r + skl(c, otherPops.s_1)
                        - (2 * cList[key] * s_1 ));
```

```
L        : real
         := (s_1 / soc.T_1) * pluslist(otherPops.G);
```

The term `skl(c, otherPops.s_1)` corresponds to $\sum_{i=j} c_{ij} s_j$. `c` is the list of coupling parameters c_{ij} of population i, `otherPops` is the relation attribute from a single population to the list of all populations. `otherPops.s_1` is the list of the population size (s_i) in the time step before.

On the society level we have to supervise the birth of new populations. The attribute `nextEvent` defines the 'birthday' of a new population.

```
nextEvent  : real
           := (time + expon(2, 2.0)) if time >= nextEvent_1
                                     else nextEvent_1  ;
```

`nextEvent` is exponentially distributed. The attribute `popKey` is necessary to initialize a new population.

To compute the total production T

```
T          :  real
           := pluslist(sel( fct $1 to $1 > 0.05 end,subpops.s));
```

we use the higher order function `sel`, which means the selection of all populations with a size greater than 0.05.

The complete MIMOSE model is shown in chapter 5.5.

5.4.2 The Modeling of Evolution on the Individual (Micro) Level

Theory

Still there is no explanation for the growth of T. It seems to be more natural to model the population growth with the help of individual probabilities to be born or to die within the next time step. Therefore we have to model individuals (*persons*) with the following properties:

- at every pointnotonime an individual belongs to exactly one population
- an individual can change from one population to another
- the individual birth and death probabilities depend on the population the individual belongs to.

For the sake of simplicity, we shall model our individuals as if they reproduce themselves asexually. To make our micro and macro models as similar as possible, we formulate the birth probabilities with the positive parts and the death probabilities with the negative parts of the gains and losses of equation (5.1), i.e. the individual reproducing probability in population i is:

$$birth_i^{prob}(s) = \nu \left[p_i(r_i + \sum_{i \neq j} c_{ij} s_j) + \frac{1}{T} \sum_h p_{hj} c_{hh} s_h^2 \right] \tag{5.8}$$

and the individual death probability in population i is

$$death_i^{prob}(s) = \nu \left[p_i c_{ii} s_i + \frac{1}{T} \sum_h p_h s_h (r_i + \sum_{i \neq j} c_{ij} s_j) \right] \tag{5.9}$$

where ν is a flexible parameter which makes sure that all probabilities are within [0,1] at all times; the lower ν is, the slower the population will grow and decay.

On an average (see equation 5.10 and 5.7), birth and death are equally probable in the whole system.

$$\sum_i s_i birth_i^{prob}(s) = \sum_i s_i death_i^{prob}(s) \tag{5.10}$$

Furthermore, we have to describe the mechanism how an individual leaves his or her subpopulation to join another. For every population we need a vector of 'mutation' probabilities. $mig(i, j)$ in equation (5.11) defines the individual probability for the change from population i to population j. Thus, mobility is modeled as occurring from a population with low relative size to a population with high relative size, but note that even for an empty population j, $mig(i, j)$ is greater than zero.

$$mig(i, j) \;\; = \;\; \frac{e^{(\frac{s_j - s_i}{T})}}{\sum_j e^{(\frac{s_j - s_i}{T})}} \tag{5.11}$$

The creation of new populations is now modeled in two steps: First — as on the macro level— new populations arise, but with an initial size of zero. 'Mutations', however, occur when an individual leaves his or her population to join a new population. If no individual takes over the new technology, or all individuals left a population, this population will be removed.

Development of the MIMOSE Model

In comparison with the macro model we now have to describe two birth and death processes:

1. As in the macro model new populations are created and extinct populations are removed.

2. Persons can be born and can die

First we need a new object type called `'person'` (see Figure 5.8). Every person refers to a population he or she belongs to (*ownPop*). For the 'migration' process persons need references to all populations, therefore we use the dynamic reference function `makeref(person,pop)`, this means, the reference between `person` and `pop` will be calculated new in each simulation step. Because of the birth and death process the object type `person` needs two references to itself (`newPersons` and `delPersons`). The new object type reads:

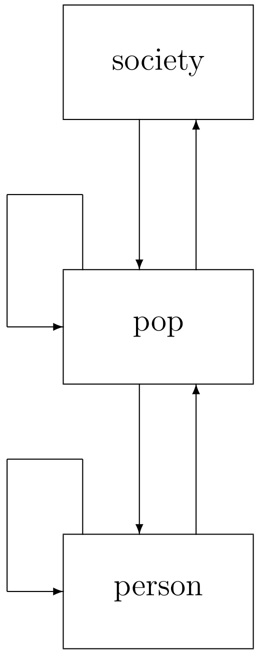

Figure 5.8: structure of the micro-model

```
person :=
{  death        :  int
                := 1 if (uniform( 1, 0.0, 1.0) < head(ownPop.deathProb))
                         || death_1  else death_2;
   birth        :  int
                := 1 if (uniform(2, 0.0, 1.0) < head(ownPop.birthProb))
                         && (~death_1) else 0;
   newPersons   :  list of person
                := copy (self(person), birth_1, [death_2 :: 0, death_1 :: 0]);
   delPersons   :  list of person
                := delete(self(person),
                          self(person) if death_2 else []);
   ownPop       :  list of pop := ... ;
   index        : ...
};
```

A new person will be born if in the time step before the attribute `birth` is true. It may happen that birth and death of a single person occur at the same time. Then a new person is created first and one simulation step later the old one is removed. Therefore a person will be removed if `death_2` becomes true.

The attributes `ownPop` and `index` regulate the change from one population to another one:

```
     ownPop       :  list of pop
                  := head((makeref(person,pop))[index]);
```

```
index       :  int
            := listpos(head(ownPop_1.mig),uniform(3, 0.0, 1.0)) + 1;
```

A random number `uniform(...)` is generated and compared to the transition rate vector (`ownPop_1.mig`). `index` is the reference index of the new population the person belongs to. Every person can change the population once in a simulation step.

The object type `society` is almost the same as in the macro model. We just changed the initial size for a new population to zero and the minimum size to one.

On the pop level we split up the attributes `G` and `L` in positive and negative terms to compute the birth and death probability (`birthProb` and `deathProb` see equation 5.10):

```
birthProb    :  real  := (gain1 + loss2) / NY;
deathProb    :  real  := (loss1 + gain2) / NY;
```

The parameter ν is modeled as global.

The attributes `mig` and `probSize` compute the transition rate vector (see equation 5.11).

```
probSize     :  real
             := s_1 / (soc.T_1);
mig          :  list of real
             := normdistrib(apply2(f1,otherPops.probSize,
                                   makelist(pop,probSize)));
```

The size `s` of a population now follows from the number of persons who join the population:

```
s         :  int
          := haselements (allPeople.ownPop, self(pop));
```

The MIMOSE function `haselements` counts all persons who refer to a special population.

The complete MIMOSE model is shown in chapter 5.6.

5.4.3 Simulation Results — an Example

It is not the purpose of this article to discuss the simulation results of this evolution model. Here, just a few remarks are given. For more detailed information see [Troitzsch 1992]. The run plotted in Figure (5.9) shows the total size T of the popu-

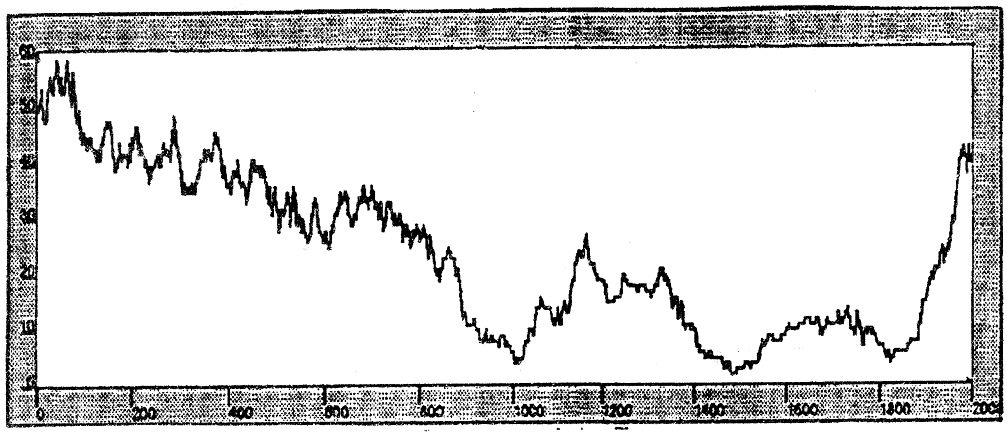

Figure 5.9: total size T

lations for about 2000 time steps. In all simulation runs carried out so far the macro state $T(t)$ seems to perform a random walk — which is not a surprise, since the sum of birth and death probabilities is zero. It may happen that a society dies out.

Figure (5.10) shows the actual number of populations for the same run. In this

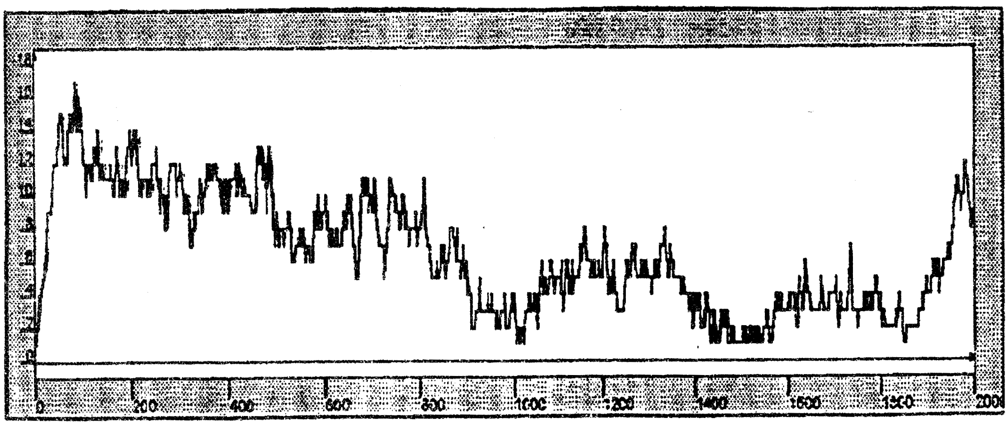

Figure 5.10: number of populations

run there seems to be a positive correlation between T and the number of populations but we could not verify this correlation in general. The principle 'that the greatest amount of life can be supported by great diversification of structure' (Charles Darwin [Darwin 1987]) does not always seem to apply for this version of an evolutionary process.

5.5 The Evolution of Technologies (Macro Model)

```
society :=
{ popKey       :  int          := (popKey_1 + 1)
                                  if time >= nextEvent_1 else popKey_1;
  subpops      :  list of pop
               := append(subpops1,
                          copy(head(subpops1), (time >= nextEvent_1),
                               [key        :: (popKey_1 + 1),
                                r          :: expon(1, 2.0),
                                p          :: uniform(2, 0.0,1.0),
                                cList_1    :: uniformlist(5,popKey,0.0,1.0),
                                s_1        :: uniform(3, 0.0,1.0);
                                sizeList_1 :: makelist((count - 1),0.0)]
                               ));
  subpops1     :  list of pop
               := delete( subpops_1, sel(fct $1 to $1.s_1 < 0.05 end,
                                         subpops_1));
  nextEvent    :  real
               := (time + expon(2, 2.0))
                  if time >= nextEvent_1 else nextEvent_1;
  T            :  real
               := pluslist(sel( fct $1 to $1 >= 0.05 end,subpops.s));
  totalSizes   :  list of int
               := append(totalSizes_1, T);
  nOfPops      :  list of int
               := append(nOfPops_1, length(subpops1));
};
pop :=
{ soc          :  list of society;
  otherPops    :  list of pop  := makeref(pop, pop);
  key          :  int;
  r            :  real;
  p            :  real;
  cList        :  list of real
               := cList_1 if (head(soc.popKey) = length(cList_1))
                          else append(cList_1, uniform(6, 0.0, 1.0));
  sizeList     :  list of real  := append(sizeList_1, s);
  c            :  list of real  := cList[otherPops.key];
  G            :  real
               := p * s_1 * (r + skl(c, otherPops.s_1)
                  - (2 * cList[key] * s_1));
  L            :  real          := (s_1 / soc.T_1) * pluslist(otherPops.G);
  s            :  real          := s_1 + DT * (G - L);
}
```

5.6 The Evolution of Technologies (Micro Model)

```
society :=
{ popKey       :  int          := (popKey_1 + 1)
                                  if time >= nextEvent_1 else popKey_1;
  subpops      :  list of pop
               := append (subpops1,
                          copy(head(subpops1), (time >= nextEvent_1),
                               [key        :: (popKey_1 + 1),
                                r          :: expon(1, 2.0),
                                p          :: uniform(2, 0.0,1.0),
                                cList_1    :: uniformlist(3,popKey,0.0,0.3),
                                s_1        :: 0,
                                sizeList_1 :: makelist((count - 1),0.0)]
                               ));
  subpops1     :  list of pop
               := delete( subpops_1, sel(fct $1 to $1.s_1 <= 0 end,
                                         subpops_1));
  nextEvent    :  real
               := (time + expon(2, 2.0))
                  if time >= nextEvent_1 else nextEvent_1  ;
  T            :  real
               := pluslist(sel( fct $1 to $1 > 0 end,subpops.s));
  totalSizes   :  list of int  := append(totalSizes_1, T);
  nOfPops      :  list of int  := append(nOfPops_1, length(subpops1));
};


pop :=
{ soc          :  list of society;
  otherPops    :  list of pop  := makeref(pop, pop);
  key          :  int;
  r            :  real;
  p            :  real;
  cList        :  list of real
               := cList_1 if (head(soc.popKey) = length(cList_1))
                          else append(cList_1, uniform(6, 0.0, 0.3));
  sizeList     :  list of real := append(sizeList_1, s);
  c            :  list of real
               := cList[otherPops.key];
  gain1        :  real
               := p * (r + skl(c, otherPops.s_1)
                  - (s_1 * cList[key]));
  gain2        :  real
               := p * s_1 * cList[key];
  gain3        :  real
               := gain2 * s_1;
  gain4        :  real
               := gain1 * s_1;
  loss1        :  real
```

```
                := pluslist(otherPops.gain4) / (soc.T_1);
  loss2         :  real
                := pluslist(otherPops.gain3) / (soc.T_1);
  birthProb     :  real
                := (gain1 + loss2) / NY;
  deathProb     :  real
                := (loss1 + gain2) / NY;
  probSize      :  real
                := s_1 / (soc.T_1);
  mig           :  list of real
                := normdistrib(apply2(f1,otherPops.probSize,
                                      makelist(pop,probSize)));
  s             :  int
                := haselements (allPeople.ownPop, self(pop));
  allPeople     :  list of person
                := sel (fct $1 to $1.death_1 = 0 end, makeref(pop, person));
}

person :=
{  death        :  int
                := 1 if (uniform( 1, 0.0, 1.0) < head(ownPop.deathProb))
                         || death_1
                     else death_2;
   birth        :  int
                := 1 if (uniform(2, 0.0, 1.0) < head(ownPop.birthProb))
                         && (~death_1)
                   else 0;
   newPersons   :  list of person
                := copy (self(person), birth_1, [death_2 :: 0, death_1 :: 0]);
   delPersons   :  list of person
                := delete(self(person),
                          self(person) if death_2 else []);
   ownPop       :  list of pop
                := head((makeref(person,pop))[index]);
   index        :  int
                := listpos(head(ownPop_1.mig),uniform(3, 0.0, 1.0)) + 1;
};

/************* function ****************/
f1        := fct a,b to exp(a - b) end;
```

5.7 Language Description

5.7.1 General View

The language design of MIMOSE is based on the structure of functional languages [Ebert 1986]. According to that, modelling with MIMOSE means *the definition and application of functions*. This is expressed by the general language structure, of which the grammar is shown in an extended Backus-Naur-Form (EBNF) in figure 5.11. The

definition	=	def ";" .
def	=	assign_expr [":" type] [":=" expr] .

Figure 5.11: definitions in MIMOSE

language is centered on the expression (*expr*), with which all modelling activities can be carried out (i.e. object type definition, function abstraction, function application) and which can be named with an identifier (*assign_expr*). Therefore, this identifier is merely an abbreviation of the expression. During a simulation multiple evaluations of functions are performed which describe the model behaviour.

5.7.2 Types

Each data object (i.e. literals, function applications, function abstractions) has a data type which determines a set of values an object may assume and the allowed operations on this set. Type definitions, given by the user, enable static type checking and the usage of type inference algorithms, to complete type informations [Schacht 1992]. Figure 5.12 shows the grammar for typing in MIMOSE. Beside the use of base data types (*real, int*), the identifiers of object types (*refid*) can be used as type identifiers too. These identifiers are treated as abbreviations of cartesian products, constructed by all the correspondent object attributes. The base data type (*ref*) describes the set of all object types. Additionally, the structured data type list (*list of* ...) is allowed. A function abstraction is typed by assigning the data types of its input and output parameters (*type "#" ... "⟶" type*). Furthermore type variables ($\uparrow$*id*) are allowed. Examples of typing in MIMOSE are given in the following subsection.

5.7.3 Expressions

As mentioned in subsection 5.7.1 the expression is the central language construct in MIMOSE. Its grammar in figure 5.13 shows the variety of its usage, which will be described in detail now:

Beside ordinary identifiers MIMOSE allows identifiers which denote object attributes from the past:

Identifier	*Examples*
id	`x : int := att1;`
id "_" num	`x : int := att1_2;`

The positive integer number which is added to a character string for the characterization of a "history" attribute will denote the *difference of time steps* of this attribute from the present, if each simulation step is assumed as the present. Figure 5.14 shows

```
type  =  "int"
      |  "real"
      |  "ref"
      |  refid
      |  ↑id
      |  "list" "of" type
      |  type "," type
      |  type "#" type
      |  type "⟶" type.
```

Figure 5.12: Types in MIMOSE

```
expr         =  literal
             |  assign_expr
             |  "[" sign_literal_tup_list "]"
             |  monop expr
             |  expr dyop expr
             |  id "(" expr_list ")"
             |  expr "if" expr "else" expr
             |  "case" expr "of" case_seq "end"
             |  fct_expr
             |  "{" def_seq "}"
             |  "≪" expr_list "≫"
             |  "(" expr ")" ["[" expr "]" ].

assign_expr  =  identifier ["[" [expr] "]" ] .

identifier   =  id [ "_" num ]
             |  id [ "_" num ] [ "." id ] [ "_" num ]

fct_expr     =  "fct" id_list "to" expr
                [ "where" def_seq ] "end" .
```

Figure 5.13: Expressions in MIMOSE

the relation between the absolute time scale, which is fixed on the "system variable" *count*, and the relative time scale.

According to the data types of MIMOSE, literals are integer or real numbers, from which lists can also be constructed.

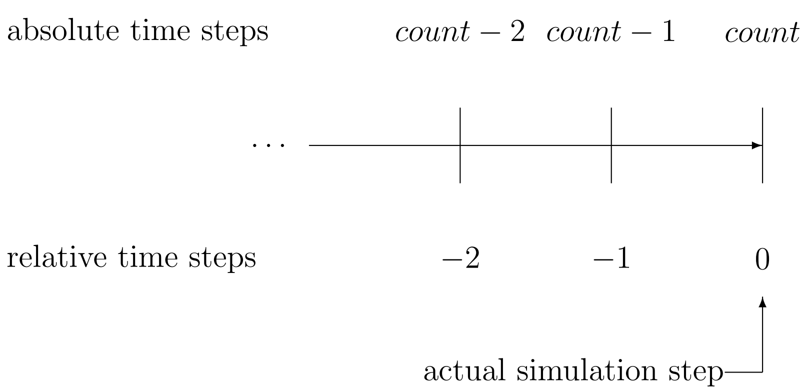

Figure 5.14: Absolute/relative time scale

Constants	*Examples*
literal	`x : int := 1;` `x : real := 2.0;`
“[” sign_literal_tup_list “]”	`x : list of list of int` `:= [[1, 2], [3, 4]];`

Beside some special functions (i.e selection of object attributes (*id* “.” *id*), the selection of a sublist (*id “[” literal “* { *“,” “literal”* } *]”*)), and the function application with one, two (prefix, infix) and more arguments are permitted. Additionally, the composition of functions and the condition expression allow structured expressions.

Funktion applications	*Examples*
id “. ” id	`x : real := y.att1;`
id “[” literal { “,” literal } “]”	`x : list of real := y[3,5];`
monop expr	`x : real := −y;`
expr dyop expr	`x : real := 2 + y;`
id “(” expr_list “)”	`x : real := sqrt (2 + y);`
expr “if” expr “else” expr	`x : real := 1 if a > b else 2;`

The definition of object types, which is required by the special purpose of MIMOSE, extends the usual scope of applicative languages. It allows the formal description (structure, behaviour) of identified entities of reality. The usage of object type definitions within an object type is not permitted.

MIMOSE provides an abstraction mechanism for the extension of the simulation system by the user. Function definitions (including recursive definitions) can be used globally as well as within an object type.

Object type definitions/Function definitions	*Examples*
"{" def_seq "}"	`obj := { att1 : int := att1_1 + 3; att2 : real := a + b };`
"fct" id_list "to" expr ["where" def_seq] "end"	`f : int # int ⟶ int := fct a, b to c where c : int := a + b end;`

5.7.4 Base Functions

In MIMOSE a lot of base functions are already implemented, which can be used without definition (T1 = {*int, real*}, T2 = {*int, real, list of* T2}):

- *Arithmetic functions*
  These functions are defined by integer and real numbers. The result of a function, of which the arguments have different data types, is converted into a real number[8].

  add (+), sub (−), times (*), div (/), mod (%)
  : T1 × T1 ⟶ T1

  minus (−) : T1 ⟶ T1

  sin, asin, sinh, cos, acos, cosh, tan, tanh, exp, log, log10, sqrt, gamma
  : T1 ⟶ *real*

  round,trunc
  : T1 ⟶ *int*

- *Logical functions*
  The result of these functions is either 1 (TRUE) or 0 (FALSE), and each argument which does not equal 0 is treated as 1 (TRUE).

  and (&&), or (||)
  : T1 × T1 ⟶ *int*

  not (˜) : T1 ⟶ *int*

  lt (<), le (<=), gt (>), ge (>=), eq (=), ne (<>, ! =)
  : T1 × T1 ⟶ *int*

[8] Additional operators in brackets indicates the usage of that function in infix notation too.

- *List functions*
  Beside the construction of constant lists (i.e. [[1,2],[3,4]]) and the selection of sublists (y[3,2]), additional list functions are available.

head	:	*list of* T2 $\longrightarrow$ T2 result: first element of the input list
tail	:	*list of* T2 $\longrightarrow$ *list of* T2 result: input list without the first element
append	:	*list of* T2 $\times$ T2 $\longrightarrow$ *list of* T2 ,
	:	T2 $\times$ *list of* T2 $\longrightarrow$ *list of* T2 result: concatenation of the input list and the element
concat	:	*list of* T2 $\times$ *list of* T2 $\longrightarrow$ *list of* T2 result: concatenation of the two input lists
makelist	:	*int* $\times$ T2 $\longrightarrow$ *list of* T2 result: creation of a new list
length	:	*list of* T2 $\longrightarrow$ *int* result: number of list elements
isempty	:	*list of* T2 $\longrightarrow$ *int* result: TRUE if the list has no elements
haselements	:	*list of* T1 $\times$ T1 $\longrightarrow$ *int* result: counts how often the second argument appears in a list
listpos	:	*list of* T1 $\times$ T1 $\longrightarrow$ *int* result: first position of an element in a list
pluslist	:	*list of* T1 $\longrightarrow$ T1 result: sum of the list elements
timeslist	:	*list of* T1 $\longrightarrow$ T1 result: product of the list elements
minlist	:	*list of* T1 $\longrightarrow$ T1 result: minimal list element
maxlist	:	*list of* T1 $\longrightarrow$ T1 result: maximal list element
skl	:	*list of* T1 $\times$ *list of* T1 $\longrightarrow$ T1 result: inner product
normdistrib	:	*list of* T1 $\longrightarrow$ *list of* real result: standardization of a list

elem : *list of* T2 $\times$ *int* $\longrightarrow$ T2
result: list element with index given in the second argument

Lists are also suitable to define matrix functions, because two-dimensional lists can be treated as matrices.

matmult : *list of list of* T2 $\times$ *list of list of* T2 $\longrightarrow$ *list of list of* T2
result: multiplication of the two matrices

invert : *list of list of* T2 $\longrightarrow$ *list of list of* T2
result: inversion of the input matrix

pdf : *list of list of* real $\longrightarrow$ *list of list of* real
pdfextrem : *list of list of* real $\times$ *list of list of* real $\longrightarrow$ *list of list of* real
result: *pdf* and *pdfextrem* estimate non-normal probability density functions [Herlitzius 1989]

- *"Higher order" Functions*
Function definitions in MIMOSE are data objects just as constants or function applications. This leads to the concept of "higher order" functions, in which arguments and values could be functions as well. This kind of functions are especially well suited for the implementation of control structures (i.e. loops). Some "higher order" functions have already been implemented as base functions [Strotmann 1991].

  - *sel*
    sel : (T2 $\longrightarrow$ *int*) $\times$ *list of* T2 $\longrightarrow$ *list of* T2
    result: list of elements from the input list, which fulfills the condition function in the second argument.

    The semantic of *sel* can be described in MIMOSE as follows:

```
sel := fct f, a to
       [] if isempty(a)
          else append (head (a), sel (f, tail (a)))
               if f(head (a))
               else sel (f, tail (a)))
       end;
```

  - *apply*
    apply : (T1 $\longrightarrow$ *T2*) $\times$ *list of* T1 $\longrightarrow$ *list of* T2
    result: applies the function to each list element.

    The semantic of *apply* can be described in MIMOSE as follows:

```
apply := fct f, a to
          [] if isempty(a)
           else append (f (head(a)), apply (f, tail (a)))
        end;
```

– *apply2*

apply2 : (T1 $\longrightarrow$ *T1*) $\times$ *list of* T1 $\times$ *list of* T1 $\longrightarrow$ *list of* T1
result: connect both lists with the help of the function.

The semantic of *apply2* can be described in MIMOSE as follows:

```
apply2 := fct f, a, b to
          [] if isempty(a)
           else append (f (head(a),head(b)),
                            apply2 (f, tail (a), tail (b)))
        end;
```

– *insert*

insert : (T1 $\longrightarrow$ *T1*) $\times$ *list of* T1 $\longrightarrow$ *list of* T1
result: connect the elements of the lists with the help of the function.

The semantic of *insert* can be described in MIMOSE as follows:

```
insert := fct f, a to
          [] if isempty(a)
           else append (f (head(a),head(tail(a))),
                            insert (f, tail(tail (a))))
        end;
```

dgl : (*list of* real $\longrightarrow$ *list of real*) $\times$ *list of* real $\times$ int $\times$ int $\longrightarrow$ *list of* real
result: dgl describes a runge-kutta method to approximate a differntial equations [Strotmann 1991]

- *The generation of random numbers*
The lack of variables and the use of identifiers as abbreviations of expressions leads to *referential transparency* of MIMOSE models. This means, that the evaluation of an expression always gives the same result, regardless whether the expression itself or the corresponding identifier is used. This very usefull characteristic becomes disadvantageous when functions for the generation of random numbers are used. Therefore every random function has a selector (first argument) to differ the function applications. According to the type of the input parameters, the result will be real or integer numbers (i.e. *uniform* (1, 0.0, 1.0)) or integer (i.e. *uniform* (2, 0, 1)). There are always two functions for a special kind of random numbers. A function (i.e. *uniform*) to create a single number and a function (i.e. *uniformlist*) to create a list of random numbers. In this case the additional second parameter (*int*) always determines the length of the list.

uniform : *int* $\times$ T1 $\times$ T1 $\longrightarrow$ T1
result: The random number will be choosen from an interval, delimited by the values of the second and third argument.

normal : *int* $\times$ T1 $\times$ T1 $\longrightarrow$ T1
result: A normal distributed (mean: second argument; standard derivation: third argument) random number will be choosen.

expon : *int* $\times$ T1 $\longrightarrow$ T1
result: A exponential distributed (mean: second argument) random number will be choosen.

gammadist : *int* $\times$ T1 $\longrightarrow$ T1
result: A gamma distributed (order: second argument) random number will be choosen.

chisquare : *int* $\times$ T1 $\longrightarrow$ T1
result: A chi-square distributed (degrees of freedom: second argument) random number will be choosen.

prob : *int* $\times$ real $\longrightarrow$ int
result: TRUE if a random number between 0 and 1 is less or equal than the argument, else FALSE

- *The creation of relations between object types*
  In MIMOSE the relations between object types will be expressed by the definition of attributes. Therefore, the creation of such an attribute, requires some special kind of functions.

  makeref : $< \text{id1} > \times < \text{id2} > \longrightarrow$ *list of* id2
  result: references from *each* element of object type $< \text{id1} >$ to *all* elements of object type $< \text{id2} >$ (see figure 5.5 in subsection 5.3.1).

  self : $< \text{id1} > \longrightarrow$ *list of* id1
  result: references from *each* element of object type $< \text{id1} >$ to itself. The result is a list with a single element.

- *Functions to create and to delete objects during simulation time*
  To model birth and death processes it is neccessary to create and to delete concrete objects during simulation time (for example see section 5.4).

copy : *ref* $\times$ *int* $\times$ *list of write_expr* $\longrightarrow$ *list of* ref
result: If the second parameter is true a new object will be created by copiing the object the first parameter refers to. It is possible to reinitialize the values of the new object (*list of write_expr*)

delete : *list of ref* $\times$ *list of ref* $\longrightarrow$ *list of ref*
result: From the list of objects (refered in the first parameter) objects will be deleted, which are also refered in the second reference list. Result is the list of remaining objects.

5.8 Conclusions

MIMOSE introduces a new modelling technique, which improves the transparence of the "model programming process" and makes model descriptions and even the corresponding simulation results easier to understand. As shown in the development of the model of technological evolution, we saw, that MIMOSE is able to describe quite complex models. Compared to standard programming languages like PASCAL or C the simulation model is short and clear. A comparable C program for this model written by us, contains about 15000 lines of code, whereas the corresponding MIMOSE model description comprises only two pages of code. Even more important, the enlargement of a MIMOSE model causes no problems. In our example it was easy to generate the micro model from the macro version. Because of this, models and model versions can be easily compared and become discussible by colleaeagues, the scientific community, etc.

MIMOSE is a new and so far unusual way of modelling. Like every simulation or programming language it takes some time to get used to. But we think it is worth while.

The prototype version of MIMOSE, which is written in the C language and implemented on a SUN-workstation, was finished in Summer 1990. The software development was additionally supported by the use of a module for the implementation of graphs and graph algorithms [Ebert 1987], which has already been used for the implementation of other functional languages. The development of the menu- and window-oriented user interface, which is implemented by using X-WINDOWS, was completed at the end of 1990 [Hecken/Schulten 1990]. Furthermore, approximation functions for the description and simulation of (quasi-)continuous models [Strotmann 1991], for the dynamic creation and deletion of model objects, and a module for the graphic presentation of simulation results have been developed recently. The current work is concentrated on debugging and on the test of MIMOSE on different simulation models, which are concerned with problems of the social science research (collective phenomena (i.e. migration [Weidlich/Haag 1983]), chaotic behaviour [Troitzsch 1990], cooperation [Axelrod 1987]). The actual Release 1.7 of MIMOSE is available on

- SUN 3/80
- SUN 4/SPARC

- MIPS
- UNIX-386/486 PC (X11R4, INTERACTIVE UNIX)

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Chapter 6

Andreas Klee and Klaus G. Troitzsch, Koblenz: Chaotic Behaviour in Social Systems: Modeling with GEMM

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Abstract

This paper[1] formalizes the behaviour of three interacting populations using two different modelling approaches, once on the macro level, once both on the individuals' level and on the populations' level. In both cases the social system displays chaotic behaviour for certain combinations of the parameters. Combinations of the parameters, however, for which chaotic behaviour occurs must be interpreted in a completely different manner. Once more (cf. e.g. [Rap84, p. 22] [Bar73, p. 307]) it is made obvious that different approaches to formalizing the same verbal hypotheses will yield different results in most cases.

The paper applies the MIMOSE model specification technique and its user interface GEMM, thus showing that MIMOSE is appropriate to meet the modeller's needs even in

[1]The most important parts of this paper were also presented at the International Conference on Social Science Methodology which was organized by the Dipartimento di Politica Sociale of the Universitá di Trento and the Research Council 33 (Logic and Metholology) of the International Sociological Association at Trento, June 22–26, 1992.

cases where several different types of objects have to be defined and linked together.

Additionally and on the basis of the micro model we show the decomposition of the MIMOSE modeling process into different phases and the graphical support of these steps by GEMM.

6.1 GEMM: a Tool for Modeling

GEMM (a **G**raphic **E**ditor for Visual Progrogramming on the Basis of the **M**odeling Language MIMOSE, see [Kl91]) was developed within the scope of the DFG-funded project MIMOSE[2] [Mö90]. The aim of MIMOSE is the design and development of software for the modelling and simulation of individual behaviour in interacting populations. GEMM supports the essential stage of the modelling process within the modelling and simulation cycle by decomposing this process into several phases. During these construction steps the modeler is given assistance by graphical means and techniques[3]. Models which are visually constructed with GEMM are then automatically transformed into analogous MIMOSE models. GEMM forms a supplement to FIBS [HS90], the user interface of the modelling and simulation software system MIMOSE, and was developed with the operating system UNIX, the programming language C and the X-Windows system.

6.2 An Example of an Opinion Formation Process Model

The following model does not claim to be a realistic reproduction of reality, in Section 6.3 it serves only as an example to demonstrate the capabilities of GEMM. The subsequent part of this section is a revised and extended version of [Tr90].

6.2.1 Hypotheses

We suppose a society consisting of three homogeneous subpopulations of, say, citizens, journalists, and scientists, discussing and working on a technological problem like the civil use of nuclear energy[4]. Among citizens, there are supporters and opponents, among journalists, there are some who write about nuclear energy topics and some who do not, and among scientists, we may find some who are doing research in better techniques ("innovation") and some who are doing research in technological risks connected with the civil exploitation of nuclear energy.

We suppose the following hypotheses:

[2]**MI**cro- and multilevel **MO**delling **S**oftwar**E**

[3]The reader interested in the psychological and physiological backgrounds of the graphical support is recommended to read [Ro88] and [Eb88].

[4]Of course, any other technological or non-technological problem in which the mass public, the media and the scientific community are involved could be supposed as well, but when I first presented this mathematical model to my students in April 1986 everybody talked about the Chernobyl accident. For a related model of nuclear energy acceptance see [Sei87, pp. 108–129] who introduces a system of only two nonlinear differential equations where chaotic behaviour cannot be observed.

X1: The more supporters (the more opponents) there are among the citizens, the slower will the rise of the number of supporters (opponents) be.

X2: The more (the less) the press reports about innovations (risks) of civil nuclear energy exploitation, the faster will the number of supporters among the citizens rise or the slower will the number of supporters fall; the more (the less) the press reports about risks (innovations) of civil nuclear energy exploitation, the faster will the number of opponents among the citizens rise or the slower will the number of opponents fall.

Y0: Exogenous influences lead to a continuous rise of the number of journalists writing about nuclear energy topics.

Y1: The more journalists write about nuclear energy topics, the slower is the rise of the number of journalists writing about these topics.

Y2: The more supporters there are among the citizens and the higher the number of scientists doing research in nuclear energy innovations is, the slower will the rise of the number of journalists writing about nuclear energy be; the more supporters there are and the higher the number of scientists doing risk research is, the faster will the rise of the number of journalists writing about nuclear energy be since then there is a conflict between mass public and scientific community which is worth reporting.

Z1: The more scientists work on nuclear energy innovations, the more scientists will change their research field to risk research.

Z2: The more supporters there are, the more innovation research and the less risk research will be done.

We are now going to formalize our qualitative hypotheses — qualitative taken in the sense that we used only ordering relations — quantitatively, i.e. we translate the ordinally scaled attributes of our three populations into attributes that are at least interval scaled. This is always a dangerous attempt since there is no *a priori* possibility to discriminate between different translations of ordinally scaled attributes into interval scaled attributes. This is why we present two different approaches in the following subsections of this section.

6.2.2 Macro Model

We first try to formalize our hypotheses using a macro approach and defining the following variables:

$x(t)$**:** the difference between the numbers of supporters and opponents among the citizens at time t,

$y(t)$**:** the difference between the numbers of journalists writing and not writing about nuclear energy topics,

$z(t)$: the difference between the numbers of scientists doing their research in innovations in nuclear energy exploitation and nuclear energy risks, respectively.

We soon arrive at the following system of differential equations:

$$\begin{array}{rclrrrrr} \dot{x} & = & & - & \gamma x & + & \kappa yz \\ \dot{y} & = & \beta y_* & - & \beta y & - & \varepsilon xz \\ \dot{z} & = & & - & \alpha z & + & \delta x \\ & & (0) & & (1) & & (2) \end{array} \tag{6.1}$$

In this system of differential equations the terms in column (1) represent the contributions of hypotheses X1, Y1, and Z1, respectively, and the terms in column (2) represent the contributions of hypotheses X2, Y2, and Z2, respectively, where we take notice of the fact that a formulation like "the more ... and the faster ..." has been translated into a product term like xz with a positive (negative) sign if the conclusion of the hypothesis reads "the faster ..." ("the slower ..."). The single term in column (0) represents hypothesis Y0 which is translated into a positive constant; a negative constant would mean that exogenous influences lead to a continuous decrease in reporting intensity.

This model has first been formalized by Haken [Hak82, pp. 336–337] who stresses its similarity with the laser equations [Hak82, pp. 236–237] but neglects the fact that this model displays chaotic behaviour since it is closely related to the famous Lorenz model[5] [Lor63] [Hak82, pp. 342–350] [Sch84, pp. 175–179] [Nic86, pp. 291–297].

Our society as it is modeled by the system of differential equations 6.1 displays the following behaviour:

$\dot{x}$, $\dot{y}$, and $\dot{z}$ vanish at the same time, i.e. the system is at equilibrium, for

$$x_{01} = 0 \qquad y_{01} = y_* \qquad z_{01} = 0 \tag{6.2}$$

and for

$$\begin{aligned} x_{02,03} &= \pm\sqrt{\frac{\alpha\beta}{\delta\varepsilon}\left(y_* - \frac{\alpha\gamma}{\delta\kappa}\right)} \\ y_{02,03} &= \frac{\alpha\gamma}{\delta\kappa} \\ z_{02,03} &= \pm\sqrt{\frac{\beta\delta}{\alpha\varepsilon}\left(y_* - \frac{\alpha\gamma}{\delta\kappa}\right)} \end{aligned} \tag{6.3}$$

The second and third equilibria are only valid for $y_* \geq \frac{\alpha\gamma}{\delta\kappa}$.

Depending on the parameters, the system will

[5]In Lorenz's original paper, the names of the variables and parameters are X, Y, Z, σ, b, and r. These and our variables and parameters may be transformed by the following equations:

$$z = \frac{X}{b} \qquad x = \frac{\sigma}{b^2}Y \qquad z = (r - Z)\frac{\sigma}{b^2}$$

$$\alpha = \frac{\sigma}{b} \qquad y_* = r\frac{\sigma}{b^2} \qquad \gamma = \frac{1}{b}$$

$$\beta = \delta = \varepsilon = \kappa = 1$$

In Lorenz's paper, the numerical example with $\sigma = 10$, $b = \frac{8}{3}$, and various r is discussed.

$0 < y_* < \frac{\alpha\gamma}{\delta\kappa}$	approach the first equilibrium (which is then the only equilibrium)
$\frac{\alpha\gamma}{\delta\kappa} < y_* < y_1$	approach the second or the third equilibrium, depending on the initial state
$y_1 < y_* < \frac{\alpha^2}{\delta\kappa}\frac{\alpha+\beta+3\gamma}{\alpha-\beta-\gamma}$	approach the second or the third equilibrium, depending on the initial state, but now on a spiral
$y_* \geq \frac{\alpha^2}{\delta\kappa}\frac{\alpha+\beta+3\gamma}{\alpha-\beta-\gamma}$	move chaotically around both the second and the third equilibria.

Our model displays several different types of behaviour, depending on whether one of the parameters is smaller or greater than a certain threshold which in turn depends on the other parameters. Either we have the case of a single equilibrium which is reached from any initial state, or we have the case of two equilibria one of which is reached depending on the initial state (with two subcases one of which leads to damped oscillation), or we have the case of chaotic behaviour in which the system moves on a chaotic attractor and never reaches equilibrium.

It is interesting to see that the threshold before chaos — $\frac{\alpha^2}{\delta\kappa}\frac{\alpha+\beta+3\gamma}{\alpha-\beta-\gamma}$ — does not depend on the coupling constants of hypotheses X2, Y2, and Z2 (save for scaling), but only on the damping constants of hypotheses X1, Y1, and Z1. With fixed damping constants, it is thus the exogenous intensity of information on nuclear energy problems that may lead to chaotic behaviour of the system once a certain threshold is exceeded.

A formulation of this macro model in the model description language MIMOSE [MSF92] is shown in section 6.5.

6.2.3 Micro Model

After having analyzed the macro development of majorities in each of the three populations our model society consists of we shall now switch over to an approach which also allows to model the individual development of each member of the three populations. Here, for the members of each population transition probabilities have to be defined. These transition probabilities are the following:

$\mu^x_{+\leftarrow-}$ the probability that an opponent becomes a supporter,

$\mu^x_{-\leftarrow+}$ the probability that a supporter becomes an opponent,

$\mu^y_{+\leftarrow-}$ the probability that a journalist begins to write about nuclear energy problems,

$\mu^y_{-\leftarrow+}$ the probability that a journalist ceases to write about nuclear energy problems,

$\mu^z_{+\leftarrow-}$ the probability that a scientist begins to do his research on innovations in nuclear energy exploitation,

$\mu^z_{-\leftarrow+}$ the probability that a scientist begins to do his research on nuclear energy risks.

All six transition probabilities are functions of the macrovariables $x(t)$, $y(t)$, and $z(t)$ or $n_{x+}(t), \ldots n_{z-}(t)$ which must be redefined in the following manner:

$n_{x+}(t)$ the number of supporters among the citizens,

$n_{x-}(t)$ the number of opponents among the citizens,

$n_{y+}(t)$ the number of journalists writing about nuclear energy problems,

$n_{y-}(t)$ the number of journalists not writing about nuclear energy problems,

$n_{z+}(t)$ the number of scientists doing their research in nuclear technology innovations,

$n_{z-}(t)$ the number of scientists doing their research in nuclear technology risks.

$$N_x = n_{x+}(t) + n_{x-}(t) \tag{6.4}$$

$$n_x(t) = \frac{n_{x+}(t) - n_{x-}(t)}{2} \tag{6.5}$$

$$x(t) = \frac{n_x(t)}{N_x} \tag{6.6}$$

and so on, such that $-1 \leq x(t) \leq 1$ holds as in Weidlich's and Haag's modelling of opinion formation and migration processes in [WH83].

It is obvious that our hypotheses from subsection 6.2.1 may be formalized in the following manner (with ν a flexibility parameter):

$$\mu^x_{+\leftarrow-} = \nu \exp\left[-\gamma x + \kappa yz\right] \tag{6.7}$$

$$\mu^x_{-\leftarrow+} = \nu \exp\left[-(-\gamma x + \kappa yz)\right] \tag{6.8}$$

$$\mu^y_{+\leftarrow-} = \nu \exp\left\{\beta(y_* - y) - \varepsilon xz\right\} \tag{6.9}$$

$$\mu^y_{-\leftarrow+} = \nu \exp\left\{-[(\beta(y_* - y) - \varepsilon xz]\right\} \tag{6.10}$$

$$\mu^z_{+\leftarrow-} = \nu \exp\left[-\alpha z + \delta x\right] \tag{6.11}$$

$$\mu^z_{-\leftarrow+} = \nu \exp\left[-(-\alpha z + \delta x)\right] \tag{6.12}$$

With this formalization we again follow the approach adopted by Weidlich and Haag; of course, different formalizations of the transition probabilities are possible. For a survey on such formalizations see for example [LW81, pp. 131–147, esp. p. 135] where Lumsden and Wilson present some "human assimilation functions" of which ours are nonlinear "nonsaturable 'trend watching'" functions. Of course, our case is a little more complicated than Lumsden's and Wilson's since we are looking at three different interacting populations whose members are "trend watchers" internally, but acting just against the trend of their respective population; the coupling between the populations has no match in Lumsden's and Wilson's concept.[6]

Thus we have an inhomogeneous Marcov process on the individual level. With the help of the master equation and by generalizing the derivations in [WH83, pp. 90–99], the stationary probability density function of the macro state variables x, y, and z

[6] Lumsden and Wilson mention an example ("fashion in women's dress") where two populations, "couturiers" and women interact in their attempt at leading in innovation and at gaining status, respectively [LW81, pp. 169–176]. If we introduce a third population — the editors of women's magazines — we might have another application of our model — Lumsden and Wilson seem to believe that the data on women's fashions originally gathered by Richardson and Kroeber [RK40] for the period from 1788 to 1936 may have been generated by a chaotic process.

and its maxima may be derived (which is only true for transition probabilities of the Weidlich-Haag type, for other types of transition probabilities a stochastic simulation seems to be the only way to get results, see for example [FS90, p. 348–349]). The latter — which for the sake of simplicity are again called x, y, and z instead of x_{max} etc. — obey the following system of differential equations:

$$\begin{aligned} \dot{x} &= \sinh u - x \cosh u & u &= -\gamma x + \kappa y z \\ \dot{y} &= \sinh v - y \cosh v & v &= \beta(y_* - y) - \varepsilon x z \\ \dot{z} &= \sinh w - z \cosh w & w &= -\alpha z + \delta x \end{aligned} \tag{6.13}$$

This system of differential equations (which obviously cannot be solved analytically) displays the following behaviour:

For $\beta > 0$ we have either one or three fixed points, depending on y_*. One fixed point (which is stable if and only if it is the only fixed point) has the coordinates $(0, y_{01}, 0)$ where y_{01} is the solution of the equation

$$\beta(y_* - y_1) = \text{ar tanh } y_1 \tag{6.14}$$

The coordinates of the two other fixed points — if they exist — are (x_{02}, y_{02}, z_{02}) and $(-x_{02}, y_{02}, -z_{02})$, respectively, i.e. their positions are symmetrical with respect to the x-z-plane. In so far we find conditions similar to the modified Lorenz model of subsection 6.2.2. Numerical calculations, however, show that these fixed points are stable (see also figure 6.4). With all parametes save y_* fixed and increasing y_*, the eigenvalues of the Jacobian at these two fixed points are complex with negative real parts of increasing absolute value.

For negative y_* we always have one single fixed point which is a spiral sink. Also in so far we have conditions similar to those of the modified Lorenz model.

Thus our micro model does not display chaotic behaviour for $\beta > 0$ and/or $y_* > 0$. In these three quadrants of β-y_* parameter subspace, the results of macro and micro models are not even qualitatively equal.

For $\beta < 0$ the model changes qualitatively. Under this condition, journalists behave like the persons in Weidlich's and Haag's simple opinion formation model [WH83, pp. 18–53], i.e. they adapt to the actual majority among the journalists, while in our original hypothesis Y1 we postulated a saturation effect among the journalists.

In this case, chaotic behaviour appears in the micro model, too. A necessary, but not yet sufficient condition for chaotic behaviour is:

$$\beta < -1 \qquad \text{and} \qquad y_* < 0$$

since for negative β the system of differential equations may have more than one fixed points on the y-axis, depending on whether the line $f_1 = \beta(y_* - y)$ intersects the curve $f_2 = \text{ar tanh } y$ once, twice, or three times. The threshold case — one intersection and one tangent point — is defined by

$$f_1(y_1) = f_2(y_1) \qquad \text{and} \qquad \frac{df_1}{dy}(y_1) = \frac{df_2}{dy}(y_1)$$

i.e. for

$$-\beta = \frac{1}{1 - y_1^2}$$

The tangent point now has coordinates $(y_1, \mathrm{ar\ tanh}\ y_1)$ in y-f-space, where

$$y_1 = \sqrt{1 + \frac{1}{\beta}}$$

Since the tangent obeys $f_1 = \beta(y_* - y)$ we can calculate that y_a (the value of y_* leading to exactly two fixed points on the y-axis) must fulfill the condition

$$\beta \left[y_a - \sqrt{1 + \frac{1}{\beta}} \right] = \mathrm{ar\ tanh} \sqrt{1 + \frac{1}{\beta}} \tag{6.15}$$

which holds for

$$y_a = \frac{1}{\beta} \mathrm{ar\ tanh} \sqrt{1 + \frac{1}{\beta}} + \sqrt{1 + \frac{1}{\beta}} \tag{6.16}$$

At the same time, equation 6.15 yields $1 > 1 + \frac{1}{\beta} > 0$, i.e. $\beta < -1$.

For $y_* > y_a$ the system of differential equations has exactly one fixed point which is a spiral sink, for $-y_a < y_* < y_a$, it has three fixed points on the y-axis (and two additional fixed points off the y-axis), for $-y_a > y_* > y_c$ there is only one fixed point on the y-axis and two additional spiral saddles off the y-axis: the case of chaotic behaviour which is shown in figures 6.1 and 6.2 for the following parameters:

$$y_* = -0.4 \quad \beta = -1.2 \quad \alpha = \gamma = 0.05 \quad \kappa = \varepsilon = \delta = 4.0$$

The attractor shown in these figures is rather similar to the well known Lorenz attractor; figure 6.3 shows the dependence of subsequent maxima of y on their predecessors; this mapping, too, makes it clear that the attractor of our system is strange.

For $y_* < y_c$ the chaotic behaviour disappears, and we find the same behaviour as for $y_* > 0$ and $\beta > 0$. Note that the calculation of y_c necessitates the evaluation of the Jacobian at the fixed points off the y-axis — which seems to be impossible.

Besides the regions of β-y_* parameter subspace which can at least partially be determined by mathematical analysis, this subspace contains regions which can only be discovered by numerical methods, i.e. by simulating the system of differential equations for varied parameter combinations, as has been done to prepare figure 6.4.

For $\gamma = \alpha = 0.01$, $\kappa = \varepsilon = \delta = 4$, the system of differential equations has different numbers and kinds of fixed points in different regions of the β-y_* parameter subspace shown in figure 6.4:

A : one spiral sink,

F : one sink (in the spandrel-shaped regions near the origin in quadrants 1 and 3),

B : one spiral sink and four saddles,

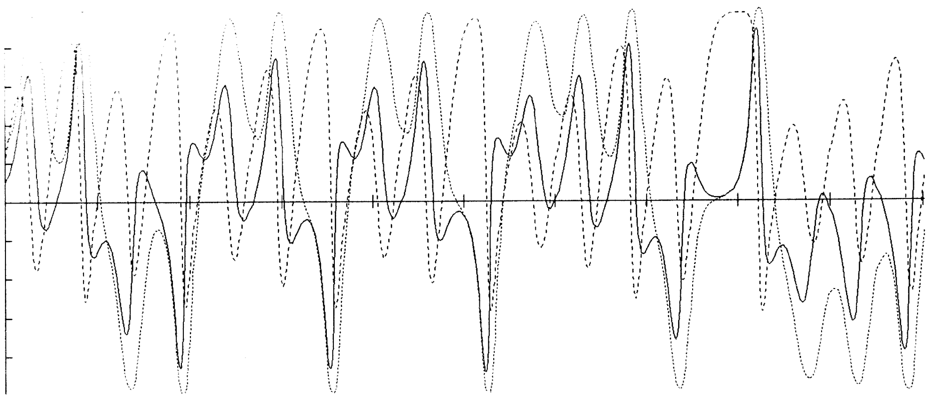

Figure 6.1: Most probable trajectory of the stochastic multilevel process of this subsection (state variables as functions of time)

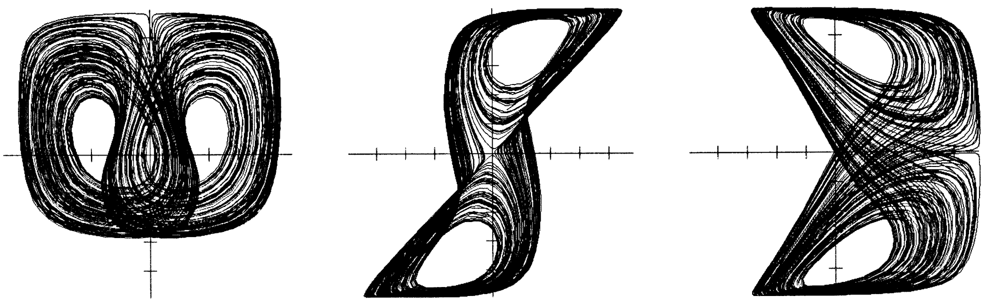

Figure 6.2: Most probable trajectory of the stochastic multilevel process of this subsection (xy-, xz-, and yz-subspaces)

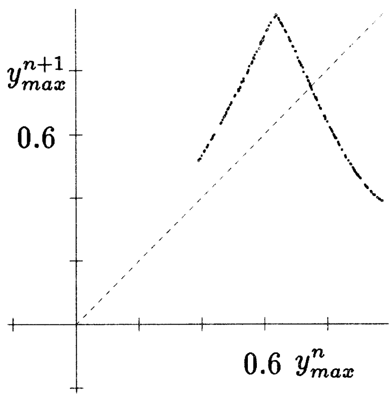

Figure 6.3: Return map of subsequent maxima of y (y_{max}^{n+1} as function of y_{max}^{n})

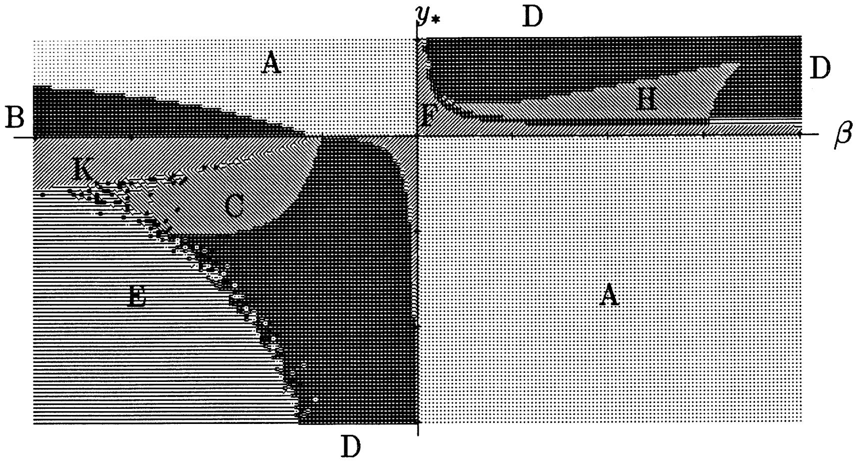

Figure 6.4: Regions of different system behaviour in β-y_* parameter subspace

K : one spiral sink, one saddle, and three spiral saddles,

D : two spiral sinks, one saddle,

H : one sink and two spiral sinks (in the triangular region below D in quadrant 1),

C : one saddle and two spiral saddles — no sink ! — (chaotic behaviour),

E : unknown for numerical reasons.

Moreover, between regions D and F in quadrants 1 and 3 there are very narrow zones in which the system displays some additional types of behaviour. Only the boundaries between regions A and B, B and K, K and C/E, as well as A and F may be described analytically, while even the numerical description of the boundaries between E and C/D is very difficult.

In regions A, F, B, and K the system approaches exactly one equilibrium, in regions D and H the system approaches one of two or three possible equilibria, while in region C an equilibrium is never reached. The equilibrium surfaces are shown in figures 6.5, 6.6, and 6.7, where stable equilibria are marked with straight lines, and unstable equilibria with dotted lines. Above regions A and F we see exactly one equilibrium surface for all three variables x, y, and z, while for all other regions exactly three equilibrium surfaces can be seen for the variables x and z. Here, the equilibria on the zero level are always unstable, while the two other surfaces — save for the chaotic region C — represent stable equilibria. The equilibrium surfaces above the regions beside A and F seem to be even more complicated. The equilibria above the "unknown" region E seem to have coordinates near ± 1, but as mentioned above, the numerical analysis is not exact enough for this region.

6.2.4 Stochastic Realizations

In this subsection we have to analyze whether a single realization of the stochastic process described by the transition probabilities in equations 6.7 to 6.12 and the micro-macro-link in equations 6.4 to 6.6 behaves in a manner similar or comparable to the

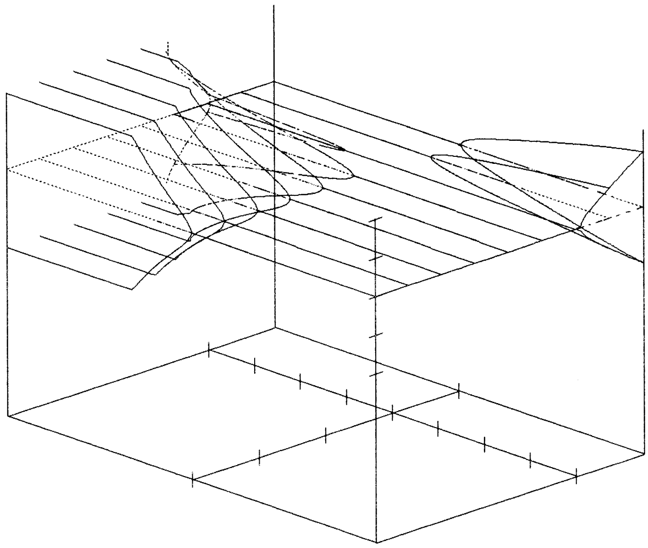

Figure 6.5: Equilibrium surface. Control parameters are $\beta \in [-4, 4]$ and $y_* \in [-3, 1]$, internal parameter shown is $x \in [-1, 1]$

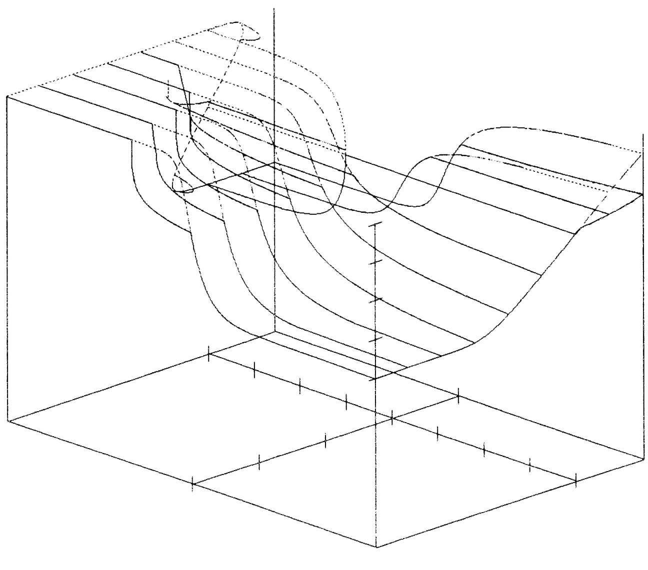

Figure 6.6: Equilibrium surface. Control parameters are $\beta \in [-4, 4]$ and $y_* \in [-3, 1]$, internal parameter shown is $y \in [-1, 1]$

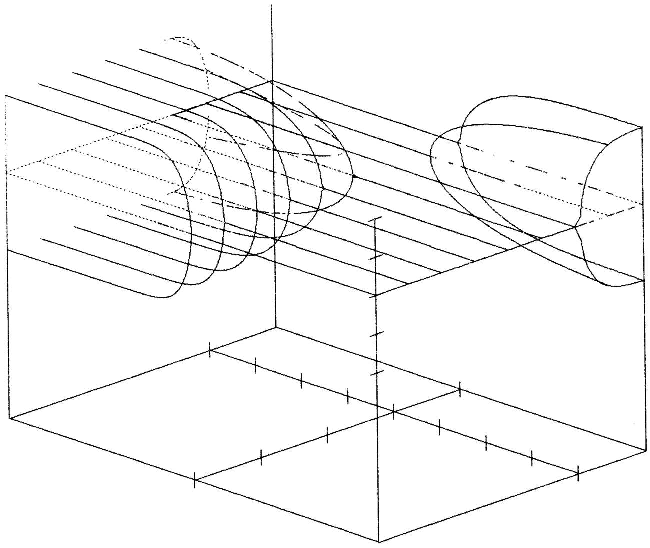

Figure 6.7: Equilibrium surface. Control parameters are $\beta \in [-4, 4]$ and $y_* \in [-3, 1]$, internal parameter shown is $z \in [-1, 1]$

stationary probability density functions derived in subsection 6.2.3 and shown in figures 6.1 and 6.2.

For this sake we specify a simulation model in the simulation specification language MIMOSE described in Möhring's contribution to this volume (pages 47ff.).[7] The specification process is described in the subsequent section 6.3, the resulting MIMOSE program is to be found in section 6.6.

The simulation results are shown in figures 6.8 and 6.9. They are very similar to the results of our semi-analytical simulation shown in figures 6.1 and 6.2. This makes it clear that the master equation approach is an appropriate means to analyze multilevel inhomogeneous Marcov processes of the type discussed in this paper.

If we simulate a greater number of stochastic realizations of our process and take expectations and variances experimentally, we could conjecture that the trajectory of the experimental expectations and the trajectories of the maxima of the stationary probability density function match well, as can be shown for the migration model with limit cycle discussed by Weidlich and Haag [WH83, pp. 111]. In the case of chaotic behaviour, however, the match is rather poor [Tro89, pp. 189–190]. Regarding nothing but such experimental expectations and variances, we should conclude that our process will become stationary in both mean and variance after a phase of damped oscillation — which is not the case for any single realization. In fact, even the stochastic realization of our chaotic process depends sensitively on its initial conditions. While in non-chaotic processes initial conditions and fluctuations may obviously be neglected due to the fact that they always flow into stable fixed points or limit cycles, realizations of chaotic processes move very wide apart after short time such that they are soon uniformly distributed over the whole strange attractor.

[7]The actual simulation results shown in figures 6.8 and 6.9 have been obtained with the author's

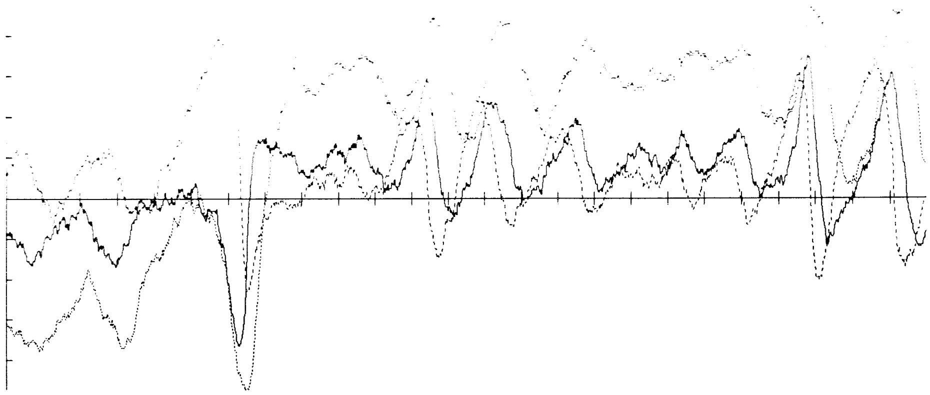

Figure 6.8: One realization of the stochastic process in three populations (state variables as functions of time)

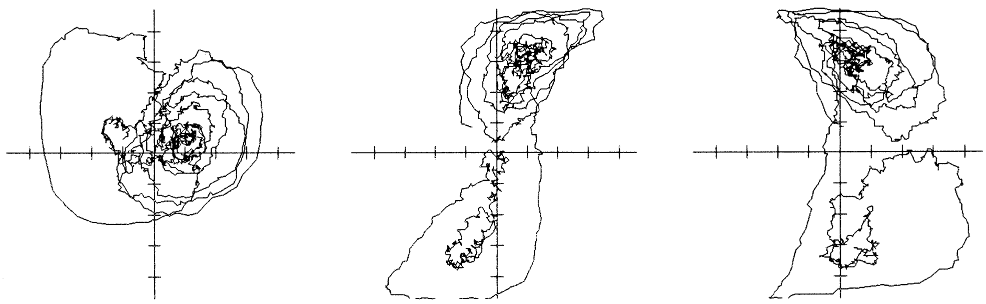

Figure 6.9: One realization of the stochastic process in three populations (xy-, xz-, and yz-subspaces)

6.2.5 Conclusions

This section has shown that modelling results may depend sensitively on the modelling techniques applied. Whether we neglect or observe the microstructure of a society makes an enormous difference, even if the model society is as simple as it was in the example taken here. Obviously, at least one of the two approaches presented here leads to completely wrong results even if it is taken for the qualitative prediction of behaviour types only — and this is obviously the macro approach.

Aggregation has to be done explicitly as has been done in the multilevel (micro) model by means of the master equation technique. Here the microanalytical simulation yields results that are qualitatively the same as the semianalytical results of explicit aggregation. Implicit aggregation as used in subsection 6.2.2 does not allow disaggregation[8] and hence does not allow even a qualitative test whether aggregation succeeds or not.

standalone program IPSA.

[8]It is an open question whether the aggregation performed between equations 6.7–6.12 and equation 6.13 can be taken as a model for disaggregating equation 6.1 to a set of transition probabilities or "human assimilation functions". The idea to proceed in this manner was first proposed by Andreas Flache. Calculations performed to this end had no satisfying results so far.

Even models as simple as the ones presented in this paper may display a wide variety of possible behaviour types some of which are even chaotic, forbidding any quantitative predictions of their future development; in some cases it may even be impossible to calculate the boundaries between regions of different behaviour types in parameter space. Thus any attempt to predict phenomena in complex (multilevel) systems very soon exceeds the bounds of possibility.

6.3 Supporting the Modeling Phase with GEMM: A Phase Model for Modeling

In the previous section we have provided the theoretical basis (in the sense of qualitative descriptions of state transition functions) for a micro model of an opinion formation process in a social system. In the following we demonstrate how GEMM supports the further modelling process.

The analysis of the modelling language MIMOSE yields the phase model as shown in Figure 6.10 which is supposed to be suitable to reduce the complexity of the modelling process. Furthermore, the modeler is provided with a methodological instruction for model construction. A detailed explanation of the step-by-step construction method is given in the next sections.

6.3.1 Definition of Object Types

In our micro model of an opinion formation process in a social system we describe a `society` consisting of three subpopulations (`population`, `media` and `science`). Each subpopulation is composed of a group of individuals (`citizens`, `journalists` and `scientists`, respectively). At this stage it should be intuitively clear that these objects differ by their internal structure or by distinct opinion formation processes. We obtain therefore seven different object types during the first step of modelling as shown in Figure 6.11. GEMM's support of this construction phase consists of a graph editor, thus enabling the user to transform the model into a graph where object types are represented by vertices and dependencies between object types (as we shall see in Section 6.3.3) by edges.

6.3.2 Definition of Attributes

In the second phase a structure editor (as shown in Figure 6.12) is available to describe the internal structure of the object types, i.e. defining attributes and their respective range for each object type.

On the level of the individuals we find the object types `citizen`, `journalist` and `scientist`. Each `citizen` either supports or opposes nuclear energy which is expressed by an attribute `att` with the range (`pro`, `contra`). The object types `journalist` and `scientist` are constructed analogously.

Let us now look at the object types `population`, `media` and `science`. For each of these object types we define an attribute `state` with range (`real`) which represents the attitude of the respective subpopulations.

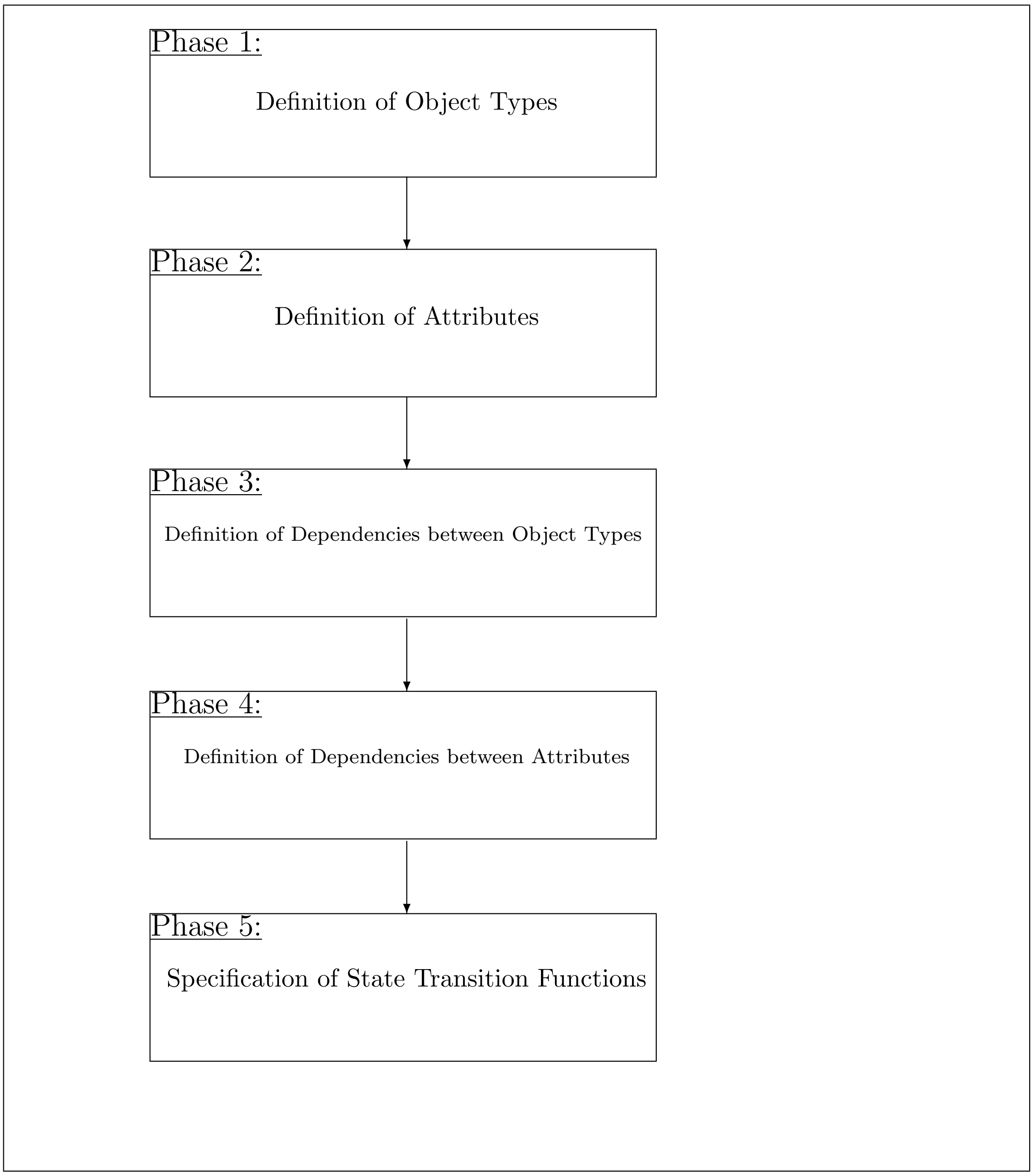

Figure 6.10: A phase model of modelling

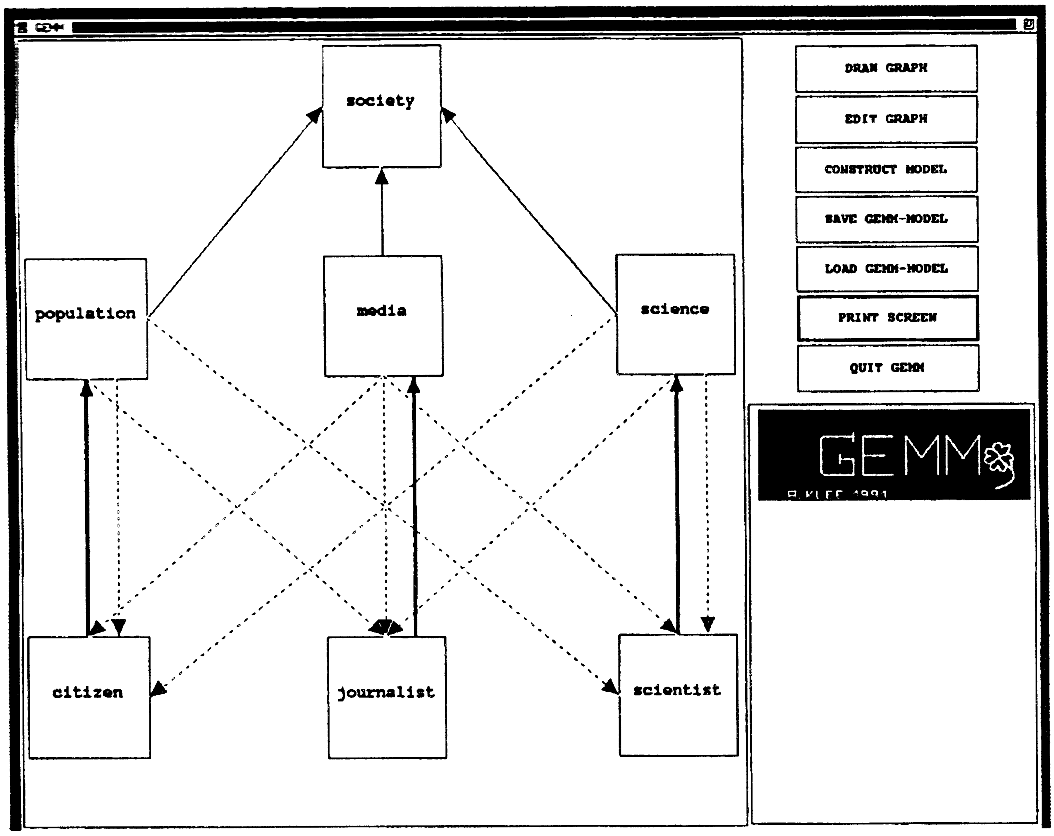

Figure 6.11: The GEMM graph editor

In MIMOSE models the modeler usually defines an object type for collecting data or for evaluating statistics. In our case we use the object type `society`[9] to store the history of states of the subpopulations which we can model through the three attributes `hisPopulation`, `hisMedia` and `hisScience`.

6.3.3 Definition of Dependencies between Object Types

The dependencies between object types are represented by edges in the graph which connect the respective object types. The description of relations is possible as shown in Table 6.1.

It is obvious that the state of a subpopulation depends on the individual opinions of its members, e.g. consider the data flow from `citizen` to `population` of Figure 6.11.

According to the hypotheses of Section 6.2 we need the state of each subpopulation to describe the attitude of each individual, e.g. consider the data flow from `population`, `media` and `science` to `citizen` in Figure 6.11.

The arrows from `population`, `media` and `science` to `society` mark the data flow for collecting the history of the several subpopulations.

GEMM translates this graphical descriptions of dependencies between object types into corresponding attributes[10], which are automatically registered for the respective

[9]In the following we do not consider the object type `society` any more.

[10]These attributes are identifiable by the prefix GEMM in front of an object type name, e.g. `GEMMpopulation`.

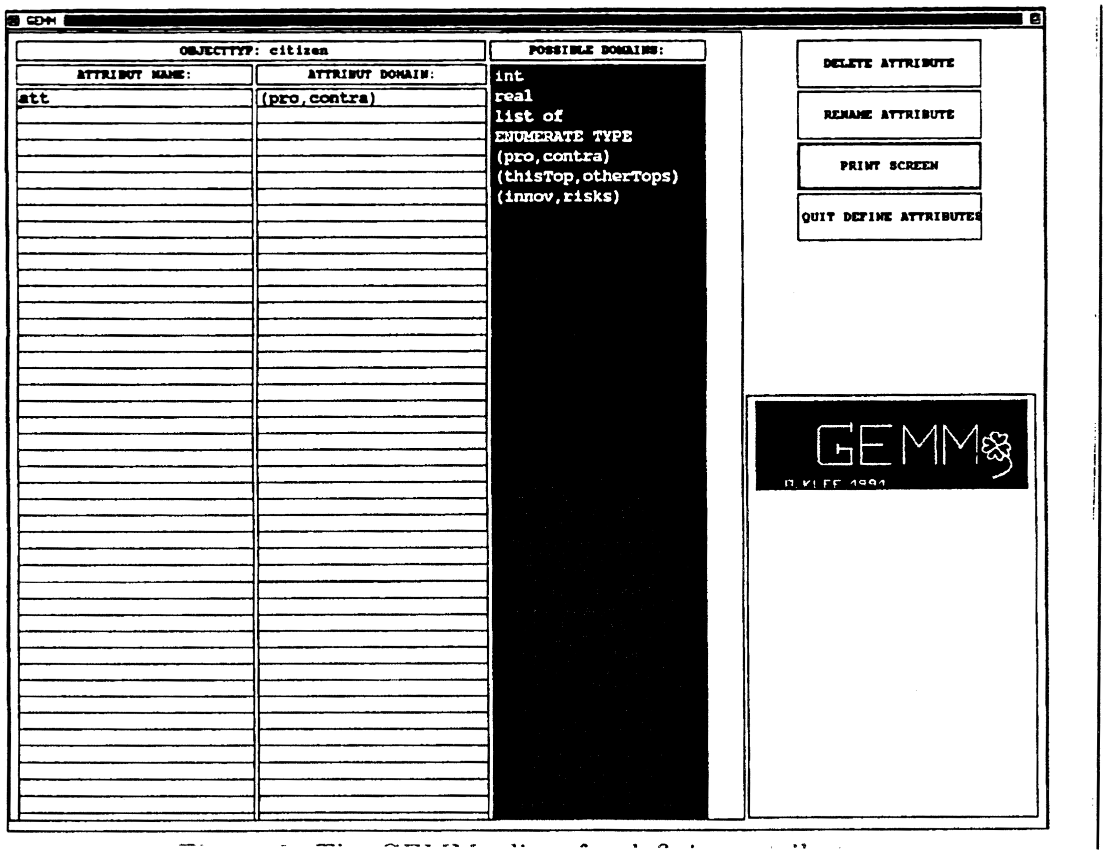

Figure 6.12: The GEMM editor for defining attributes

Graphic Representation:	**Semantic:**
B ——→ A	Object type A uses at least one attribute value of a single object belonging to object type B at the current time step.
B ▬▬➡ A	Object type A uses at least one attribute value of several objects belonging to object type B at the current time step.
B – – – → A	Object type A uses only attribute values of a single object belonging to object type B with respect to former time steps.
B ▬ ▬ ▬ ➡ A	Object type A uses only attribute values of several objects belonging to object type B with respect to former time steps.

Table 6.1: Description of relations

object types.

6.3.4 Definition of Dependencies between Attributes

After we have described the dependencies between object types we are now able to define the dependencies between attributes with the support of another GEMM structure editor as shown in Figure 6.13.

Let us consider the object types `population` and `citizen`; for the pairs `media` —

`journalist` and `science` — `scientist` there exist analogous solutions.

We learn from equations 6.7 and 6.8 that the change of opinion of a `citizen` depends on the respective attribute value `state` of `population` $x(t)$, `media` $y(t)$ and `science` $z(t)$ of the last time step. The selection of formula 4 or 5 as the state transition function depends on the former[11] attitude of the citizens.

The state transition function `fCitAtt` which yields the value of attribute `att` (attitude of `citizen`) must therefore be provided with arguments `att_1`, `GEMMpopulation.state_1`, `GEMMmedia.state_1` and `GEMMscience.state_1`. The last step of describing these dependencies between attributes is shown in Figure 6.13.

In Section 6.3.2 we have defined the attribute `state` for the object type `population`. Its state transition function `fPopState` depends only on the attitudes of all the objects of type `citizen`, so that we provide `fPopState` with the argument `GEMMcitizen.att`.

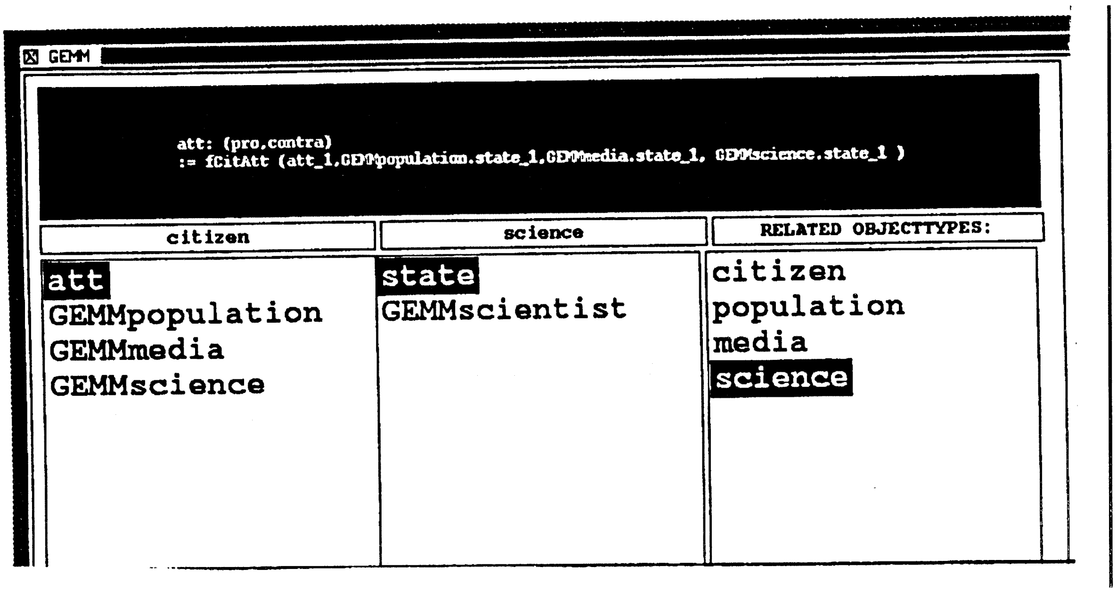

Figure 6.13: The GEMM editor for defining dependencies between attributes

6.3.5 Definition of State Transition Functions

GEMM provides the user with a text editor to describe the mathematical expression of the state transition function. The system automatically generates a frame, the so called signature, as shown as the black boxes in Figure 6.14 which the user can not manipulate with the text editor. The signature especially comprises the name, the domain and the range of the state transition function. The modeler can describe the function body by means of the text editor. For that purpose consider the function `fCitAtt` in Figure 6.14.

This state transition function reflects equations 6.7 and 6.8 as a stochastic variant in the model specification language MIMOSE. We similarly transform the equations 1, 2 and 3 for the state transition function `fPopState` as shown in Figure 6.15.

At the end of the modelling process GEMM automatically transforms the model into a MIMOSE text as shown in section 6.6.

[11]Values of the last simulation step are expressed through the suffix _1.

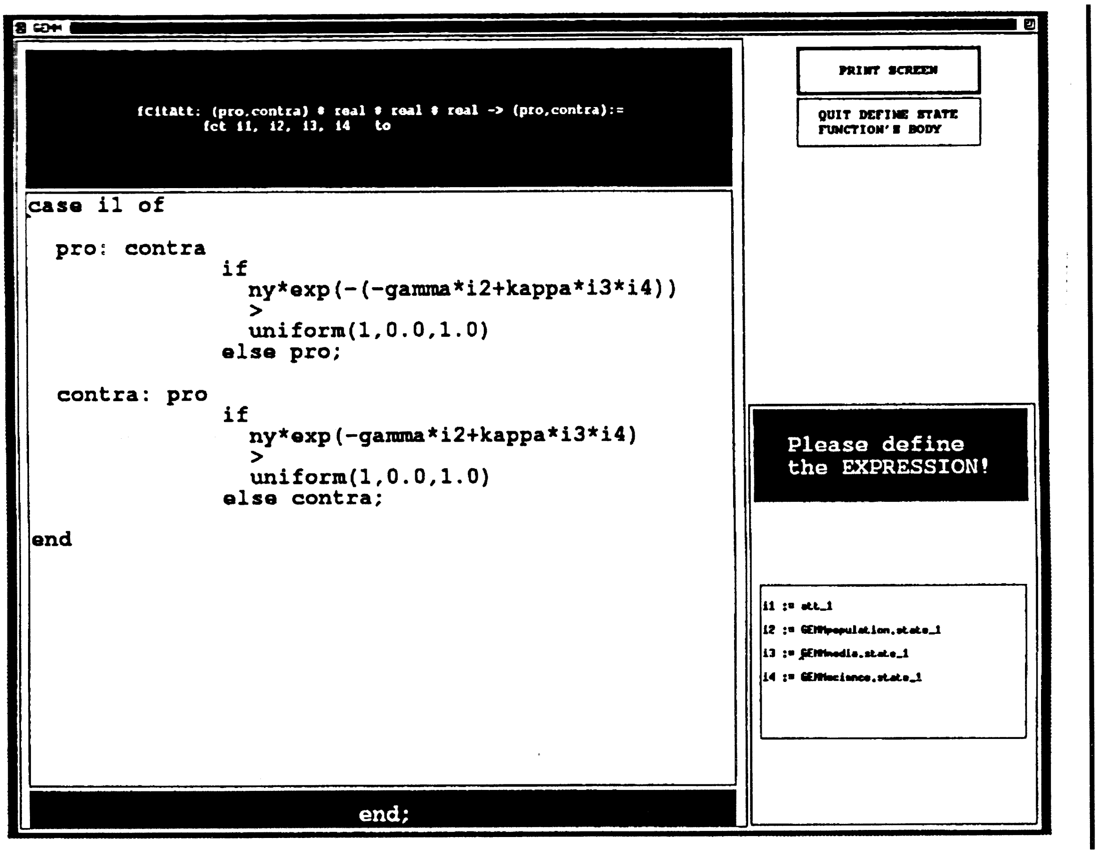

Figure 6.14: The state transition function of `fCitAtt`

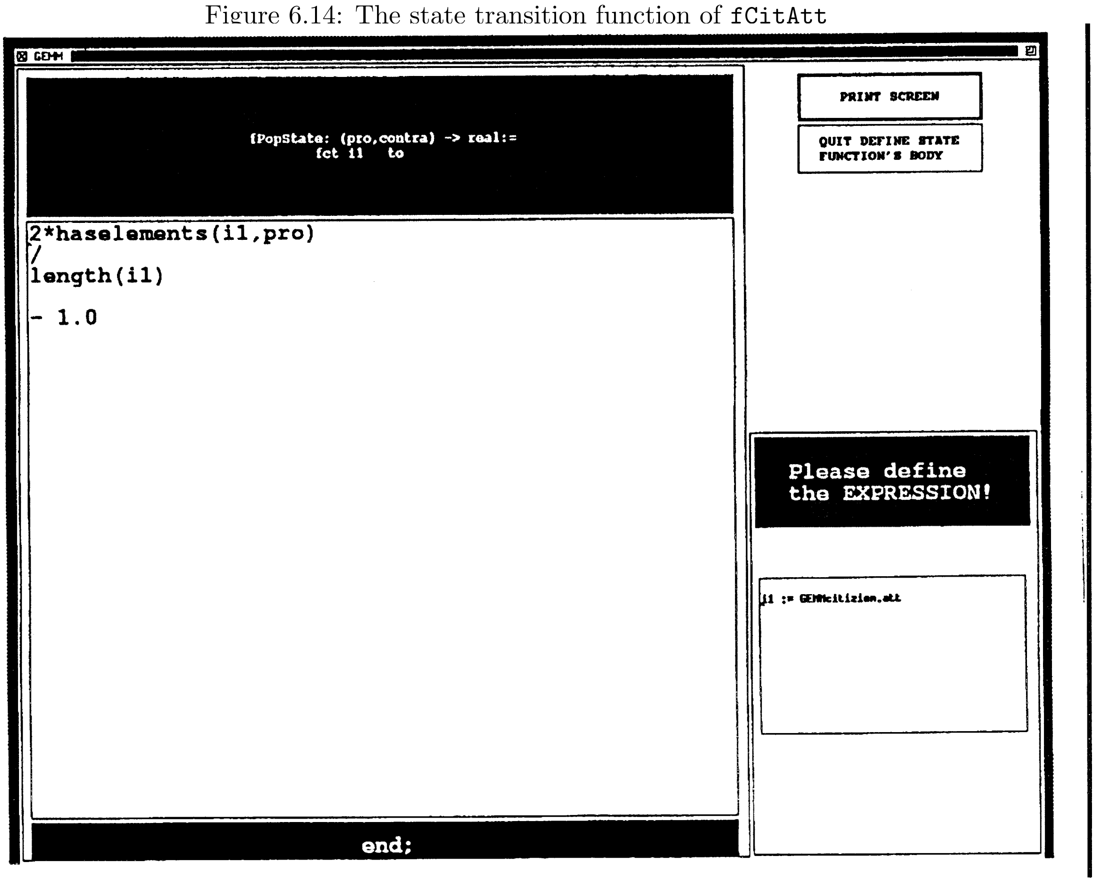

Figure 6.15: The state transition function of `fPopState`

6.4 Conclusions

The previous section has demonstrated that GEMM **reduces the complexity** of a MIMOSE modelling process by decomposing it into several phases. In addition to that the GEMM phase model offers a **methodological instruction** for modelling and relieves the user of unimportant details during the several steps of construction.

GEMM especially supports the **visual specification** of MIMOSE model structures and thus uncovers the dependencies between object types which were hidden in the MIMOSE model despription up to now. By revealing such inherent structures GEMM increases the **transparency** and **comprehensibility** of usually very complex MIMOSE models.

Thus GEMM seems to be a useful tool to facilitate the approach to the MIMOSE simulation system.

6.5 MIMOSE: Macro Model

```
/******************************************************************/
/*                                                                */
/*              Chaotic Behaviour in Social Systems               */
/*                                                                */
/******************************************************************/



chaos := {
  att : dif;
  outx : list of real := append (outx_1, att.x);
  outy : list of real := append (outy_1, att.y);
  outz : list of real := append (outz_1, att.z)
};

dif :=
{ x : real := x_1 + DT * (-gam * x_1 + kappa * y_1 * z_1);
  y : real := y_1 + DT * (beta * (Y - y_1) - eps * x_1 * z_1);
  z : real := z_1 + DT * (-alpha * z_1 + delta * x_1)
};
```

6.6 MIMOSE: Micro Model

```
/**********************************************************************/
/*                                                                    */
/*     Chaotic Behaviour in  Social Systems (Micro Model)             */
/*                                                                    */
/**********************************************************************/


/**********************************************************************/
/*              Definition of Object Types                            */
/**********************************************************************/

society:=
{
 hisPopulation: list of real
               := fSocHistory( hisPopulation_1,GEMMpopulation.state );

 hisMedia:      list of real
               := fSocHistory( hisMedia_1,GEMMmedia.state );
```

```
 hisScience:    list of real
                := fSocHistory( hisScience_1,GEMMscience.state );

 GEMMpopulation: population;
 GEMMmedia: media;
 GEMMscience: science
};


population:=
{
 state: real
        := fPopState( GEMMcitizen.att );

 GEMMcitizen: list of citizen
};


media:=
{
 state: real
        := fMediaState( GEMMjournalist.att );

 GEMMjournalist: list of journalist
};


science:=
{
 state: real
        := fSciState( GEMMscientist.att );

 GEMMscientist: list of scientist
};


citizen:=
{
 att: (pro,contra)
      := fCitAtt( att_1,GEMMpopulation.state_1, GEMMmedia.state_1,
                  GEMMscience.state_1 );

 GEMMpopulation: population;
 GEMMmedia: media;
 GEMMscience: science
};


journalist:=
{
 att: (thisTop,otherTops)
      := fJourAtt( att_1,GEMMpopulation.state_1, GEMMmedia.state_1,
                   GEMMscience.state_1 );

 GEMMmedia: media;
 GEMMpopulation: population;
 GEMMscience: science
};


scientist:=
{
 att: (innov,risks)
      := fSciAtt( att_1,GEMMpopulation.state_1, GEMMmedia.state_1,
                  GEMMscience.state_1 );

 GEMMpopulation: population;
 GEMMmedia: media;
 GEMMscience: science
```

```
};

/*********************************************************************/
/*              Definition of State Transition Functions             */
/*********************************************************************/

fSocHistory: list of real # real -> list of real:=
             fct i1, i2   to
             append(i1,i2)
end;

fPopState: (pro,contra) -> real:=
           fct i1   to
           2*haselements(i1,pro) / length(i1)
           - 1.0
end;

fMediaState: (thisTop,otherTops) -> real:=
             fct i1   to
             2*haselements(i1,thisTop) / length(i1)
             - 1.0
end;

fSciState: (innov,risks) -> real:=
           fct i1   to
           2*haselements(i1,thisTop) / length(i1)
           - 1.0
end;

fCitAtt: (pro,contra) # real # real # real -> (pro,contra):=
         fct i1, i2, i3, i4   to
         case i1 of
                  pro: contra
                         if
                                 ny*exp(-(-gamma*i2+kappa*i3*i4))
                                 >
                                 uniform(1,0.0,1.0)
                         else pro;

         contra: pro
                         if
                                 ny*exp(-gamma*i2+kappa*i3*i4)
                                 >
                                 uniform(1,0.0,1.0)
                         else contra;

         end
end;

fJourAtt: (thisTop,otherTops) # real # real # real -> (thisTop,otherTops):=
          fct i1, i2, i3, i4   to
          case i1 of
                thisTop: otherTops
                         if ny*exp( -(beta*(ystar-i3) - epsilon*i2*i4) )
                            >
                            uniform(1,0.0,1.0)
                         else thisTop;

                otherTops: thisTop
                         if ny*exp( beta*(ystar-i3) - epsilon*i2*i4)
                            >
                            uniform(1,0.0,1.0)
                         else otherTops;
          end
end;

fSciAtt: (innov,risks) # real # real # real -> (innov,risks):=
         fct i1, i2, i3, i4   to
                case i1 of
```

```
                    innov: risks
                            if ny*exp(-(-alpha*i4+delta*i2))
                               >
                               uniform(1,0.0,1.0)
                            else innov;

                    risks: innov
                            if ny*exp(-alpha*i4+delta*i2)
                               >
                               uniform(1,0.0,1.0)
                            else risks;
            end
end;
```

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Chapter 7

Yuly M. Borodyansky: Computer Simulation of Nonalgorithmic Procedures

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Abstract

Construction investigation of alphabetic operators based on works of Alan M. Turing and A. Markov deals with algorithmic procedures only. In the early sixties it was proved that more general operators cannot be built in a framework of constructivism. This approach seems to lead to fundamental difficulties in artificial intelligence problems connected with the taboo of constructivism. Social actions and creative intelligence appear to include nonalgorithmic procedures.

To simulate nonalgorithmic procedures, a sequential extension of a constructive algorithm set is made. A metric of the algorithm set is introduced by using the partial metric of the word space.

A transcendent algorithm is treated as limit of a series of algorithms that converges to a nonalgorithmic procedure.

It is suggested that processes of learning and socialization can be described as a system of transcendent algorithms.

"Go there God knows where,
bring God knows what..."

from Russian folk tale

The well-known problem put forward by A. Turing in [Tur 56] "Can the Machine Think ?" is transformable into a paradoxical question — "is it possible to make the

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computer to do what cannot be done by it, in principle?" when one is convinced in incorrectness of the obvious solution.

The paradox is apparent. What is impossible to be done can be done.

The entire question is how adequately can the ideal answer be approached, what approximation sheold then be used in seeking for the solution of an unsolvable problem.

Here we consider the possibility to go beyond the intellectual abilities of the computer by an approximate simulation of nonalgorithmized procedures.

7.1 An insight into a history

The alphabet operators proved to be and still are a convenient model for man's intellectual activity. The search for the instruments used to realize these models has been helpful in creating the computers.

A number of the brilliant papers in the field of cybernetics in 50's were devoted to a simulation of intellectual activities both of a man and the entire social groups.

All this was operative in in the attempts to create new systems for the alphabet operators but the equivalence between newly born systems and the existing ones, for instance, the "basic" one and Turing computers, has just been proved [Tur 37]. Gradually it became clear that within the framework of the constructive mathematics it is impossible to create the alphabet operators (in the sense of [Glu 64]) which are not reducible to normal algorithms [Mar 54] or Turing computers, i.e., nonalgorithmized (non-renormalized by [Glu 64]).

A.N. Kolmogorov and V.A. Uspenski in their paper [KU 58] furnished the problem. But at the same time the above paper played the role of a "Trojan horse" having set as a basis for the universal discrete information processor model the description of the man's conscious activity identifying implicitly this model with possibilities to work out information by the man in general.

Thus "a white thesis" of cybernetics formulated in either of the form by the founders of algorithmic system whose essence — "all that can be done by man's drain can be described by the algorithm", triumphed.

It seemed that very soon the computers will prove theorems, play chess, translate, compose music and verses and, generally, replace people, if and only if their memory and speed of response increase!

Computer generations changed, their resources increased many times, the program products get transformed from theoretical-illustrative toys into boundless but efficient monsters. Systems of programs, a number of scientific directions for their analysis and synthesis to solve the applied programs have been given rise.It then hasn't been taken into account that the systems of programs prove to be the necessary structurized utilization of algorithmic principles used in information processing and that the computers of any generation, either mono or macro-processing ones, effect only a particular case of algorithms represent able as finite automats.

In a concept dealt with intellectual simulation pure theoretical papers were stepped aside since in spite of expectations the desired targets were going too far to be attained. Under the press of general utilization of cybernetics they also have got the pragmatic features.

And the interest in the eternal question “is the computer capable of thinking?” (let it be not the modern one but a hypothetical computer in a project!) was gradually losses.

Coming back to this question posed, in general, incorrectly, we should focus at two forms of information and at two types of thinking.

7.2 Gnosiologic platform

The author shares the view that a subject reflects an entire set of actions produced on it by the external medium as on the object of the real world rather than their discretized image (in view of a so-called fixed resolution of the organs of feelings). A set of actions on man will be determined as a *total* information at the finite time moment (interval) for the given subject.The total information cannot be given adequately as a word above certain alphabet.

The second type of information is a discretized total information “projection” onto the subject consciousness in the form of a word above the alphabet.

Such a division of information corresponds to a traditional division of thinking into unconscious — subconsciousness and conscious — consciousness.

The first type of thinking — subconsciousness — represents a subject as an operator over the total information expressed in certain nondiscrete image.

The second type — consciousness — represents a subject as an alphabet operator constructed in a general form in [KU 58].

Thus, two subject different qualitatively should be present in a single man.

These two processes are closely correlated and serve to effect a single efficiency function — the survival. Subconsciousness is the survival of a subject as an individual, and consciousness — as a communication instrument of mankind.

The first suggesting correlation between subconsciousness and consciousness is a discretized “projection” of the result of subconscious activity — realization of our reaction to total information coming from the environment (the result, rather than the motive!).

The second correlation is a discretization of total information by subconsciousness to adopt a decision taken by consciousness — our consciously “thought over” reaction to external actions. This is just the correlation that is identified with resolution of our organs of feelings by pure analogy with resolution of physical devices representing materialized models generated by consciousness, whereas our organs of feelings are not such — these are generated by our consciousness!

Thus, there hold at least two transformations of total information into a discrete one — peculiar operators-“lattices” effected by subconsciousness.

The following question is natural: what is this scheme for, if all abilities of a subject were explained within each of the information processing versions suggested above?

The duplicity in thinking is due to the efficiency function for survival *of a nonsovereign subject in the continuous infinite medium.*

Correctness of the model for survival of a man as an algorithmized subject with fixed discretization perception to the continuous medium actions seems to be problematic enough. The role of subconsciousness is more favorable for this purpose.

But the subject is not sovereign.It survives only in contact with the other subjects preserving at the same time its individuality. The necessity to be individuality on the one hand and to be in contact — on the other, resulted in consciousness.

The contact between individuals is dynamic. Mutual understanding is a process of constant training expressed in the dynamic control of subconsciousness by operators-"lattices" in each of contacting individuals to maximize the adequacy of a combined reaction to the external medium when the interaction is minimized.

A number of facts suggest that an inverse information exchange between consciousness and subconsciousness takes place — so-called operators-"antilattices". In particular, following this scheme it is natural to explain the phenomenon of people-"counters" or the process of obtaining mathematical results. An exclamation ascribed to the outstanding C.F. Gauss — "I know that the theorem is true but I do not know how to prove it" is a brilliant illustration that the written proof of any theorem represents a discrete chain of considerations, justifying post factum the result, and carrying, in fact, no information about the process of its obtaining; that is, probably, why the construction of "exhaustive".

The essence of man's intellectual activity is seen by the author just in such a scheme.

In this case the behavior not only of a single individual but of the entire social groups could hardly be simulated adequately enough within the constructive approach. The arranged group of people is distinguished in a high degree of consistency between its "lattices". It is larger than a simple set of mutual understanding (mutual training) the actions of a social group exhibit the forms of the joint behavior which are not described at the level of consciousness.

The author's views on intellectual activity of a man in general and the problems of its simulation in particular, presented in the form of thesis, premise our further treatment to emphasize in gnosiologic context one more attempt to simulate the process of approaching to the adequate understanding of our thinking was made.

This paper is an attempt to return to the "initial point of reference" going, naturally, beyond the frame of constructivism but preserving the possibility to use modern in certain sense (the more so future) computers as the usual means to simulate the operators suggested below.

It should just be noted that a simple formal introduction of nonalgorithmized alphabet operators is not difficult from a "technical" view point and would hardly be helpful. We suggest here certain extension of a set of algorithms above the given alphabet following in the "internal manner" from the properties of algorithms when a partial metric is introduced on a set of words.

7.3 Partial order ratio on a set of words

The treatment is based on normal algorithms [Mar 54].

The basic notations.

- $A = \{e, \alpha_1, \ldots, \alpha_m\}$ — is the alphabet with a separated letter e.
- $\mathcal{H}$ — is a set of all normal algorithms above A.

- Q_i — is the algorithm from $\mathcal{H}$ with an applicability region P_i.

Small latin letters p and q with and without indices denote the words above A, and ξ and ζ are arbitrary letters from A, different, generally, from e.

Definition 1.

Modulus of the word $p = \zeta_1, \zeta_2 \dots, \zeta_n$ will be called a positive proper binary fraction of the form $0, k_1 k_2 \dots k_n$ where $k_i = 0$ $(i = 1, \dots, n)$ if and only if $\zeta_i = e$ and will be denoted as $\mid p \mid$.

Definition 2.

For an arbitrary pair of words p and q, there will be assumed $p > q$ if $\mid p \mid > \mid q \mid$.

It follows from definition 2 that $\alpha_i > e$, $i = 1, \dots, m$.

Definition 3. (The rule of assigning an empty symbol).

For any word p $p = pe$.

Thus, two words are equal if they are coincident letterwise, or differ in the number of symbols e at the end of the word.

It is easy to show that the relation introduced gives partial order (transitively or anti symmetrically) and a set of words is dense with respect to the given order, i.e. for an arbitrary pair of words p_1 and p_2 such that $p_2 > p_1$ there exists a word q such that $p_2 > q > p_1$.

Definition 4.

Two words are comparable if they are coupled by order relation.

7.4 Distance and intervals on a set of words

Let's take the algorithm for subtraction of two words — a certain generalization of subtraction of proper positive binary fractions where each letter of the alphabet (except e) plays the role of an individual binary unity incommensurable with the other ones, and e — the role of zero.

Definition 5.

The distance $/\rho/$ between two comparable words will be determined as the modulus of their difference:

$\rho(p_1, p_2) = \mid p_2 - p_1 \mid$ where $p_2 > p_1$

Setting $\rho(p_1, p_2) = \rho(p_2, p_1)$ we can show by simple calculations that the distance introduced in this way gives partial metric on a set of words.

Definition 6.

For any pair of comparable words $p_2 > p_1$ a set of all comparable pairwise words satisfying the condition $p_2 \geq q \geq p_1$ is called the cut $[p_1, p_2]$.

Taking into account that letters are not comparable there exists a set of cuts for a pair of words $p_2 > p_1$.

7.5 Convergence on a set of words

Let us construct two sequences:

$\{q_n\}$ where $\{q_n\} = pe \dots e\xi$, (e $n-1$ times) $n = 1, 2, \dots$,ξ is an arbitrary fixed letter from A different from e;

$\{p_n\}$ where $\{p_n\} = qe\zeta \dots \zeta$, ($\zeta$ n times) $n = 1, 2, \dots$,ζ is an arbitrary fixed letter from A different from e;

It follows from definitions 1 and 5 that

$$\lim_{n\to\infty} \rho(p_n, p_{n+1}) = 0$$

and

$$\lim_{n\to\infty} \rho(q_n, q_{n+1}) = 0$$

The constructed sequence q_n and p_n satisfy classical principles of convergence. It is natural to regard the word p as the limit of monotonously decreasing sequence q_n, and the word $q\zeta$ — as the limit of monotonously increasing sequence p_n,and to the notations: $\lim_{n\to\infty} q_n = p$, $\lim_{n\to\infty} p_n = q\zeta$.

We get our consideration constricted setting in the examples of sequences $p = q\zeta$ and $\xi = \zeta$. We obtain a system of embedded cuts $\{[p_n, q_n]\}$ begin a system of neighborhoods of the word p, and

$$\lim_{n\to\infty} p_n = \lim_{n\to\infty} q_n = p$$

Thus, for a specific form of sequences the analog of the general limit is constructed namely:

Definition 7.

The word p is called the limit of the sequence $\{p_n\}$ and is denoted as $\lim_{n\to\infty} p_n = p$ if:

a all elements of the set $\{p, \{p_i\}\}$ are comparable;

b there exists such a system of neighborhoods $\{[p_n^*, q_n^*]\}$ of the word p, that for any neighborhood $[p_j^*, q_j^*]$ there exists the number n_j , that for all $n \geq n_j$ $p_j^* \leq p_n \leq q_j^*$

The limit constructed means contensively that in a converging sequence with increasing n the coincident initial cut of the neighboring wordterms should get increased, and their difference moves continuously “into the tail”.

Within the framework of the given paper consider only this particular example.

In fact, we lay here the basis to construct the analog of a numerical sequence theory within the introduced partial metric.

7.6 Convergence on a set of algorithms

Definition 8.

A sequence of algorithms $\{Q_i\}$ converges if:

a $P_i \subseteq P_{i+1}$,$i = 1, 2, \dots$;

b for any k , for all $p \in P_k$at $n \geq k$ all sequences $\{Q_n(p)\}$ converge.

Definition 9.

Alphabet operator Q with applicability region P is the limit of a converging sequence $\{Q_i\}$ and is denoted as $\lim_{n\to\infty} Q_n = Q$

a $P = \bigcup_{i=1}^{\infty} P_i$;

b $\lim_{n\to\infty} Q_n(p) = Q(p)$, $p \in P_n$;

The necessity to deal with an alphabetic operator rather than with an algorithm is due to the fact that, as it will be shown in next section, the set of all normal algorithms above the given alphabet is not closed relative to the introduced notion of the limit.

7.7 Transalgorithms

Theorem

There exists a converging sequence of algorithms whose limit is not an algorithm.

Proof.

Let certain standard way of recording algorithm as a word above A, without using symbol e, be accepted with the given alphabet A.

Consider the sequence $\{Q_n\}$ where Q_n is the algorithm applicable to recordings of those algorithms only, that are not self-applicable and their recording length is no more than n. Due to the finite number of these algorithms, such a Q_n exists starting from certain minimum n_0.

We then require that $Q_n(p) = pee\xi \dots \xi$ (ξ t times) where $t = n + 1 - r$ (r is the recording length of p).

It is clear that:

a $P_i \subseteq P_{i+1}$, $i = n_0, n_0 + 1, \dots$;

b $P = \bigcup_{i=n_0}^{\infty} P_i$ is a set of recordings of those algorithms only are which not selfapplicable.

c for any k and any $p \in P_k$ at $n \geq k$ $\lim_{n\to\infty} Q_p = pe\xi$by the definition 7.

Having defined the operator Q with applicability region P as the alphabet operator of the form $Q(p) = pe\xi$ for any $p \in P$ we get by definition 9:

$$\lim_{n\to\infty} q_n = q$$

As it is known from a theory of algorithms(for instance [Glu 64]) there is no such an operator in the set of normal algorithms. So, the theorem is proved.

Let us call the alphabet operators being the limits of algorithm sequences and not being algorithms — transalgorithms.

We denote through $\mathcal{H}_*$ the closure of the set $\mathcal{H}$ with respect to the introduced convergence. Then for algorithmically unsolvable problems reducible to that of non-self-applicability it is possible to formulate the following consequence:

In $\mathcal{H}_*$ for any algorithmically unsolvable problem there exists a transalgorithm that solves it.

Generally, as it was mentioned earlier, this result in it self does not justify the entire technique suggested to obtain the operator Q. It can simply be said: "Let such an operator Q be...", then in $\mathcal{H} \bigcup \mathcal{Q}$ the statement of the consequence is valid. The main result is in the statement following from theorem:

For any algorithmically unsolvable problem reducible that of non-self-applicability there exists the algorithm that solves it with arbitrary pre assigned accuracy.

To clarify the notion *accuracy* we transform the sequence of algorithms $\{Q_i\}$ from the proof of theorem into the sequence $\{Q_i^*\}$ by the following rule:

a applicability region of all Q_i^* is P^* — a set of recordings of all algorithms above A;

b $Q_i^*(p) = Q_i(p)$ for $p \in P_i$;

c $Q_i^*(q) = q_{i+2}$ for $q \in \{P^* \setminus P_i\}$ where q_{i+2} is the initial cut of the word q of length $i+2$.

Taking into account that in algorithm recording the symbol e in construction is not used it is clear that Q_i^* perform the same functions in recognizing non-self-applicability as Q_i but have the following property — for any n, all $p \in P^*$ and all $k = 1, 2, \ldots$ $\rho(Q_n^*(p), Q_{n+k}^*(p)) \leq 10^{-(n-1)}$

Hence $\rho(Q_n^*(p), Q^*(p)) \leq 10_{-(n-1)}$ for all $p \in P$, where $\lim_{n\to\infty} Q_n^* = Q^*$. I.e.$\rho(Q_n(p), Q(p))$ tends to zero uniformly over all $p \in P$ in view of a general estimate independent of p.

It can then be said that $\rho(T_n, T) \leq 10_{-(n-1)}$ where T_n is the range of values Q_n and this estimate can be taken as the *accuracy* measure of the solution of the problem by algorithm Q_n^* .

The possibility to realize transalgorithm by certain computer program with the pre assigned accuracy (in the sense of the introduced metric) is shown.

In conclusion we also note that for the construction performed it is important that the metric on the set of algorithms and the distance between the output words were generated by algorithms, i.e., by algorithms rather than by the chosen order rule on the set of input words.

Artificial enough order on the set of input words, the algorithm of word subtraction equally with the metric based on them prove to be the induced means for further constructions and can be different depending on the "technique" of construction.

7.8 Interpretation of nonconstructiveness of transalgorithms

Constructively the problem of consciousness is to represent in constructive things connectively, even pseudo constructively. The preceding treatment was compiled according to this principle.

The paper is of a statement character and it does not give the ways to construct transalgorithm approximations. Let us make an attempt to the approach to a possible approximate computer simulation.

In constructiveness of the transalgorithms suggested can intuitively be represented as the algorithms with infinite substitution tables. The transalgorithm work consists in checking the applicability of an infinite number of substitutions to the word at the finite time moment i.e., use is made of the phenomenon of infinity which is strictly forbidden by the constructivism.

It is convenient to represent the approximation of transalgorithm by an algorithm as the "cut off" of certain "essential" table of substitutions given transalgorithm.

It is quite natural to assume that transalgorithm table is constructed according to the principle of decreasing significance of terms (substitutions)with increasing distance from the beginning. This approach is in full agreement with the convergence on the set of words. The transalgorithm approximation consists then in rejecting the "tail" of the table starting from certain number of substitution (as infinite fraction approximation by the finite one).

In fact, this is the principle of constructing the numeric converging series being a classical (if not a single!) model in our consciousness of counting actual infinity.

The approach based on the explicit use of actual infinity should be distinguished from those based on the use of potential infinity which does not go beyond the constructivism.

First of all, this is one of the fundamental theoretic-applicable directions in cybernetics — the approximate numerical methods constructed on the converging iterative procedures.

The iterative algorithm is given by the finite number of substitutions and the potential infinity of its work is in the input data, i.e., the condition of its shutdown with each application is coded in an input word . Just these words make up its applicability region.

To amplify similarity with transalgorithms the work of the iterative algorithm can be represented in a somewhat different way, in two stages:

- at the first one the algorithm, according to the given shut-down criterion, "accumulates" the number of substitutions due to iteration cycles recording;

- at the second stage it applies, generally speaking, an infinite (but in each specific case finite) substitution table.

This scheme of approximate calculations is analogous to the approximations of transalgorithm, however, there exist the essential differences.

The work of the iterative algorithm consists in applying to the input word all its substitutions, in the limit — the infinite number. The transalgorithm work involves finding out certain finite set of the "necessary" substitutions in the infinite table and getting the result instantaneously.

An instantaneous character of the transalgorithm work generates a different nature of its accuracy than the iterative procedure. The latter accumulates constantly the accuracy for each input word giving in the limit the absolute answer. Approximations of the transalgorithm work results (in any case the example suggested), such as "yes", "no", "don't know" have the structure of the type "almost for all" when the subset of words from the applicability region to which the answer "don't know" is given, decreases continuously.

A different approach to the construction of nonalgorithmized procedure is possible when the algorithm (with the finite number of substitutions!) interprets for the finite time interval an infinite sequence of letters — a kind of an infinite word.

With the metric introduced above on the set of words it is clear what is meant under the approximate simulation of the procedure similar to using the approximate value of an infinite fraction, for instance $\sqrt{2}$. When we say "take up $\sqrt{2}$ ..." the finite approximation is usually meant.

The classical algorithm used to calculate the square root is inapplicable to the input word "2" because it will never be stopped. But applicable is its "double" — the algorithm of an approximate calculation of the square root, whose applicability region is in the words such as: *peq* where *p* means the initial number recording, e is separator, q — giving the accuracy of calculation.

The statement of the problem of using a potentially infinite input word generated by the algorithm is identical to the case of potentially infinite iterative calculations.

We show this by the example of calculating $\sqrt{2}$.

Let there be certain algorithm Q applicable to the words d_i, $i = 1, 2, \dots$,where d_i is the value of $\sqrt{2}$ with accuracy to the i-th symbol. Then Q can be changed by the equivalent algorithm being a superposition of the algorithm Q and that of an approximate calculation of the square root with the applicability region $P \in 2ei$, $i = 1, 2, \dots$, where i is the given accuracy of calculation of $\sqrt{2}$.

Thus, the potential infinity of the input word is reducible to a potential infinity of the iterative algorithm steps.

However, it is not clear whether the procedure to use actually the infinite word by an algorithm is adequate to that introduced by transalgorithms since under the transition to infinity "surprises" are possible.

7.9 Is the computer capable of thinking?

As it follows from the scheme of establishing the equivalence between a man and a computer, as the information processors, in [Tur 56], A. Turing restricted himself to the region of conscious thinking. In this case the answer can be in the affirmative "yes".

Otherwise, if under thinking one means the reflection of the entire set of actions of the external medium the answer should be in the negative "no".

But between these "yes" and "no" in view of multiformity of the interpretation of the problem there exists the entire spectrum of solutions or "almost solutions".

We show here the possibility of one of such "almost solutions" , how one can approximately "force" the computer to do what lies beyond the scope of its abilities (with the estimate of such an approximation).

The constructions given should rather be referred not to simulation of thinking (unconscious one) but to the problem of simulation of the operators-"lattices" at the level of transition from a discretized countable model to the constructive finite one.

In principle, it is possible to construct the computer model of consciousness, the model of our model of thinking, a kind of the "derivative" of the function of the individuum adaptation. I.e., it is possible to simulate thinking to the extent at which consciousness reflects adequately the reality.

If we proceed from a two-level thinking the subject should have at least two sources for the criterion of adequacy of the reflection — the conscious experience and unconscious one transmitted to the consciousness through the operators-"lattices" as a component of discretized models. Semantics of these models (and also our classification of the objects) are off the consciousness sphere, it is the function of the operators-"lattices".

As far as the computer cannot be a model of a subject as a single whole it is deprived of the source of unconscious experience and, thus, the possibility to simulate independently classification and semantics. Not mentioning the motivation of actions — the survival which is explicitly off the consciousness.

Thus, we ascertain that only conscious models of thinking can be effected at the computer and to the extent we'll be able to "explain" to it the realization of motivation and semantics.

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Chapter 8

Yuly M. Borodyansky and Mark S. Burgin: Social Processes and Limit Computations

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Abstract

One of the principal ideas of modern sociology states that any social process is a consecutive change of states of the whole society or of some its systems. Modeling of social processes by traditional algorithmic systems or by their standard program realizations on computers leads to the following contradiction. A usual algorithm produces some result only when the device that realized this algorithm halts i.e. finishes its functioning. In social systems, on the contrary, results exist only when the system continues its functioning. Such contradiction shows that for modeling social processes it is necessary to use such schemas and constructions that differ from usual algorithms. It was found and is demonstrated further that necessary properties have schemas of limit computations and algorithms. Limit "algorithms" in a definite sense occupy intermediate place between discrete finite algorithmic methods and continuous infinite analytical ones. This gives one more argument showing perspectives of such approach. So this work is a study of such limit schemata and of various relations between them.

8.1 Limit Turing Machines

As the base model of the algorithm we take Turing machine [Tur 37] with one head and three tapes — one working tape, one input tape and one output tape. All tapes are one-sided.

Let A be an arbitrary alphabet (finite or countable), A^* be the set of all words on A, A^{**} be the set of all infinite sequences (so to say, "infinite words") with elements from A and $A^0 = A^* \bigcup A^{**}$. We suppose that on the set A^0 or only on the subset A^* some Hausdorff topology τ is defined [Kell 55] and that A is at the same time the input, working and output alphabet of all Turing machines studied in this paper.

We use the following denotations:

- letters p and q from the Latin alphabet (maybe with indices) denote words from A^*;
- ST denotes a Turing machine of the mentioned above type;
- P_{ST} denotes the applicability domain of the Turing machine ST, here $P_{ST} \subseteq A^*$;
- N as usually denotes the set of all natural numbers;
- if $p_0 \in A^*$ and $i \in N$ then $ST(p_0)$ denotes the result of applying ST to the word p_0 and $ST(p_0, i)$ is the state of the output tape of the machine ST after this machine has performed i steps of its calculations after being applied to the word p_0;
- CST denotes the set of all Turing machines ST.

We consider such sequences of words that are consecutive states of the output tape of some functioning Turing machine ST.

Definition 1 A limit Turing machine (LT) relative to the topology τ is some machine that is organized and is functioning like ST but has another (not by the halt instruction) definition of its result: if in the topological space $(A\ , \tau)$ the limit $\lim_{i\to\infty} p_i = p$ exists for for the sequence $\{p_i = LT(p_0, i); i = 1, 2, \ldots\}$ then p is the result of application of LT to the word p_0, i.e. $LT(p_0) = p$.

Let CLT_τ denotes the class of all LT relative to the topology τ.

Let us consider some examples of LT.

Example 1 Let us take discrete topology τ_0 on A i.e. each point $p \in A$ is an open set in this topology. Then we have $\lim_{i\to\infty} p_i = p$ if $p = p_i$ beginning from some $i = i_0$. For such topology an LT will be an inductive Turing machine. These machines were studied in [Bur 83] , [Bur 87] . They make possible to model different important processes including functioning of computers with display output, inductive inference and argumentation, knowledge acquisition (for example, language learning), and other inductive processes.

Example 2 Let us take some coding by A the set R of all real numbers. The standard topology on R induces the definite topology τ on A . In this situation limit Turing machines give a precise mathematical model for numerical methods. These methods are widely used for solving different problems (including sociological ones) by means of computers.

An example of such methods is given by a calculation of $\sqrt{2}$ or anyother irrational radical. While any ST that calculates such radical is applied only to few positive numbers, the analogous LT is applied to all positive numbers and all calculations are performed in the standard topology of real numbers.

The used notion of "analogous" machine has to be more precise for our further discussion.

Definition 2 Two Turing machines T_1 and T_2 (either from CST or CLT) are equivalent ($T_1 \simeq T_2$) if

1. $P_{T_1} = P_{T_2}$;
2. $(T_1(p) = T_2(p))$ for all $p \in P_{T_1} = P_{T_2}$

where as above P_{T_i} designates the applicability domain of the machines T_i.

It is necessary to remark that there are different kinds of equivalence of algorithms (like ST or LT). The kind of equivalence defined above may be called functional. It is determined by morphisms of named sets corresponding to algorithms [Bur 90]. Unformally functional equivalence shows that machines T_1 and T_2 have the same computating abilities if the are equivalent.

Let us consider the computational power of ST and LT.

Theorem 1 *For any topology τ on A and for an arbitrary ST there exists such LT (relative to the topology τ) for which $ST \simeq LT$.*

For the special case of LT — inductive Turing machines discussed in the first example — this theorem was proved in [Bur 83].

Corollary 1.1 For any topology τ on A there exists an inclusion μ of the set CST into the set CLT_τ and for any ST we have $ST \simeq \mu(ST)$.

It is known that all universal classes of so called recursive algorithms (Post systems, recursive partial functions, Minsky machines [Min 67], Kolmogorov algorithms [KU 58], normal Markov algorithms, multi head, multitape and multidimenisional Turing machines etc.) are equivalent to the class CST or in other words they have the same computating abilities. Thus we have :

Corollary 1.2 For any class CA of all recursive algorithms and for any topology τ on A there exists an inclusion ρ of CA into the set CLT_τ and for any $T \in CA$ we have $T \simeq \rho(T)$.

Definition 3 A set $X subset A_*$ is called decidable in the limit or L-decidable of there exists such LT that for any $p \in X$ we have $LT(p) = 1$and for any $q \notin X$ we have $LT(q) = 0$

Theorem 2 *Any recursively enumerable set [Rod 67] is L-decidable.*

Proof From the theory of recursive partial functions [Rod 67] follows that any recursively enumerable set X is equal to P_{ST} for some ST. Let us consider an arbitrary ST and some $p \in P_{ST}$. Let t_p be the time (the number of paces made by ST) that ST is functioning after being applied to p.

Informally the proof of the theorem consists of the "construction" of such LT that $LT(p, t) = 1$ if $p \in P_{ST}$ and $t \geq t_p$; in other cases $LT(q, t) = 0$. It means that for the given ST we construct everywhere applicable LT that being applied to some word p from P_{ST}works parallel with ST applied to p and till the halt of ST the head of LT prints on the working tape the same symbols as the head of ST on its working tape. At the same time on the output tape the head of LT prints 0. So the working tape of LT has the same sequence of states as the working tape of the considered ST. After the ST halts the LT continues its work so that it prints on the output tape 1 and then does not change this value.

When $q \notin P_{ST}$ then the ST either halts but in the forbidden state or ST continues its functioning without halting. In the first case after the ST halts the LT continues its functioning but does not change the state of the output tape. In the second case the LT continues its work as before having on the output tape 0.

Thus for any $p \in A^*$ the sequence of the output tape states converges in the topology τ for the constructed LT because its members does not change beginning from some number of this sequence.In addition to this on the construction of LT we have $LT(p) = 1$ for $p \in P_{ST}$ and $LT(q) = 0$ for $q \notin P_{ST}$. This concludes the proof.

As not all recursively enumerable sets are decidable [Rod 67] we have the following result.

Corollary 2.1 There exist L-decidable sets that are not decidable.

Corollary 2.2 There exist such LT that is not equivalent to any ST i.e. the class CLT_τ is bigger than the class CST.

This result may be interpreted as one more formal confirmation of the stated above considerations about nonalgorithmizability in the usual sense of some processes of information processing and as an argument of possibilities to model these processes by algorithmic approximations. The same is true also for social processes because they give results without halting.

8.2 Alphabetical operators

We suppose that A^* is a topological space with the same topology τ as before.

Definition 4 An alphabetical operator Q on A^* with the topology τ determines a partial mapping from A^* into A^* and is defined as follows:

1. a definite sequence $\{ST_i; i = 1, 2, \ldots\}$ is uniquely associated with Q;

2. for the applicability domain of Q the inclusion $P_Q \subseteq \bigcup P_{ST_i}$ exists;
3. $Q(p)$ is defined for those and only those $P_Q \in \bigcup P_{ST_i}$ that beginning from some i the limit $\lim_{i\to\infty} ST_i(p) = p_0$ exists and in this case we put $Q(p) = p_0$.

In chapter 7 the concept of transalgorithm was introduced. This construction is naturally incorporated into a more general concept of an alphabetical operator. Really, any transalgorithm is defined by some sequence $\{ST_i; i = 1, 2, \ldots\}$ but with some extra conditions on it (including the definite fixed topology τ on A^*).

Remark 1 There exists a natural way to transform any ST into some alphabetical operator Q. Such operator is defined by the sequence $\{ST_i; i = 1, 2, \ldots\}$in which $ST_i = ST$ for all $i = 1, 2, \ldots$. For the operator Q we have $P_Q = P_{ST}$.

Remark 2 Any sequence $\{ST_i; i = 1, 2, \ldots\}$ defines some alphabetical operator Q. It is possible that the set P_Q of its applicability is empty.

Let us show that any set of words from A^* is the set of values of some alphabetical operator. Namely, we have

Theorem 3 *For any $X \subseteq A^*$ such alphabetical operator Q exists that $\{Q(p) \mid p \in P_Q\} = X$.*

Proof The set X is countable as a subset of the countable set A^*. This means that a one-to-one mapping $\nu : N \to X$ exists. In other words, all elements of X may be enumerated i.e. $X = \{q_1, q_2, \ldots, q_n, \ldots\}$, although it is possible that this enumeration cannot be realized by a constructive procedure.

In the same way we can fix some one-to-one mapping $\alpha : N \to A^*$ that enumerates A^*. En this case the enumeration α can be chosen constructive [Mal 65].

Then by theorem 3 from the second chapter of [Mal 65] for any finite collection of words $Y_n = p_1, p_2, \ldots, p_{n+1}$ such ST exist that $ST(\alpha(k)) = p_k$ for all $k \le n$ and $ST(\alpha(k)) = p_{n+1}$ for all $k > n$. Thus for all $i = 1, 2, \ldots$ and each collection $X_i = q_1, \ldots, q_{i+1}$we take such ST and consider the sequence $ST_i; i = 1, 2, \ldots$. While for a given collection Y_n a process of elaboration of ST is constructive the whole sequence $\{ST_i; i = 1, 2, \ldots\}$ may be noncomputable.

Nevertheless the alphabetical operator Q defined by this sequence gives the solution of the problem i.e. $Q(\alpha(k)) = q_k$ for all $k = 1, 2, \ldots$ and $Q(A^*) = X$.

Corollary 3.1 The set of all alphabetical operators is uncountable and includes the set CST as its proper subset.

Like it is made in classical theory of algorithms we can introduce the concept of Q-enumerable set i.e. of such set C that it consists of the results of the application of some alphabetical operator Q. From theorem 3 we have:

Corollary 3.2 Any $X \subseteq A^*$ is Q-enumerable.

8.3 Limit Turing machines and alphabetical operators

As LT differs from ST only by the definition of the result, so the class CLT_τ of all LT on A^* with the topology τ will be countable like the set CST. At the same time the set CQ_τ that consists of all alphabetical operators on A^* with the topology τ is uncountable (corollary 3.1).Besides the set CST may be included into the set CQ_τ. Naturally appears the question about interrelations between the sets CQ_τ and CLT_τ.

Let some natural enumeration n of the set CST be given.

Definition 5 An alphabetical operator Q is called constructive and denoted by KQ if the set of indices according to the enumeration ν in the sequence $\{ST_i; i = 1, 2, \ldots\}$ that determines Q is computable.

In other words, an alphabetical operator Q is constructive if there exists an algorithm T that constructs all ST_i for some sequence $\{ST_i; i = 1, 2, \ldots\}$ that determines Q with accuracy to the equivalence relation introduced in definition 3. We can use only such LT that does not halt on any of the input words from A^*.

Theorem 4 *For any LT there exists an equivalent KQ.*

Really, for a given LT we construct a sequence $\{ST_i; i = 1, 2, \ldots\}$ of everywhere defined ST_i. Each of the ST_i functions in the same way as LT making i first steps and then ST_i halts. Thus we have the equality $\lim ST_i(p) = LT(p)$.

That means by definition 4 that the sequence $\{ST_i; i = 1, 2, \ldots\}$ determines the alphabetical operator Q which is equivalent to LT and is constructive because all ST_i from the sequence have been constructively determined.

Theorem 5 *An alphabetical operator Q is equivalent to some LT if there exists a computable sequence $\{ST_i; i = 1, 2, \ldots\}$ that determines Q and includes only everywhere defined ST_i.*

Proof Necessity of the theorem's condition follows from theorem 4.

Sufficiency. Let there exists such computable sequence $\{ST_i; i = 1, 2, \ldots\}$ that determines operator Q. We construct such LT that at first for any $p \in A^*$ processes p in the same way as ST_1. After ST_1 stops LT writes on the output tape the word $ST_1(p)$ and continues its functioning. Then LT starts to process p in the same way as ST_2 . After ST_2 stops LT writes on the output tape the word $ST_2(p)$ and continues its functioning. Then LT starts to process p in the same way as ST_3 and so on.

Thus on the output tape LT writes the succession of words $ST_1(p)$, $ST_2(p)$,$ST_3(p)$ and so on. This LT never halts and the limit value of this output words (if the limit exists) is the result of the application LT to the word p. So on the construction this LT is equivalent to the given operator Q.

In the paper the most general class of alphabetical operators is described. From it the subclass of constructive operators is extracted. Such alphabetical operators have constructive description being thus superrecursive algorithms. Their computatioonal abilities are greater than computational abilities of classical algorithms due to the nonconstructivity of the results definition. The dynamics of such operators is more adequate to the information processing in many cases as well as to the different social processes.

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Chapter 9

Mark S. Burgin: Procedures of Sociological Measurement

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9.1 Introduction

Any study of a social system is based on acquiring knowledge about properties of this system. There are different ways of acquiring such knowledge. The first is realized by the process of argumentation. The second consists of various computations. The third way is experimental and includes measurements. In sociology the procedures of measurement are of great importance. But in order to present true information these procedures must be grounded, and mathematical methods play an important role in this task.

There exists a great variety of such methods but they may be grouped in few classes. The main of them are: 1) analytical or descriptive ones using definite mathematical apparatus (differential and integral calculus, functional analysis, graph theory, set theory etc.); 2) algorithmical or procedural methods using algorithmic and procedural structures. These structures may be taken from the classical theory of algorithms. But in many cases an appropriate schema of superrecursive limit algorithms discussed in the two previous papers appears more adequate.

In this paper we discuss existing schemata of sociological measurements and elaborate a new one showing that it is more adequate and effective for sociological studies. As a mathematical basis for this approach the theory of named sets and the theory of abstract properties [Bur 85] is used because according to [Pfa 71] the object of any measurement are properties.

9.2 Definitions

Unformally a named set X is a triple of the form (X, α, I) where X and I are some sets (systems, objects etc.) from some chosen classes Ens and Set, correspondingly, and $\alpha : X \to I$is a correspondence between X and I [Bur 90]. The set X is called the support, the set I is called the set of names and α is called the naming relation of the named set X.

But such a denomination “a named set” is an obstacle for those who are acquainted with the conception of a set. They think that either a named set is some kind of sets or it is generated exclusively from sets. That is why a named set was called by a new name “a triad” [Bur 91] although the concept remained the same.

It is an important peculiarity that in the theory of named sets a name is understood in a considerably broader sense that usual. In logic or natural languages, a name is regarded as some word possibly expressing an essential feature of the object which is named. Such names, i.e. words or word expressions, will be called linguistic names. In the theory of named sets, no additional conditions are imposed on names but a single one that they should be corresponded to objects. Thus names are not necessarily different word expressions as was shown above. In conceptual systems where names are usually linguistic they may be bigger or more sophisticated that the entities named by them. For example, in any encyclopedia, different words are named by more or less complex texts exits explaining their meaning. These texts play the role of some specific names for words. We have the same situation with terms and their definitions. While terms are names of some conceptual or material entities, their definitions, which are more complicated, are names of these termini.

Even material objects may be used as names of words. Such an inverted situation appears, for example, when a child is learning a language. Somebody (his mother, his father etc.) shows a tree and says: ”This is a tree”. But for a child just the word ”a tree” is the first or the initial entity because she/he is learning words and the real tree is the second entity or a name of the first entity in our terminological setting.

Thus the statement that something is a name, only fixed the position of this entity in a triad but not the nature of this entity. As a consequence the same entities may be names in some triads, i.e. may belong to reflectors of these triads, and be elements of the supports of another triads.

It is necessary to remark that many kinds of triads are essentially indecomposable, indissoluble units. As a consequence, any of the objects X, f, I does not exist independently outside such a triad.

Other kinds of triads are decomposable into three parts. But all these parts are either triads themselves or they consist of triads.

An abstract property P is a triple (a named set [Bur 90]) of the form $P = (U, p, L)$ were U is a universe of objects for which the property P is considered, L is a partially ordered set that is called the scale of the property P and $p : U \to L$ is a partial function.

Different characteristics, parameters, qualities, indices and properties in the intuitive sense may be modeled by abstract properties. For example, such characteristic of a social group as its unity is usually represented (implicitely) as some abstract property with a range scale.

9.3 Properties and Scales

The concept of abstract property allows to analyze and to unify such important construction of sociology as a scale. Sometimes even in one book different understandings of this construction appear. In such a way in [WS 83] a scale is defined as some algorithm that is used to apply some number to the observed object. But further it is written that for estimation of measured property one sometimes uses graphic scales and that a scale is an interval of the straight line that is divided into some parts and supplied with verbal or numerical signs. Thus in one case a scale is an algorithm while in another case it is an interval i.e. something very different. In [KKT 78] a scale is some mapping of an arbitrary empirical system with relations into a numerical system with relations. But in sociological studies, as different authors (cf., for example, [WS 83]) remark, the numbers that are utilized are not always numbers according to the standard meaning. More often they are only signs of grades.

So the most admissible understanding of a scale is according to the theory of abstract properties. In this case the scale is the partially ordered set in which some sociological property or characteristic takes its values. Moreover if we take other meanings of the notion "scale" we see that they are also connected with the construction of abstract property but not exactly with its scale: sometimes with some kinds of abstract properties as in [KKT 78], sometimes with the procedure realizing the mapping p.

The concept of abstract property provides the possibility to give the precise definition of a sociological measurement that may be used for a general definition of measurement (as in [Mer 77]). *Measurement is a process of determination of abstract properties' values for some object by means of the direct interaction with these objects.*

9.4 Interaction and Measurement

Presence of interaction is the essential characteristic of measurement that separates measurements from other kinds of determination of abstract properties' values such as calculations (computations) or argumentation (deduction).

Thus the structure of measurement is reflected in the interaction named set [Bur 90] having the form $M = (X, m, D)$ where X is the measured (studied) system, D is the measuring system (for example, a group of sociologists) and m is the interaction between X and D.

In addition to the interaction named set M an attribute named set A also corresponds to the same measurement. Usually the procedure is a follows [KKT 78]. An empirical system U is associated with the measured object (system) Q.The system U has the form (S, R) where S is some set and R is the system of relations on S. Then some number system $V = (T, Q)$ is chosen. In it T is some set of numbers and Q consists of some natural relations on T. For example Q may be the order relation. According to this approach a measurement is a homomorphism $f : U \to V$. Thus we have the attribute named set $A = (U, f, V)$.

As it is shown in [Bur 84] any relation may be represented by some abstract property. Thus any relational system (like U or V) is represented by some functional named set. From this follows that U and V in the attribute named set are also definite named sets.

Such named sets in which the support and the set of names are also named sets are named sets of the second order. So the attribute named set A has the second order. It gives the functional model of the measurement under consideration.

The construction of a named sets representation gives a more general model for measurement because it makes possible to estimate measurements.

Nevertheless in existing definitions of measurement (in general ones [Mer 77] as well as in special sociological ones [WS 83]) the necessity of interaction is not mentioned and measurement is considered to be a mapping or a procedure or a process. In the given above definition measurement is an action of realization of an operation or a process of measurement. In its turn some definite procedure is corresponded to such process. Using one procedures we can carry out many measurements. Measurement procedure may be considered as classical (recursive) algorithms or limit (superrecursive) algorithms.

Using models of sociological processes and measurement procedures we can elaborate a more adequate schema of sociological investigations because any measurement and especially in sociology needs some time to be realized. But during this time and during realization social processes go on. Thus we have interactions between two kinds of processes. The first are related to the measured system and the second correspond to the measuring system. If the measured system is modeled by a limit computation like the one realized by a limit Turing machine then we have the precise value of some property we need to use some limit measurement procedure.

As a simple model of such situation we can take the set CLT of all limit Turing machines described in chapter 8 and considered as models of social processes. Some set of classical Turing machines ST is used to extract the results of functioning machines from CLT.

Theorem *The precise value of the result of an arbitrary LT may be obtained only by a measurement realized by an adequate alphabetical operator.*

This shows in what cases strict models of measurements give only superrecursive algorithms. Unformally, this is so if the situation is connected with measuring tendencies and other dynamically properties.

It is necessary to remark that sometimes the existence of interaction is difficult to detect. For example, when a sociologist uses observation as the method of sociological investigation he can make some measurements like determination of the influence of the age structure for some community on its behavior. Although in this case interaction is implicit it exists because observation is such an interaction that is active only from one side.

The monographic method [Yad 87] of sociological studies gives the second example. In it a sociologist analyzes different monographs and papers in order to obtain the necessary information. This case is quite different because an interaction exists only between the sociologist and the utilized object. This object is literature. That is why for such methods it is better to speak about estimation or evaluation but not about measurement.

There are different estimation and evaluation procedures. In many aspects they are like the ones corresponding to measurement but they do not necessarily include interaction with the objects under investigation.

Existence of interaction in measurement has the consequence: in sociology like in microphysics the system that realizes the measurement procedure may influence the measured system and thus change to some extent the results of the measurement. From this follows that measurement procedures as well as processes of interaction demand a thorough study and modeling.

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Chapter 10

Serge V. Chernyshenko: Comparative Analysis of Discrete and Continuous Models in Socio-ecology

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Abstract

The applicability of differential equations to description of integer values dynamics is investigated. It is shown that a differential model may be interpreted as a continuous analog of a stochastic flow. The method of reconstruction of cuasi Poisson's flow on the base of multi-dimension differential equations is proposed. Mathematical correctness of the algorithm is proved.

The system has been studied by a computer simulation and a discrete nature of processes has been taken into account. The proposed schema has been applied to the ecological Volterra models and it has been determined that qualitative and quantitative differences in the behaviour of discrete and continuous ones take place.

10.1 Introduction

Nonlinear models allow one to study some phenomena of self-organization in complex systems. The basic nonlinear model for investigation evolutionary processes appears to be proposed by Manfred Eigen and Peter Schuster [ES 79]. In the present book synergetic models of sociodynamics were considered by Wolfgang Weidlich, Klaus G. Troitzsch, and Igor Chernenko.

Catastrophe Theory and Modified Eigen Hypercycle were used for simulation of social and ecological cataclysms ([Che 91b, CC 92]). Most of synergetic models are based on a continuous approximation of discrete real processes in the complex systems considered. Nevertheless such models give us a possibility to study functional patterns of self-organized systems and simulate a set of nonlinear phenomena.

It is sure that taking into account the discrete nature of real processes could allow us to modify nonlinear continuous models and investigate specific effects such as loss of stability caused by discretization [Cherny 89].

As will be shown below discrete properties essentially predetermine quality changes of system behaviour as well as metamorphoses of phase portrait and catastrophic phenomena.

This approach could be used to elaborate techniques of social diagnostic and forecasting of social cataclysms.

Some possible applications of this approach in socio-economics will be considered in section 10.4 and 10.5.

10.2 The stochastic interpretation of differential models

Let us suppose that the variables x_i have discrete nature, but their dynamics can be approximately described by a system of differential equations

$$\frac{dx_i}{dt} = F_i(x_1, \ldots, x_N), i = \overline{1, N} \tag{10.1}$$

The variable x may get only integer values (or discrete values being described by a series with a constant step). In what follows variables x_i will be treated as the size of the i-th group or subpopulation.

The moments of changing of the group sizes are presumed to compose a stochastic flow $\{t_1^{(i)}, t_2^{(i)}, \ldots, t_k^{(i)}, \ldots; i = 1, N\}$. A time interval between two consecutive changes of the group size is considered as stochastic value η_i. It depends on the group sizes $x_1, \ldots, x_N$. The frequency $\eta_i(x_1, \ldots, x_N)$ of the stochastic flow is taken as a basic feature of the stochastic value $\lambda_i(x_1, \ldots, x_N)$.

In accordance with the definition of the stochastic variable $\eta_i(x_1, \ldots, x_N)$ one can write down

$$x_i(t + \eta_i(x_1(t), \ldots, x_N(t))) = x_i(t) \pm 1.$$

We choose the sign "plus" on the right hand side of the equations if the group size has a tendency to grow up and get "minus" when the group size decreases. The tendency of the size to change is determined by the sign of the right hand side $F_i(x_1^{(0)}, \ldots, x_N^{(0)})$ of Equation 10.1.

The stochastic process can be described as

$$x_i(t + D_i(x_1(t), \ldots, x_N(t))) = x_i(t) \pm 1. \tag{10.2}$$

or

$$\frac{x_i(t + D_i(x_1, \ldots, x_N)) - x_i(t)}{D_i(x_1, \ldots, x_N)} = \pm \frac{1}{D_i(x_1, \ldots, x_N)} \tag{10.3}$$

where $D_i(x_1, \ldots, x_N)$ is expected value of stochastic variable η_i.

The left hand side of Equation 10.3 is a finite differential approximation of the derivative x_i at point t. For small D_i Equation 10.4 is equal to

$$\frac{dx}{dt} = \pm \frac{1}{D_i(x_1, \dots, x_N)}. \tag{10.4}$$

Comparing Equations 10.1 and 10.4 one can obtain a relation

$$D_i(x_1, \dots, x_N) = \frac{1}{F_i(x_1, \dots, x_N)}$$

which reflects the connection of differential models and stochastic flows.

10.3 The algorithm of the time interval calculation

Let us consider a problem of forecasting the dynamics of the group sizes in some time interval. The group sizes at the initial moment t_0 are as follows

$$x_i(t_0) = x_i^{(0)}, i = \overline{1, N}$$

An algorithm for finding the integer sizes at moments $t > t_0$ is taken as follows.

The first step. Let us suppose that stochastic flow is a Poisson flow. In this case the stochastic variable η_i has the normal distribution. We are choosing the median as expected value of the stochastic value. In the case of an exponential law the median differs from the mathematical expectation by a coefficient $\ln 2$ whose value approximately equals to 1. But as will be shown below, the mathematical expectation is unapplicable in this case.

The frequency of the stochastic flow is

$$\lambda_i^{(1)} = |F_i(x_1^{(0)}, \dots, x_N^{(0)})| \ln 2 \tag{10.5}$$

The expected moment $\Delta t_i^{(1)}$ of the size change of the i-th group can be determined by

$$\Delta t_i^{(1)} = |F_i(x_1^{(0)}, \dots, x_N^{(0)})|^{-1} \tag{10.6}$$

Let us assume that the minimal value of $\Delta t_i^{(1)}$ is attained in the l_1-th group:

$$\Delta t_{l_1}^{(1)} = \min_i \Delta t_i^{(1)} \tag{10.7}$$

The value $\Delta t_{l_1}^{(1)}$ is taken as ΔT_1 - the duration of first time step. It is presumed that the size of l_1-th group increases or decreases by 1 (depending on the sign of $F_{l_1}(x_1^{(0)}, \dots, x_N^{(0)})$) in the moment $t_1 = t_0 + \Delta T_1$.

$$x_{l_1}(t_1) = x_{l_1}(t_0 + \Delta T_1) = x_{l_1} + \operatorname{sign}(F_{l_1}(x_1^{(0)}, \dots, x_N^{(0)})), \tag{10.8}$$

and the sizes of the other groups are not changed:

$$x_i(t_1) = x_i(t_0 + \Delta T_1) = x_i^{(0)}, i = (\overline{1, N}) i \neq l_1. \tag{10.9}$$

The second step. Because the size of one group has been changed, Equation 10.6 must be corrected. Changing of one group size influences the process of changing other group sizes. For the l_1-th group the expected moment of changing can be determined similarly to Equation 10.6

$$\Delta t_{l_1}^{(2)} = |F_{l_1}(x_1^{(1)}, \ldots, x_N^{(1)})|^{-1}. \tag{10.10}$$

but for other groups a specific algorithm is needed.

The increasing (decreasing) of the value t_1 induces a changing of the right hand sides of system 10.1, i.e. a changing the frequencies of the stochastic flows. For $t \in [t_0, t_0 + \Delta T_1]$ they are determined by Equation 10.5, and for $t > t_0 + \Delta T_1$ we have

$$\lambda_i^{(1)} = |F_i(x_1^{(1)}, \ldots, x_N^{(1)})| \ln 2, i = (\overline{1, N}), i \neq l_1. \tag{10.11}$$

Generally speaking, the stochastic flows are not the Poisson flow.

Let us define the law of distribution of the stochastic value η_i. The probability density function can be described on the base of Equations 10.5 and 10.11, similarly to the case of flow with constant frequency

$$\rho_i^{(1)}(\tau)_i = \begin{cases} \lambda_i \exp(-\lambda_i^{(0)}\tau) & \tau > \Delta T_1 \\ \lambda_i^{(0)} \exp(-\lambda_i^{(0)}\Delta T_1 - \lambda_i^{(1)}(\tau - \Delta T_1)), & \tau > \Delta T_1 \end{cases} \tag{10.12}$$

The median $D_i^{(0)}$ can be obtained from equation

$$\int_0^D \rho_i^{(1)}(\tau) d\tau = 0,5$$

in accordance with its definition. Taking into account Equations 10.5, 10.11, 10.12 one can obtain

$$D_i^{(1)} = |F_i(x_1^{(1)}, \ldots, x_N^{(1)})|^{-1} + (1 - |F_i(x_1^{(0)}, \ldots, x_N^{(0)})| \cdot |F_i(x_1^{(1)}, \ldots, x_N^{(1)})|^{-1}) \Delta T_1. \tag{10.13}$$

The expected value of $\Delta t_i^{(1)}$ (the interval between the moment $t_0 + \Delta T_1$ and the moment of change of size of the i-th group) can be determined by

$$\Delta t_i^{(2)} = D_i^{(1)} \Delta T_1 = (1 - \Delta T_1 |F_i(x_1^{(0)}, \ldots, x_N^{(0)})|) \cdot |F_i(x_1^{(1)}, \ldots, x_N^{(1)})|^{-1} i \neq l_1. \tag{10.14}$$

in accordance with Equations 10.6 and 10.7

$$\Delta T_1 < |F_i(x_1^{(0)}, \ldots, x_N^{(0)})|^{-1}$$

and $\Delta t_1^{(1)} > 0$.

Equation 10.14 is correct in the case when the expected interval is determined as the median of the stochastic variable. If the interval is determined as the mathematical expectation of η_i, the equation is not true. This fact is not obvious and reflects a specific feature of the process.

The minimal $\Delta t_i^{(1)}$ from Equations 10.10 and 10.14 is chosen as the value of the second step $\Delta T_2 = \Delta t_{l_2}^{(2)} = \min \Delta t_i^{(2)}$. Frequencies of stochastic flows are constants for interval t_2 and the change of size the of l_2-th group is expected to realize before the change of sizes of other groups.

Values $x_i^{(2)}, i = 1, \ldots, N$ are obtained similarly to Equations 10.8, 10.9. The current moment is $t_2 = t_1 + \Delta T_2$.

$(M+1)$**-th step.** The expected moment of the next change of size of the l_M-th group is determined by the following relation

$$\Delta t_{l_M}^{(M+1)} = |F_{l_M}(x_1^{(M)}, \ldots, x_N^{(M)})|^{-1} \tag{10.15}$$

For other groups

$$\Delta t_{l_M}^{(M+1)} = [1 - \sum_{v=m_i+1}^{M} \Delta T_v |F_i(x_1^{(v-1)}, \ldots, x_N^{(v-1)})|] \cdot |F(x_1^{(M)}, \ldots, x_N^{(M)})|^{-1} \tag{10.16}$$

where m_i is the number of step, when the size of the i-th group is changed for the last time.

Using the mathematical induction method it is easy to prove that $\Delta t_i^{(M+1)}$ in Equation 10.16 is positive.

1. It is obvious that $\Delta t_i^{(m_i+1)} > 0$ because from Equation 10.16 it follows

$$\Delta t_i^{(m_i+1)} = |F_i(x_1^{(M)}, \ldots, x_N^{(M)})|^{-1}.$$

2. Assuming that $\Delta t_i^{(l)} > 0$ for all $l \notin [m_i + 1, L]$ we can prove that

$$\Delta t_i^{(L=1)} > 0$$

From Equation 10.5

$$\Delta t_i^{(L+1)} = [1 - \sum_{v=m_i+1}^{L-1} \Delta T_v |F_i(x^{(v-1)})| - \Delta T_L |F_i(x^{(L+1)})|] \cdot |F_i(x^{(L)})|^{-1} \tag{10.17}$$

Since it is supposed that the size of the i-th group is not changed at the L-th step, $\min \Delta t_j^{(L)} = \Delta T_L$ is not achieved on $\Delta t^{(L)}$. Hence

$$\Delta T_L < \Delta t_i^{(L)} = [1 - \sum_{v=m_i+1}^{L-1} \Delta T_v |F_i(x^{(v-1)})|] |F_i(x^{(i-1)})|^{-1}. \tag{10.18}$$

From Equations 10.17 and 10.18 it follows that $\Delta t_i^{(L+1)} > 0$ for any L. Q.E.D.

From Equations 10.15, 10.16 the minimal interval is determined as

$$\min \Delta t_i^{(M+1)} = \Delta t_{l_{M+1}}^{(M+1)}$$

The value of the $(M+1)$-th step is ΔT_{M+1}. The value of the sizes is determined as in Equations 10.8, 10.9

$$x_{l_{M+1}}(t) = x_{l_{M+1}}(t) + \mathrm{sign}(F_{l_{M+1}}(x)), x_i(t) = x_i(t), i = (\overline{1,N}), i \neq l_{M+1}.$$

Steps repeat until t exceeds T.

10.4 "Supply and demand" model

To illustrate some possible applications of the method we will consider two-dimensional nonlinear models that can be represented in the form of Equation 10.1.

To describe a "supply and demand" process the modified Volterra model can be used:

$$\begin{cases} \dot{x} = ax - bxy - cx^2 \\ \dot{y} = dxy - ey^2 \end{cases} \tag{10.19}$$

Taking into account the structure of the model we may consider a "demand" as a "predator" and a "supply" as a "prey". While one could suppose that supply might be treated as a "prey" of demand which "absorbs" it, the more detailed analysis of equations demonstrates that the consumer is to be considered as a "prey" of supply. For example, in accordance with the model, vanishing demand leads to the extinction of supply. In the same way vanishing prey causes the disappearance of predators. On the other hand, passing from existence of "supply-predator" produces the increasing of "demand-prey" until stabilization.

The variable x describes the intensity of demand and may be measured by the number of people which are ready to buy goods at the current moment. The variable y describes the intensivity of supply and can be measured by the number of goods for sale at the current moment. Thus phase coordinates can have only integer values. So we can apply the method that was considered above.

The coefficients of Equation 10.19 can be interpreted as follows. Coefficient c determines a maximum level of demand. Coefficient a depends on the rate of population growth and the inclination to extraordinary demand that means exponential growth of demand caused by abrupt decrease of supply. Then the demand stabilizes at value a/c. Coefficient b describes the effectiveness of trade net and advertisement. Parameter d determines the intensity of the feedback relation between production and consumption. Coefficient e describes the capacity of market. Value d/e is a level of maximum consumption per person.

The classic Volterra model is represented by the following relations [Vol 31]

$$\begin{cases} \dot{x} = ax - bxy - cx^2 \\ \dot{y} = dxy - ey \end{cases} \tag{10.20}$$

These relations differ from Equation 10.19 in the last term of the second relation only. The specific form of Equation 10.19 is predetermined by the innate system mechanism which stabilizes production in case of stable demand. Model 10.20 does not account for this aspect. For this model, stabilization of supply can be realized as a result of a

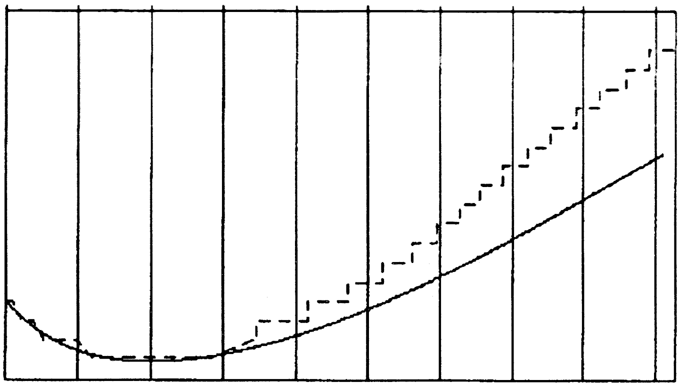

Figure 10.1: Dynamics of demand (model 10.19). Equilibrium point is node

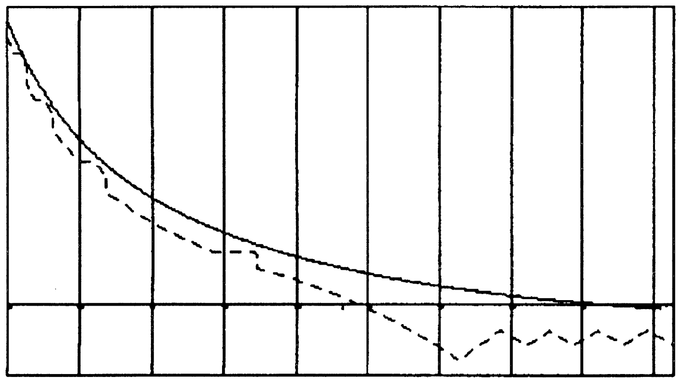

Figure 10.2: Dynamics of supply (model 10.19). Equilibrium point is node

long oscillation process only [Smi 74]. Thus Equation 10.19 is more stable and more adequate to modern economical realities.

Results of numerical experiments with the models are represented on Figures 10.1–10.10. Continuous solutions are drown by continuous lines and solutions of the corresponding discrete equations are shown by dotted lines. Figures 10.1 and 10.2 represent

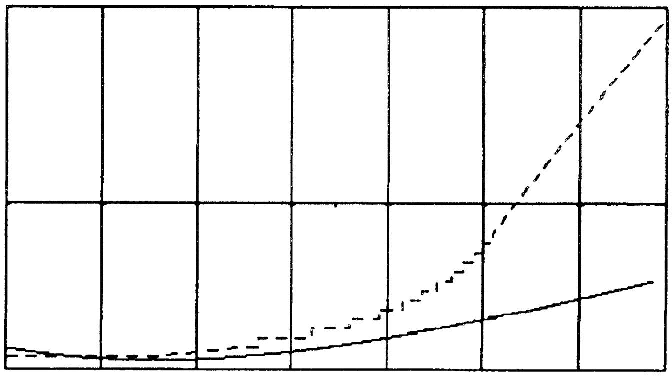

Figure 10.3: Dynamics of demand (model 10.20). Equilibrium point is node

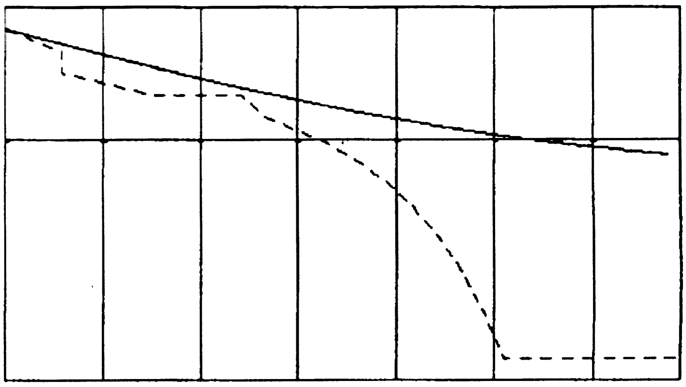

Figure 10.4: Dynamics of supply (model 10.20). Equilibrium point is node

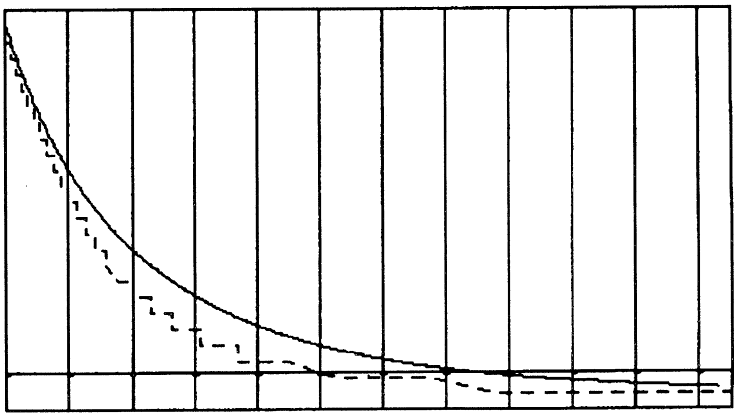

Figure 10.5: Dynamics of demand (model 10.19). Equilibrium point is focus

the solutions $x(t), y(t)$ of model 10.19 with coefficients: $a = 10$; $b = 0.5$; $c = 0.1$; $d = 0.1$; $e = 1$ (equilibrium point is $x^* = 66.(6)$; $y^* = 6.(6)$). Figures 10.3 and 10.4 display solutions of Equation 10.20 with coefficients: $a = 10$; $b = 0.5$; $c = 0.1$; $d = 0.1$; $e = 5$ (equilibrium point is $x^* = 50$; $y^* = 10$). In both cases the equilibrium points are stable nodes. Initial conditions are $x(0) = 35$; $y(0) = 15$.

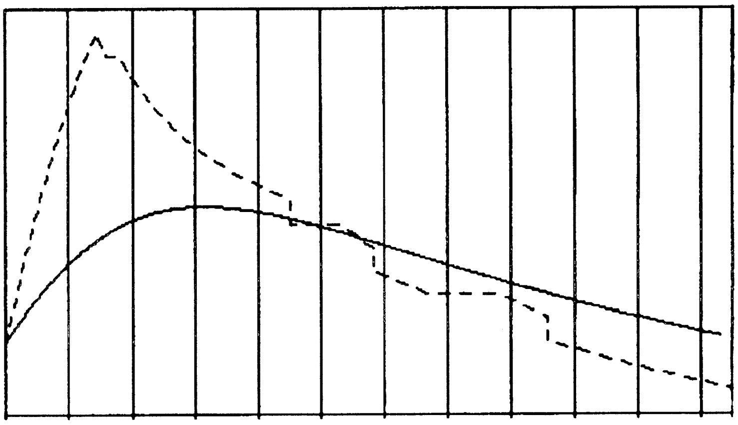

Figure 10.6: Dynamics of supply (model 10.19). Equilibrium point is focus

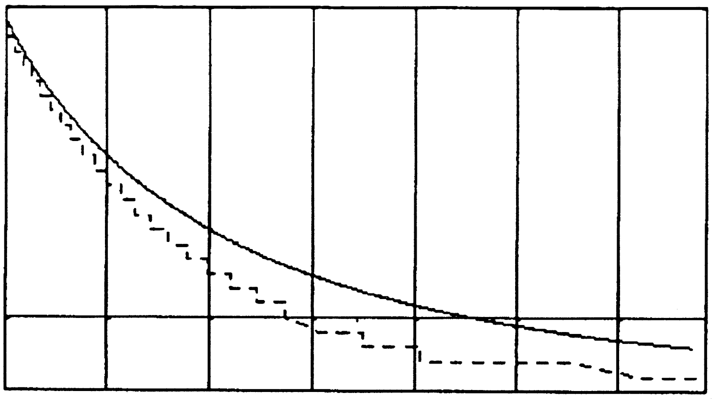

Figure 10.7: Dynamics of demand (model 10.20). Equilibrium point is focus

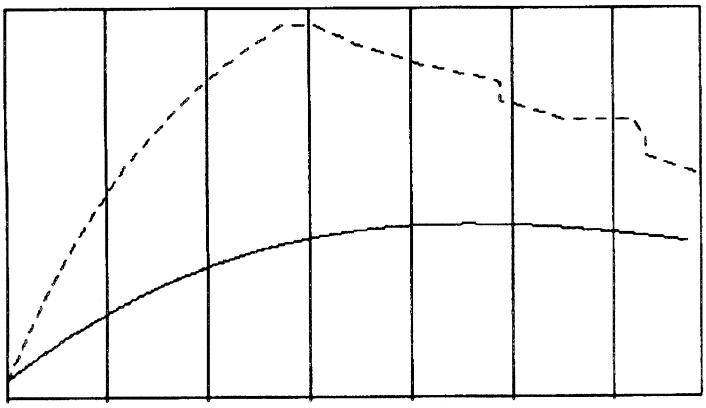

Figure 10.8: Dynamics of supply (model 10.20). Equilibrium point is focus

As can be seen from the pictures, discrete trajectories are sufficiently different from those of the continuous case, since discretization decreases stability [Casti 79]. At the beginning the discrete model describes more intensive depression of production as a result of low level of initial demand. Then the demand has rapid growth, but in some

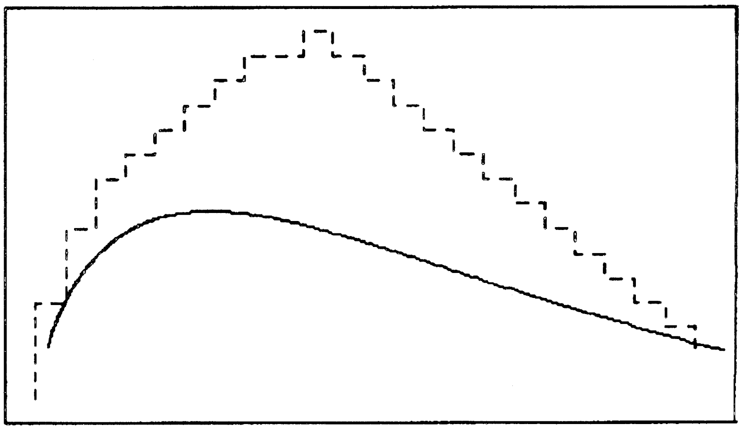

Figure 10.9: Phase portrait of solution of model 10.19. Equilibrium point is focus.

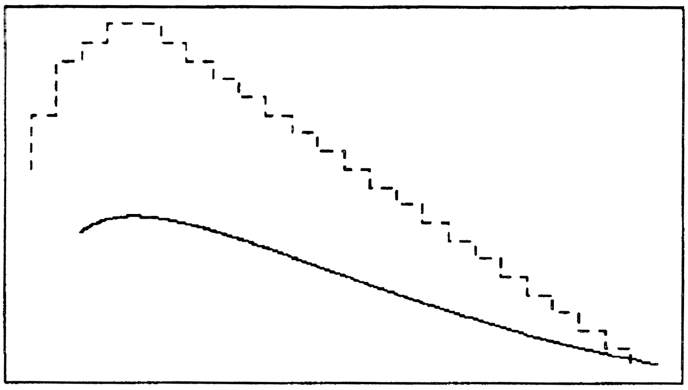

Figure 10.10: Phase portrait of solution of model 10.21). Equilibrium point is focus.

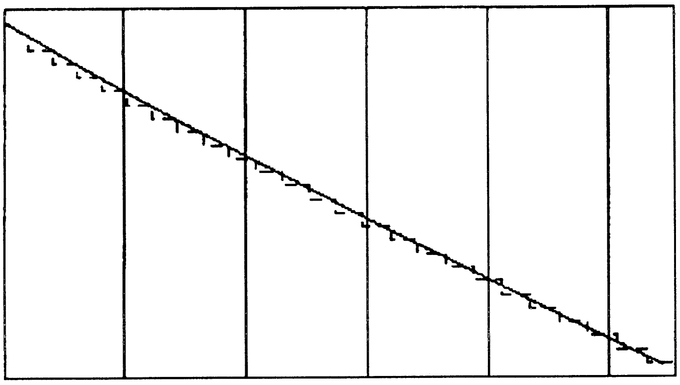

Figure 10.11: Production dynamics of first firm. Model 10.21.

cases such growth is not sufficient to stabilize the supply which diminishes to zero. The last example illustrates the following. Absence of operative reaction of producers to the situation on the market (as it takes place in model Equation 10.21) leads to an unstability of the economic system.

Figures 10.5–10.10 illustrate another case of dynamics of the models. The coefficients of Equation 10.19 are determined as follows: $a = 10$; $b = 1$; $c = 1$; $d = 1$; $e = 0.5$ (equilibrium point is $x^* = 3\frac{1}{3}$; $y^* = 6\frac{2}{3}$). In case of Equation 10.20 they are taken as follows: $a = 10$; $b = 1$; $c = 1$; $d = 1$; $e = 5$. In both cases $x^* = 5$; $y^* = 5$ and equilibrium points are focuses. Initial conditions are: $x(0) = 25$; $y(0) = 10$.

The behaviour of the discrete system is essentially different from the continuous one, but the stability of the solution is not lost. At first supply has rapid growth connected with an initially excessive level of demand while the initial level of supply also exceeds the stability level. Then the production is rapidly decreasing. After this fading oscillations are realized.

Phase portraits of systems are represented on Figures 10.7 and 10.10. At a great distance from equilibrium points discrete trajectories are sufficiently distinguishing from that of continuous model. At the vicinity of the stable point one could observe an effect of "beating" discrete solutions.

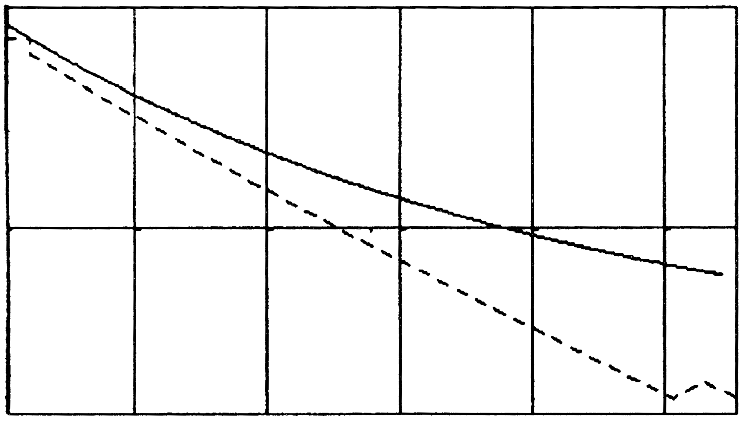

Figure 10.12: Production dynamics of second firm. Model 10.21.

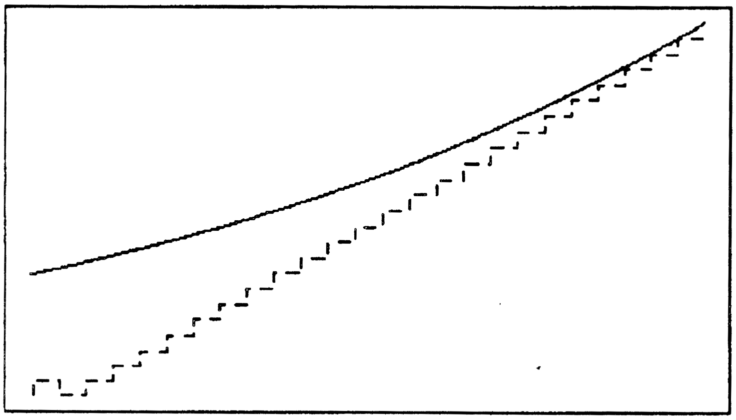

Figure 10.13: Phase portrait of solution of model 10.21

10.5 Model of competition

The classic model of competition was proposed by Volterra [Vol 31]. It could be presented in the following form

$$\begin{cases} \dot{x} = ax - bx^2 - cxy \\ \dot{y} = dy - exy - fy^2 \end{cases} \tag{10.21}$$

Let us suppose that variables x and y describe intensities of production of two competitive firms. They may be measured by integer number of goods produced per unit of time. Coefficients c and e describe interaction of firms. Their values are predetermined by level of interchangeability of goods produced by the firms, and by level of isolation of their markets. As a rule, $c = e$. Coefficients d and f depend on capacity of market for goods produced by both firms respectively. They determine level of "internal competition" (i.e. competition between goods of the same firm).

Equilibrium state of the model 10.21 can be stable if level of "internal competition" is higher than level of "external competition" (i.e. competition between goods of different firms). That is stability is determined by the following relation $bf > ce$.

This condition is true when goods of different firms have essentially different spheres of using or their markets are located on different territories. It is known that Gauze's principle [Smi 34] is applicable for economic systems. A firm with small initial capital has a chance to achieve a good position on the market if its production has unique customer quality or it is orientated towards new markets.

The coefficients of the model are $a = 10$; $b = 0.02$; $c = 0.01$; $d = 100$; $e = 0.01$; $f = 0.2$. Initial values are: $x(0) = 500$; $y(0) = 500$. The stable equilibrium point is: $x^* = 256$; $y^* = 487$. The results of calculations are represented on Figures 10.11–10.13. Although the second firm has larger market capacity for goods than the first one has, it is not essentially for model dynamics at the beginning. Both firms have equal decrease of production. It is need a large amount of time for potentials of second firm to be revealed and to demonstrate a growth of output. Solutions of continuous and discrete systems are essentially different. It is to be emphasized that such phenomena are observed at large scales of system variables.

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Chapter 11

Sergei K. Polumiyenko: The Ecosystem Structural Game-Theoretical Model

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Abstract

The class of game-theoretical models of ecosystems is considered as the base of an algorithm and program apparatus for solving human activity optimization problems. The aim is to find non-destructive strategies as well as to choose the most essential corresponding social interests in the development of industrial and other actitivies in certain biogeocenoses.

11.1 Introduction

Now there exists a rather wide class of models of heterogeneous, compound and structural organization, ecological and economic systems, Nature models the majority of which is connected with the general system law of existing and evolution. Models for such heterogeneous at their inner laws systems are developed much more less and they can't pretend to be realized in any decidable algorithmic system allowing to optimize the initial complex system. This circumstance sometimes directs to the essential reducings in simulation and analysis the human activities optimization problem.

Introducing our models we consider the following aspects of simulating as the initial.

1. Every rather complex system includes as the factors of existing and evolution heterogeneous elements reflecting its deterministic, random, and conflict features. Used now approaches of picking out the main separate system feature in the simulating process can't direct us to adequate models and results of optimization.

2. We have to find the global optimality principle. Due to the experience of real world evolution we can characterize it in general as the moving to stability in the environment of different heterogeneous troubles coming from the system elements and its subsystems. Also, the global optimality principle can't exclude another optimality principles on the more detailed system levels.

3. The majority of models presenting complex structural systems have no precise solving algorithms. But, we can find such solutions in the class of incorrect heuristic algorithms on the base of decision making experience formalizing.

4. Existing game-theoretical models are not connected with the algebraic, formal systems in terms of which we can investigate the properties of models algorithmizing .

5. Algorithmic formal systems allowing us to simulate and optimize described systems are not developed nowadays.

Here we suggest to the reader an attempt to build the models satisfying these properties and serving as the base for such algorithmic systems.

Models have descriptive features in presentation of different complex systems. We separate three classes of models depending on the available information: the descriptive - presenting the initial system objects and processes in the case of absence of any known behaviour law or global optimality principle of the system; the constructive - presenting the case when there exists such optimality principle, one of them is discussed in this paper; the normative - presenting the case when (some) corresponding to optimality principle solutions are known also. The general model is formed as the extended union of these three classes of models.

11.2 The Industry Model

We put the model components as in [Pol 90], the reader can also find a more detailed description there.

Let us suppose the following elements and their sets are given: a player of the level p — an employee $i \in I^p$; the set of his interests U_I^p; the set of strategies of player i formed as prestrategies vectors — S_i; time to fulfil the strategy s for the player p — $T_{s,i}$; the complexes of coalitions of interests and actions $\mathcal{K}_i^p$ and $\mathcal{K}_a^p$; the set of coalitional interests $U_{\mathcal{K}_i}^p$; the set of coalitional strategies $\{S_K^p\}_{K\in\mathcal{K}_a^1\subseteq\mathcal{K}_i^1}$; the set of situations — vectors of strategies $\tilde{S}^p$; production — the result of the coalition K strategy turnout — $D_K^p = D_K^p(\tilde{s})$; the capital of the player i, the latter is given to fulfil the strategy S_i^p — $G^p(i)$ — the industry funds in currency; the payoff function $H_i^p(\tilde{s})$ of the player i — the salary he gets after situation $\tilde{s}$ realization; the capital $G^p(K)$ of coalition K; the payoff function of coalition $H_K^p(\tilde{s})$ in the situation $\tilde{s}$; the work quotas vector th^p; the task th^p turnout expenditures — $C^{p,O}$ — and the profit $C^{p,1}$; $H_{\tilde{I}^{p-1}}^p(\tilde{s}^{p-1})$ — the encourage fund of the level $p-1$, $\tilde{I}^p = I^p \cup \{i^{p+1}\}$.

Let us put player's utility functions $\{\tilde{L}_i^p\}_{i\in I^p}$ [Pol 90] and similarly task turnout functions:

$$\tilde{P}^p(th^p) = \sum_{k=1}^{k_0} \gamma_k^{p,th} P(th_k^p),$$
$$\sum_{k=1}^{k_0} \gamma_k^{p,th} = 1, 0 \le \gamma_k^{p,th} \le 1, \tag{11.1}$$

where $P(th_k^p)$ is a turnout predicate. We then obtain the aggregate the level p:

$$\Gamma^p(t) = \langle i^p, F^p, \tilde{P}^p(th), C^{p,0}, C^{p,1}, S_i^p, \tilde{L}_i^p, G^p(th), H_{\tilde{I}^{p-1}}^p(\tilde{s}^{p-1}), [t_0, T], 2 \le p \le p_0\rangle, \tag{11.2}$$

the level $p-1$:

$$F^p(t) = \langle \Gamma_1^{p-1}(t) \ldots \Gamma_{n_{p-1}}^{p-1}(t) \rangle, n_{p-1} \le card(I^{p-1}(t)),$$
$$\Gamma^{p-1}(t) = \langle I^{p-1}(t), \{S_i^{p-1}\}_{i \in I^{p-1}(t)}, \mathcal{K}_i^{p-1}(t), \mathcal{K}_a^{p-1}(t),$$
$$\{\tilde{L}_i^{p-1}\}_{i \in I^{p-1}(t)}, \{G^{p-1}(K)\}_{K \in \mathcal{K}_a^{p-1}(t)}, \{H_K^{p-1}\}_{K \in \mathcal{K}_a^{p-1}(t)} \rangle.$$

Let us consider together with th^p, D^p the value $\tilde{G}^p$ of task th^p turnout. We pick out following main components of expenditures $\tilde{G}^p$ — the vectors of: neccessary natural resources Res^p and their values CR^p, equipment Eq^p, and labour $I^p = \cup_p \tilde{I}^{p-1}$. We denote summary expenditures by $Pm^p = C^{p,0}$ and by Tm^p — the term of task th^p turnout (production realization). Hence, we begin with the extended vector of task-demand

$$Th^p = (th^p, Res^p, Eq^p, \tilde{I}^p, Pm^p, Tm^p). \tag{11.3}$$

Then the difference

$$\begin{aligned} Rt^p &= \tilde{G}^p - Pm^p, \\ Pm^p &= CR^p + G^p(\tilde{I}) + H^p(\tilde{I}), \end{aligned} \tag{11.4}$$

$Rt^p = C^{p,1}$, indicates the net profit of the level p player from turnout of th^p, and if th^p is a demand vector — the profit expected after its realization.

In 11.2 we can also consider the equations of [For 61] and [For 72].

The game in eqs. 11.1–11.4 with assumptions made above we call the descriptive industry model and denote by $\Gamma_i^p(t)$.

Due to our build-up we can form the collection

$$F^{p+1}(t) = \langle \Gamma_i^{p,n}(t), 1 \le n \le n_p \rangle$$

which defines the industry model for its different branches.

We connect level $p+1$ with $F^{p+1}(t)$. On the base of vectors Th^p we form union Th^{p+1} where we use as orders and the values of financial resourses.

11.3 The Population Dynamics Model

We describe the population at the moment of time t by matrix $IP(t)$ with elements $IP_{m,l}(t)$ indicate the population quantity in m professional and l age group. The population dynamics is reflected by the following aspects: 1) birth rate $IB(t)$; 2) age scale movement $IMT_{m,l}(t)$; 3) migration $IMR_{m,l}(t)$; 4) mortality $ID(t)$; 5) occupations change $IKR_{m,l}(t)$.

We omit their detailed consideration and define that at the moment t_{k+1} the group is characterized by expression

$$IP_{m,l}(t_{k+1}) = IP_{m,l}(t_k) + CI_{m,l}(t_{k+1}),$$

where the values in changes $CI_{m,l}(t_{k+1})$ are taken for the year t_k to the beginning of the year t_{k+1}. At the initial moment t_0 we suppose

$$IP_{m,l}(t_0) = IP_{m,l,0} = const, \tag{11.5}$$

Let us define $CI_{m,l}(t_k)$ as a random value. We take as an elementary event the fact that value $CI_{m,l}(t_k)$ is equal to $y_k^{m,l}$ at the moment t_k on the given interval $[\alpha,\beta]_k^{m,l}$ of the set of integer numbers Z^1 which we define as a σ-algebra of events $y_k^{m,l} \in Y_k^{m,l}$. Let $C\kappa_k^{m,l}$ be a measure on $\mathcal{A}_k^{m,l} = [\alpha,\beta]_k^{m,l}$ and $CJ(t_k)^{m,l}$ — a result probability space $(Y,\mathcal{A},C\kappa)(t_k)^{m,l}$ with which we identify a $m-l$-group at the moment of time t_k. Taking $CI_{m,l}(t_k)$ as a function on $\mathcal{A}_k^{m,l}$, we obtain that it is a random value defined by measures $C\kappa(y_k^{m,l}) = P\{CI^{m,l}(t_k) = y_k^{m,l}\}$.

At the initial moment of time t_0 we have 11.5, at the moment $t+\tau$ suppose

$$IP_{m,l}(t_1) = IP_{m,l,0} + E(CI_{m,l}(t_1)/IP_{m,l,0}),$$

where E is the expected value taken under the condition $IP_{m,l}(t_0) = IP_{m,l,0}$. So, we obtain

$$IP_{m,l}(t_{k+1}) = IP_{m,l,0} + \sum_{j=0}^{k} E(CI_{m,l}(t_{j+1})/IP_{m,l}(t_j = y_j^{m,l}),$$

and assume that CI and IP are partially constant on the interval $[t_k, t_{k+1})$. Let

$$IP_{m,l,0} = E(CI_{m,l}(t_0)/IP_{m,l}(t_{-1})),$$

then

$$IP_{m,l}(t_{k+1}) = IP_{m,l,0} + \sum_{j=-1}^{k} E(CI_{m,l}(t_{j+1})/IP_{m,l}(t_j)) =$$
$$= \sum_{j=-1}^{k} E(CI_{m,l}(t_{j+1})/\mathcal{A}_j^{m,l}),$$

where $\mathcal{A}_j^{m,l}$ are corresponding to t_j σ-algebras. Let

$$IP(t_k) = \sum_{m,l} IP_{m,l}(t_k) \tag{11.6}$$

are random values expressing population quantity, $CJ(t_k)$ their spaces and $IP(t)$ and $CJ(t)$ — corresponding to eq. 11.6 random process and space when $t \in [t_0, T]$, we shall identify $CJ(t)$ with the set of players (population).

We suppose that in this model the player's cj strategy $s \in S_{cj}$ is only the choice of the his interests u_{cj} realization way from the existing possibilities given by the system including him and this is connected with his entry into coalitions $CK \in C\mathcal{K}_a(t)$. We identify the subset $C\mathcal{K}_a(t)$ with $m-l$-groups. The union of these groups at different m and l is identified with the whole $C\mathcal{K}_a(t)$.

Let $D^{p+1}(t)$ be a corresponding to $F^{p+1}(t)$ vector of production, services, etc fulfilled by all p-level systems from $F^{p+1}(t)$, and $CD^{p+1}(t)$ be its part directed to satisfy interests CU_{cj} of players $cj \in CJ(t)$. For all $CK \in C\mathcal{K}_i(t)$ we suppose that $\tilde{u}_{C\mathcal{K}(t)}$ is a result vector of population interests and $cu_{C\mathcal{K}_i(t)}$ is a vector of demand $\tilde{u}_{C\mathcal{K}(t)}$ satisfaction by production $CD^{p+1}(t)$, $CP(cu_{C\mathcal{K}_i}(t))$ is built as function 11.1,

$$DP((\tilde{u},cu)_{C\mathcal{K}_i(t))} = CP(\tilde{u}_{C\mathcal{K}_i(t))} - CP(cu_{C\mathcal{K}_i(t))},$$

and $C\mathcal{K}_a(t)$ is a complex of action coalitions of players cj.

Following [AuSh 74], we shall consider the strategies under the assumption that individual players cannot change the situation as a whole. Let cs_t and $c\eta_t$ are the vectors of coalitional strategies and corresponding to them measures on $CK \in C\mathcal{K}(t)$. Having put that $C\eta_t$ is a consistent extension [Vor 62] of measures $c\eta_t$, we define a probability space of situations cs_t - CS_t. Similarly to the above ones but for coalitions only we define utility functions (of interests $\tilde{u}_{CK}$ satisfaction) of coalitions $CK \in C\mathcal{K}(t)$:

$$L_{CK}(\tilde{u}_{CK}(cs_t)) = \sum_{m=1}^{m_0} \gamma_{CK}^m R(\tilde{u}_{CK}^m(cs_t)), \tag{11.7}$$

$0 \le \gamma_{CK}^m \le 1$, $\sum_{m=1}^{m_0} \gamma_{CK}^m = 1$, where R and $\tilde{u}_{CK}$ are the functions of random values. Let $CU_{CK}^m(t)$, $CU_{CK}(t)$ be formed as $CS_{CK}(t)$, $CS(t)$ probability spaces built on $\tilde{u}_{CK}^m$, $\tilde{u}_{CK}$ with measures $c\nu_{CK}^m(t)$, $c\nu_{CK}(t)$, and $CU(t)$, $c\nu(t)$ are corresponding to $\tilde{u}_{CK}$, $c\nu_t$ is a consistent extension of $c\nu_{CK}(t)$. Function $R(\tilde{u}_{CK}^m(cs_t))$ is an expected value taken at $C\eta_t$ and $c\nu_t$:

$$\begin{aligned} R(\tilde{u}_{CK}^m(cs_t)) &= \sum_{h=1}^{h_m}\sum_{g=1}^{g_m} P_{hg}(\tilde{u}_{CK}^m) c\nu_t^g C\eta_t^h = \\ &= \sum_{h=1}^{h_m}\sum_{g=1}^{g_m} P_{hg}(\tilde{u}_{CK}^m) c\nu_{CK}^g(t) C\eta_{CK}^h(t), \end{aligned} \tag{11.8}$$

where $C\eta_t^g$, $c\nu_t^h$ are probabilities of strategy h appearance and of coalition CK interests satisfaction by it in situation g, $P_{hg}(\tilde{u}_{CK}^m)$ is a corresponding to them predicate value, $C\eta_t$ and $c\nu_t$ are stochastically independent, and $c\nu_{CK}^g(t)$, $c\eta_{CK}^h(t)$ are the elements of extensions $c\nu_t$ and $C\eta_t$. According to eqs. 11.7–11.8 we obtain

$$\begin{aligned} CL_{CK}(\tilde{u}_{CK}(cs_t)) &= \sum_{m=1}^{m_0}\sum_{h=1}^{h_0}\sum_{g=1}^{g_0} \gamma_{CK}^m P_{hg}(\tilde{u}_{CK}^m) c\nu_t^g C\eta_t^h, \\ 0 \le \gamma_{CK}^m \le 1, \sum_{k=1}^{m_0} \gamma_{CK}^m = 1. \end{aligned}$$

The game

$$\Gamma_d(t) = \langle CJ(t), CU_{C\mathcal{K}(t)}, CS(t), \{CL_{CK}\}_{CK \in C\mathcal{K}(t)}, [t_0, T]\rangle,$$

modified at each transition between the intervals of game playing $[t_k, t_{k+1})$ of segment $[t_0, T]$ division we call the descriptive population dynamics model.

11.4 The Ecosystem Model

We shall identify each of the existed biogeocenosis components with the fictive player ir of the set IR. The players vector under $\delta_{IR} = card(IR)$ then reflects all biogeocenosis resources. Let $DR(t) \subset IR$ be a subset of biogeocenosis polluting players.

Let $RS_0 = RS(t_0)$ be the initial state. We suppose at each $t \in [t_k, t_{k+1})$ are given vectors of resources $Res^{p+1}(t)$ and $RF(t)$ taken and left in biogeocenosis, and vector $RF(t_0) = RF_0$. We assume for the components of these vectors it is fulfilled:

$$RF(t_k) = RF_0 - \sum_{j \le k} Res^{p+1}(t_j), \tag{11.9}$$

and 11.9 is partially constant on the intervals $[t_k, t_{k+1})$. We denote by $RD^{p+1}(t)$ a subvector of $D^{p+1}(t)$ included Nature polluting components. As before, $RD^{p+1}(t_0) = RD_0^{p+1}$ is its initial state, $RD^{p+1}(t)$ is partially constant and we have

$$RD^{p+1}(t_k) = RD_0^{p+1} + \sum_{j \le k} RD^{p+1}(t_j). \tag{11.10}$$

We suppose that $RF(t)$ is the result of players $IR \setminus DR$ activity, divide the set IR into the subsets of distructing and recreating players, and consider the Nature reaction at the activity of $F^{p+1}(t)$ subsystems reflected by $D^{p+1}(t)$, $Res^{p+1}(t)$. The set of player ir strategies at the moment of time $t \in [t_k, t_{k+1})$ we denote by $RS_{ir}(t)$.

Together with deterministic sets $RS_{ir}(t)$ we shall also consider probability spaces of strategies $SR_{ir}(t) = (RS_{ir}, \sigma S_{ir}, R\theta_{ir})(t)$ for players $ir \in JR(t) \subset IR(t)$. We can obtain a common expression for strategies from $RS_{ir}(t)$ and $SR_{ir}(t)$, using the concept of random transitions families described in [Vor 64]. We omit the neccessary construction and suppose that $RS(t)$ and $R\theta(t)$ are corresponding to them probability space and measure.

Let $R\mathcal{K}_i(t) = R\mathcal{K}_a(t) = R\mathcal{K}(t)$ are constant for each $t \in [t_k, t_{k+1})$ complexes of coalitions of interests and actions.

Due to interconnections of all the players in IR, we consider realization of payoff functions through coalitional strategies only. Let $H_{RK}(R\theta_t)$ be a profit of coalition RK in situation $R\theta_t$ when $t \in [t_k, t_{k+1})$ and

$$H_{RK}(R\theta_t) = \sum_{RK \in R\mathcal{K}(t)} H_{RK}(RS_t) R\theta_t(RS_t), \tag{11.11}$$

with eqs. 11.10–11.11 we have

$$\begin{aligned} RF(t_k) &= RF(t_0) - \sum_{j \le k} Res^{p+1}(t_j) + \sum_{j \le k} Re^{p+1}(t_j) = \\ &= RF(t_k) - Res^{p+1}(t_k) + Re(t_k), \end{aligned} \tag{11.12}$$

where $Re(t_k)$ is a biogeocenosis (self) recreation/distruction vector. We obtain

$$RF_{ir}(t_{k+1}) = RF_{ir}(t_k) + H_{ir}(R\theta_{t_k})\alpha_{ir}, \tag{11.13}$$

where α_{ir} is a coefficient of the natural and cost value of resource ir and H_{ir} expresses then the value of self-recreation of the resource ir (self-distruction when it has a negative value). For players from $DR(t)$ it (DH_{ir}) will express the value of biogeocenosis pollutents self-distruction. We transform 11.10 as 11.12, 11.13:

$$\begin{aligned} RD^{p+1}(t_{k+1}) &= RD^{p+1}(t_0) + \sum_{j \le k} RD^{p+1}(t_j) - \sum_{j \le k} ED(t_j) \\ &= RD^{p+1}(t_k) - ED(t_k), \\ RD_{ir}^{p+1}(t_{k+1}) &= RD_{ir}^{p+1}(t_k) + DH_{ir}(R\theta_{t_k})\beta_{ir}, \end{aligned} \tag{11.14}$$

where $ED(t_k)$ is a vector of spontaneous pollutents distruction, β_{ir} is a coefficient of the natural and cost value of the pollutent ir. Let us unite eqs. 11.13–11.15:

$$\begin{aligned} DF_{ir}^{p+1}(t_k) &= RF_{ir}(t_k) - RD_{ir}^{p+1}(t_k), \\ DF_{ir}^{p+1}(t_{k+1}) &= DF_{ir}^{p+1}(t_k) + H_{ir}(R\theta_{t_k})\alpha_{ir} - DH_{ir}(R\theta_{t_k})\beta_{ir}, \end{aligned} \tag{11.15}$$

DF is defined as a consequence of component ir polluting. The game

$$\Gamma_r(t) = \langle IR, DR(t), R\mathcal{K}(t), RS(t), R\theta(t), \{H_{RK}\}_{RK \in R\mathcal{K}(t)}, [t_0, T] \rangle,$$

which satisfies expressions 11.10–11.15 we call the descriptive natural resources dynamics model.

Let $\tilde{F}^{p+1}(t) = \langle F^{p+1}(t), \Gamma_d(t), \Gamma_r(t) \rangle$. We identify $\tilde{F}^{p+1}(t)$ components with fictive players I^{p+1}, II^{p+1}, III^{p+1}, assuming that they represent the interests of corresponding subsystems. At the moment of time t their state is expressed by a collection of elements

$$MS(t) = \langle Th^{p+1}(t), Rt^{p+1}(t), CJ(t), CS(t), CH(t), IR, DR(t), R\theta(t), DF^{p+1}(t) \rangle. \tag{11.16}$$

In 11.16 we omit dependent values and identify it with the situation defined by strategies of players I^{p+1}, II^{p+1}, III^{p+1}. We can identify the values $Rt^{p+1}(t)$, $CH(t)$, $DF^{p+1}(t)$ with the players I^{p+1}, II^{p+1}, III^{p+1} payoff functions. Hence, we construct the game

$$\Gamma R(t) = \langle I^{p+1}, II^{p+1}, III^{p+1}, MS(t), \mathcal{K}R(t), Rt^{p+1}(t), CH(t), DF^{p+1}(t), [t_0, T] \rangle, \tag{11.17}$$

which we call the $(p+1)$-level descriptive ecosystem model ($\mathcal{K}R(t)$-coalitions complex).

11.5 Coalitional Stochastic Games

The players of a conflict system described by eq. 11.17 and developing at the segment of time $[t_0, T]$ may take actions of the following types: 1) deterministic; 2) random; 3) random influences (the reaction) realizing at the level of players sets as a whole. Here we introduce coalitional stochastic games to describe these actions on a certain level of the model 11.17.

Determined coalitional game in the most general form has solutions in the case of coalitional strategic game [Vor 67] and cooperative game [AuSh 74].

Let us introduce them:

a. coalitional game [Vor 67]:

$$\Gamma_S = \langle I, \mathcal{K}, \{S_i\}_{i \in I}, \tilde{S}, \tilde{\mathcal{K}}, \{H_K\}_{K \in \tilde{\mathcal{K}}} \rangle, \tag{11.18}$$

where I is the set of players, $\mathcal{K}$ the set (complex) of coalitions of actions, S_i the set of individual player's strategies, $\tilde{S}$ the set of situations, $\tilde{S} \subseteq \prod_{i \in I} S_i$, where $\tilde{\mathcal{K}}$ is a certain aggregate of coalitions from $\mathcal{K}$ such as $\bigcup_{K \in \tilde{\mathcal{K}}} K = I$, where $\{H_K\}$ is the set of payoff functions.

b. cooperative non-strategic game [AuSh 74]:

$$\Gamma_C = \langle \mathcal{K}_i, \{G(K)\}_{K \in \mathcal{K}_i} \rangle, \tag{11.19}$$

where $\{G(K)\}$ are the subsets functions (measures) on the complex of interests $\mathcal{K}_i$ reflecting the coalitions payoffs (values according to [AuSh 74]). Let us use some definitions and results of papers [Vor 67], [PtDn 85], [Bar 81] to put the controls corresponding to the points 1–3 above.

1. We identify the deterministic actions of players with the measurable space of permissible controls $(Y, \mathcal{Y})$ [PtDn 85].

2. Let $(E, \mathcal{E})$ be the measurable space of the system 11.17 states and y_t, $t \in [t_0, T]$ are the functions from $Y^{[t_0,T]}$, the space of all the functions defined on $[t_0, T]$, $\mathcal{Y}^{[t_0,T]}$ is the minimal σ-algebra containing all cylindric sets of $Y^{[t_0,T]}$. There exist the limits $y_{t-0} = \lim_{s \to t-0} y_s$. Let us denote $y_0^{t-0} = \{y_s, t_0 \le s < t\}$.

We call [Bar 81] the deciding function ν_t the family of probability measures $\nu_t(A \mid x_0^t, y_0^{t-0})$ defined at $A \in \mathcal{Y}$, $x_0^t \in E^{[t_0,T]}$ (the space of all functions on $[t_0, T]$ with values from E), $y_0^{t-0} \in Y^{[t_0,T]}$, which satisfies the condition: for all $t \in [t_0, T]$ and $A \in \mathcal{Y}$ $\nu_t(A \mid x_0^t, y_0^{t-0})$ is a $\mathcal{E}_t \times \mathcal{Y}_{t-0}$-measurable function of (x_0^t, y_0^{t-0}). Here $\mathcal{E}_t$, $(\mathcal{Y}_t)$ is σ-algebra born by cylindric sets with the bases above the $[t_0, T]$, $\mathcal{Y}_t = \bigcup_{s<t} \mathcal{Y}_s$. We put the family of such measures to correspondence with the whole set of players I and identify it with the random actions of the type 2).

The family of deciding functions $\nu = \{\nu_t, t \in [t_0, T]\}$ we call the strategy.

Further we put ν_t to correspondence with the prestrategies [Pol 90] of players $i \in I$, denote them by ν_t^i, and call ν_t^i the simple pure individual prestrategy of the player i. Also we suppose that ν_t^i defines the distribution on $Y^{[t_0,T]}$ and it is possible to identify it with the set of random prestrategies.

The function ν_t that has the property $\nu_t(y_t, A \mid x_0^t, y_0^{t-0}) = X_A(y_t \mid x_0^t, y_0^{t-0})$ where X is an indicator of y_t is called nonrandomized [Bar 81]. We put the set of nonrandomized controls $\{\nu_t^i\}$ to correspondence with the deterministic prestrategies of player $i \in I$, i. e. we consider it as the set of $\mathcal{E}_t^i \times \mathcal{Y}_{t-0}^i$-measurable functions. Let $\mathcal{E}_t^{in} \times \mathcal{Y}_{t-0}^{in}$ and $\mathcal{E}_t^{ir} \times \mathcal{Y}_{t-0}^{ir}$ are the minimal σ-algebras corresponding to the products of measures ν_t^i at nonrandomized and random prestrategies of the player i, and ν_t^{in} and ν_t^{ir} are the measures corresponding to them. We denote by $s\nu_t^i$ the pair (ν_t^{in}, ν_t^{ir}) and call it the simple pure individual strategy of the player i. Let $\mathcal{E}_t^i, \mathcal{Y}_{t-0}^i$ are the minimal σ-algebras containing $\mathcal{E}_t^{in} \times \mathcal{E}_t^{ir}$, $\mathcal{Y}_{t-0}^{in} \times \mathcal{Y}_{t-0}^{ir}$. We suppose that prestrategies are stochastically independent and $s\nu_t^i$ for every $i \in I$ and $t \in [t_0, T]$ are $\mathcal{E}_t^i \times \mathcal{Y}_{t-0}^i$-measurable. We denote by

$$S\nu_t^i = \bigcup_{\nu_t^{in} \in s\nu_t^i} s\nu_t^i$$

the set of strategies of players $i \in I$.

Following [Vor 67] let $\tilde{\nu}_t^{in}$ be the probability measure on the set of measurable vector-functions $\{\nu_t^{in}\}$ and $\tilde{\mathcal{E}}_t^{in}$, $\tilde{\mathcal{Y}}_{t-0}^{in}$ are the σ-algebras corresponding to them. Vector-measure $\theta_t^i = (\tilde{\nu}_t^{in}, \nu_t^{ir})$ defined on the minimal σ-algebra $\tilde{\mathcal{E}}_t^i \times \mathcal{Y}_{t-0}^i$ containing $\tilde{\mathcal{E}}_t^{in} \times \mathcal{E}_t^{ir} \times \tilde{\mathcal{Y}}_{t-0}^{in} \times \mathcal{Y}_{t-0}^{ir}$ we call the simple mixed strategy of the player i. We put this strategy to correspondence with the controls of the type 1) and denote by $\tilde{E}_i$ and $\tilde{Y}_i$ corresponding basic sets. On the measure θ_t^i we may build the joint distribution on the players prestrategies and, hence, define the measure θ_t^{i*} on the spaces $(SE_t^i, S\mathcal{E}_t^i), (SY_t^i, S\mathcal{Y}_{t-0}^{in})$ — the joint mixed strategy of the player i at the moment t. Due to our build-up we have

$$SE_t^i \subseteq \prod_{S\nu_t^i} E^{\nu_t^i}, SY_t^i \subseteq \prod_{S\nu_t^i} Y^{\nu_t^i},$$

where $E^{\nu_t^i}$, $Y^{\nu_t^i}$ are the sets corresponding to prestrategies ν_t^i of the player i.

Let us denote by $(SY_t, S\mathcal{Y}_{t-0})$ the measurable space of the players strategies sets where $SY_t = \prod_{i\in I} SY_i$, and $S\mathcal{Y}_{t-0}$ is the minimal σ-algebra containing the product $\prod_{i\in I} SY_{t-0}^i$. Let $(SE^{[t_0,T]}, S\mathcal{E}_t)$ be the measurable space corresponding to $(SY^{[t_0,T]}, S\mathcal{Y}_{t-0})$. This introduced set we call the set of situations (compare with [Vor 67]) and put it to correspondence with the controls of the type 3).

We call the controlled object (compare with [Bar 81], [GhSk 77]) the family of probability measures $\mu^t(B \mid \tilde{x}_0^t, \tilde{y}_0^{t-0})$ defined at $B \in S\mathcal{E}_t$, $\tilde{x}_0^t \in B^{[t_0,T]}$, $\tilde{y}_0^{t-0} \in SY^{[t_0,T]}$, and satisfying the condition: for all $t \in [t_0, T]$, $B \in S\mathcal{E}_t$ $\mu_t(B \mid \tilde{x}_0^t, \tilde{y}_0^{t-0})$ is a $S\mathcal{Y}_{t-0}$-measurable function of $y(t)$.

So, the aggregate $\Sigma = SY, SE, \{\theta_t^{i*}\}_{i\in I}, \mu_t, t \in [t_0, T]$ we shall identify with the elements of a certain level of the model 11.17.

We have a compound process $(\xi(t), \eta(t))$ with values in $S\mathcal{E}_t \times S\mathcal{Y}_{t-0}$ defined by measures μ_t and θ_t^{i*} correspondingly. To build a basic process $\xi(t) = \xi(t, \eta(t))$ and its joint distribution we have to know the process state and the control $\eta(t)$ at the same moment of time t, but $\eta(t)$ is also defined through the value of $\xi(t)$. If we suppose that the control delays the process we can define the $\xi(t)$ behaviour with deterministic or with random control as well. The procedure of such process building up is discussed in [GhSk 77].

The control $y(t) \in Y$ we call stepped [Bar 81], [GhSk 77] if $y(t)$ is partially constant on $[t_0, T]$. The control $\nu_t(B \mid \tilde{x}_0^t, \tilde{y}_0^{t-0})$ we call stepped [GhSk 77] if measure ν_t for all $x \in E^{[t_0,T]}$ is defined on the stepped functions.

Further we suppose that players of the system 11.17 use partially constant strategies from SY_t, and t_k are the moments of their changing, and as well as in coalitional actions research in terms of coalitional static deterministic games [Vor 62]–[Vor 67] we substitute initial coalitional nonrandomized strategies to the mixed and consider them as the families of random transitions.

Let the moment of time $t \in [t_k, t_{k+1})$ be fixed and the coalitions complex $\mathcal{K} = \mathcal{K}_a$ of players I is regular [Vor 62]. Then [Vor 62] there exist the consistent extension of mixed strategies θ_t^{i*} of players $i \in K$, $K \in \mathcal{K}$, which we denote by θ_t^{K*}, and the consistent extension θ_t^{K*} on all $K \in \mathcal{K}$ and $K \subset I$ denoted by θ_t^*. We call θ_t^{K*} the (simple) mixed coalitional strategy and θ_t^* the (simple) mixed situation (at the moment of time t). Players in coalitions may refuse their mixed coalitional strategies. That's why, let us define the following family of probability measures.

The pure strategy θ_t^i of the player $i \in I$ is defined on the measurable space $(Y_i, \tilde{\mathcal{Y}}_{t-0}^i)$ and on the space $(E_i, \tilde{\mathcal{E}}_t^i)$ corresponding to it. The subset $S\nu_t^i$ of measures $s\nu_t^i$ of the set of all the functions $Y_i^{[t_0,T]}$ with values Y_i we call the set of pure strategies of the player i and identify it with the set of strategies S_i introduced in the chapter 2. Then $S\nu_t^K = \prod_{i\in K} S\nu_t^i$ is the set of pure coalitional strategies to which we put to correspondence the spaces $(E_K, \tilde{\mathcal{E}}_t^K)$ and $(Y_K, \tilde{\mathcal{Y}}_{t-0}^K)$. The spaces $(SY_i, S\mathcal{Y}_{t-0}^i)$ and $(SE_i, S\mathcal{E}_t^i)$ correspond to the mixed strategy θ_t^{i*} of the player i, and, hence, the generalized measurable spaces $(SY_K, S\mathcal{Y}_{t-0}^K)$ and $(SE_K, S\mathcal{E}_t^K)$ to the mixed coalitional strategy θ_t^{K*} of the coalition K. Here $S\mathcal{Y}_{t-0}^K$, $S\mathcal{E}_t^K$ are the systems of σ-algebras with the system of measures $\{\theta_t^{i*}\}_{i\in K} = \theta_t^{K*}$.

Following [Vor 64] we put the situation to the correspondence with the consistent extension θ_t^{K*} on all $K \in \mathcal{K}$ and the generalized measurable spaces $(S\nu_t^I, S\mathcal{Y}_{t-0}^{\mathcal{K}})$ and $(SE_t^I, S\mathcal{E}_t^{\mathcal{K}})$ where

$S\nu_t^I$ is the set of pure situations, $S\mathcal{Y}_{t-0}^{\mathcal{K}}$ is the system of σ-algebras corresponding to $i \in I$, and $(SE_t^I, S\mathcal{E}_t^{\mathcal{K}})$ the states space corresponding to $S\nu_t^I$.

Let us put every player $i \in I$ to correspondence [Vor 62] with the generalized measurable space $(S\nu_t^i, S\mathcal{Y}_{t-0}^i)$ and consider for a certain coalition $K \subset I$ the product $S\nu_t^K$. Let

$$P_t^K = \prod_{i \in K} A_i \times S\nu_t^{I\setminus K}, A \in S\mathcal{Y}_{t-0}^i, S\nu_t^{I\setminus K} \in S\mathcal{Y}_{t-0}^{I\setminus K}.$$

We consider σ_t^K as the minimal subsets $S\nu_t^K$ σ-algebra containing P_t^K. We denote by $(S\nu_t^I, \Sigma_t^{\mathcal{K}})$ the generalized measurable space including the set $S\nu_t^I$ and σ-algebras σ_t^K, $K \in \mathcal{K}$ and take it as the initial.

Let t be fixed. According to [Vor 64] we can build on the space $(S\nu_t^I, \Sigma_t^{\mathcal{K}})$ the family of random transitions satisfying the properties of introduced strategies.

Following [Vor 67] we identify the set of situations $S\nu_t^I$ with the family of random transitions (realizing at the moment of time t), denote them by η_t, and call the precise situations. Let us define the payoff functions of coalition K in situation η_t by expression:

$$H_K^*(\eta_t) = \sum_{sy \in SY} H_K(sy)\eta_t(sy), \eta_t \in \tilde{\eta}_t,$$

where $H_K(sy_t)$ is a payoff function of coalition K in the pure situation sy_t, η_t is the family of precise situations.

The aggregate

$$\Gamma_S^0(t) = \langle I, \mathcal{K}, \tilde{\eta}_t, \tilde{\mathcal{K}}, \{H_K^*\}_{K \in \tilde{\mathcal{K}}} \rangle \tag{11.20}$$

at fixed t we call the static coalitional stochastic game.

On the game 11.20 we may build the static cooperative stochastic game by defining the following functions on the sets of coalitional strategies:

$$\nu_t(K) = \max_{S\nu_t^K} \min S\nu_t^K(H_K^*(\eta_t)). \tag{11.21}$$

The aggregate

$$\Gamma_C^0(t) = \langle I, \mathcal{K}_i, S\nu_t^I, nu^t \rangle \tag{11.22}$$

at fixed t we call the static cooperative stochastic game.

It is proved that the games 11.20 and 11.22 have solutions (stable situations and Shapley value correspondingly, look below).

Let us consider dynamic coalitional stochastic games. There are possible three following cases.

1. There are fixed: the set of players I, complexes of coalitions $\mathcal{K}$ and $\tilde{\mathcal{K}}$, the set of payoff functions $\{H_K^*\}_{K \in \tilde{\mathcal{K}}}$ at all the segment $[t_0, T]$. We have the aggregate

$$\Gamma_S^1(t) = \langle I, \mathcal{K}, \tilde{\eta}_t, \tilde{\mathcal{K}}, \{H_K^*\}_{K \in \tilde{\mathcal{K}}}, t \in [t_0, T] \rangle. \tag{11.23}$$

2. The set of players $I(t)$ and, hence, the complexes of coalitions $\mathcal{K}(t)$ and $\tilde{\mathcal{K}}(t)$ change in time. The payoff functions don't depend on time at $[t_0, T]$. We have

$$\Gamma_S^2(t) = \langle I(t), \mathcal{K}(t), \tilde{\eta}_t, \tilde{\mathcal{K}}(t), \{H_K^*\}_{K \in \tilde{\mathcal{K}}(t)}, t \in [t_0, T] \rangle. \tag{11.24}$$

3. The payoff functions depend on time in the conditions of p. 2 also. We denote them by $\{H_K^t\}_{K\in\tilde{\mathcal{K}}(t)}$ and suppose that at every t they are given as in eqs. 11.23–11.24. We obtain the aggregate

$$\Gamma_S(t) = \langle I(t), \mathcal{K}(t), \tilde{\eta}_t, \tilde{\mathcal{K}}(t), \{H_K^t\}_{K\in\tilde{\mathcal{K}}(t)}, t \in [t_0, T]\rangle. \tag{11.25}$$

The aggregates in 11.23–11.25 we call the dynamic coalitinal stochastic (DCS-) games. The game in 11.25 is the general form of the DCS-game.

We can build the cooperative variants of the games in 11.23–11.25 build by the analogy with the game in 11.22 and functions in 11.21.

In accordance with the game 11.20 build-up we can define the stepped games for 11.23–11.25 and put for them the following payoff functions:

for eqs. 11.23–11.24:

$$H_K^*(\eta_T^*) = \sum_{t\in[t_k,t_{k+1})_{k=1}}^{n-1} H_K^*(\eta_t^*), \tag{11.26}$$

and for 11.25

$$H_K^T(\eta_T^*) = \sum_{t\in[t_k,t_{k+1})_{k=1}}^{n-1} H_K^t(\eta_t^*), \tag{11.27}$$

where η_t^* is ϕ_t-stable situation [Vor 67] on the interval $[t_k, t_{k+1})$, η_T^* the aggregate of ϕ_t-stable situations corresponding to the segment $[t_0, T]$ division. Due to eqs. 11.25–11.26 we obtain the outcome payoff of coalition in the DCS-game in the ϕ_t-stable situation η_T^* at all $[t_k, t_{k+1})$. Also, if coalition ceases the game playing before the moment T its payoff is defined by the summation till the last interval. It is proved that the games in eqs. 11.23–11.25 with functions in eqs. 11.26–11.27 have stable situations (see below).

The process $\xi(t) = \xi(t, \eta(t))$ state with the DCS-game realized control given by situations $\tilde{\eta}_t$, $t \in [t_0, T]$ is defined by procedure [GhSk 77]. The optimal process $\xi^*(t)$ corresponds to the situation η_T^* and its optimality is induced by the stable situation η_T^*.

As above, we can define dynamic cooperative stochastic games on the functions 11.26–11.27, 11.21, their stepped games and, then, the single Shapley value also.

There were considered coalitional stochastic games with asymptotically continuous time as well.

11.6 Multilevel Coalitional Games

Above we have described the components of the multilevel coalitional/cooperative game which serves as the ecosystem model. Now we shall consider optimality principles realizing in it, i. e. the conditions under which there exist the solutions of the game.

At first let us consider coalitional games when we have only the single level in model. From 11.18 and 11.19 or from the games in eqs. 11.21–11.25 and their cooperative variants we obtain the game

$$\Gamma = \langle \Gamma_C, \Gamma_S \rangle \tag{11.28}$$

with the following payoff functions in situation $\tilde{s}$:

$$\tilde{H}(K,\tilde{s}) = H_K(\tilde{s}) + G(K)$$

where Γ_C we call the cooperative and Γ_S the strategic component of Γ.

Let ϕ_C be the mapping (Shapley value) which gives the solution to the Γ_C and ϕ_S to the game Γ_S (existence of them is proved in [AuSh 74] and [Vor 67] correspondingly). We can describe the ϕ_C-solution as the previos, forecasted values of coalitions payoffs. The procedure of its finding includes all the antagonistic games solving in which each coalition K plays against the coalition of $I \setminus K$ players. Principle ϕ_C in this procedure gives the minimal guaranteed payoff depending on the set of situations only. Mapping ϕ_S gives "inner" distribution of payoffs independent from the individual behavior of every player in coalitions from $\tilde{\mathcal{K}}$ in accordance with each situation of the game.

We take vector $\tilde{\phi} = (\phi_C, \phi_S)$ as the optimality principle of the game 11.28 and may consider the procedure of its solving separately at components Γ_C and Γ_S correspondingly.

At the definition of 11.18 its coalitions complex $\mathcal{K}$ is formed before the beginning of the game. That's why, let us define 2 stages of the game 11.28 solving:

- forming the coalitions complex $\mathcal{K} = \mathcal{K}_a$ in the cooperative component Γ_C, (after it we delete "weak" or non-atomic players in $\mathcal{K}_i \supseteq \mathcal{K}_a$);
- redistribution of payoffs in Γ_S.

The game eq. 11.28 at this assumptions has $\tilde{\phi}$-optimal solution.

The characterization of presentations of different and nonconflict systems also in the game 11.28 is described in [Kon 86], [KPS 87].

As before, we identify the element of the second level with the aggregate

$$\Gamma^{2,1} = \langle i^2, S_i^2, G^2(i), H_i^2 \rangle,$$

where S_i^2 is the set of strategies of the player i^2, $G^2(i)$ and H_i^2 are his payoff functions.

The aggregate

$$\Gamma^{2,2} = \langle \Gamma, \Gamma^{2,1} \rangle \tag{11.29}$$

we call the general two-level coalitional game.

This game we shall use to describe the interactions between levels in the above general model.

Let us define the connections between payoff functions in 11.29 (the same we put for Γ^{p+1} and F^p), and the rules of the game 11.29. We denote by $\tilde{K}_j^p$ all the coalitions of the level p taking part in situations forming, i. e. to each situation $\tilde{s}^{p+1}$, $\tilde{s}^p$, $\tilde{s}^p \in \prod_{i=1}^{n_p} \tilde{s}_i^p$, it is corresponded its "own" fictive coalition $\tilde{K}_j^p$ differed from the other coalitions.

Let coalition $\tilde{K}_j^p$ has the capital $G^{p,0}(\tilde{K}_j^p)$ at the level p and gets from its representative at the level $p+1$ the value

$$G_0^{p+1,p}(j^{p+1}) = G_0^{p+1,p}(\tilde{K}_j^p). \tag{11.30}$$

To the beginning of the game $\tilde{K}_j^p$ has the capital

$$\tilde{G}^{p,0}(\tilde{K}_j^p) = G^{p,0}(\tilde{K}_j^p) + G_0^{p+1,p}(\tilde{K}_j^p) + G_0^{p-1,p}(\tilde{K}_j^p), \tag{11.31}$$

distributed between included players. In accordance with the definition of cooperative game it must be fulfilled [AuSh 74]:

$$\tilde{G}^{p+1,0}(\tilde{K}_j^p) \geq \sum_{i \in \tilde{K}_j^p} G^{p,0}(i^p). \tag{11.32}$$

At first the cooperative component of 11.28 is played. Let $G^{p,\Delta}(K_j^p)$ be the payoff which coalition $K_j^p \subset \tilde{K}_j^p$ obtains as its outcome. We have [AuSh 74]

$$G^{p,\Delta}(K_j^p) = \sum_{i \in K_j^p} G^{p,\Delta}(i^p). \tag{11.33}$$

At this stage from the further game we may delete "weak" players by comparing their payoffs with certain threshold ζ. Then the game is continued at its strategic component with the payoff functions $\{H_K^p\}_{K \in \mathcal{K}}$.

As the outcome of both game components playing coalition K_j^p obtains

$$\tilde{H}^p(K_j^p, \tilde{s}^p) = H_{K_j}^p(\tilde{s}) + G^p(K_j),$$

$$G^p(K_j) = G^{p,\Delta}(K_j^p) - G^{p,p-1}(K_j^p) - G^{p,p+1}(K_j^p), \tag{11.34}$$

$$G^{p,p-1}(K_j^p) = \sum_{i \in K_j^p} G^{p,p-1}(i^p), K_j^p \subset \tilde{K}_j^p,$$

where $G^{p,p-1}(i^p)$ and $G^{p,p+1}(K_j^p)$ are the payments of player i^p to his agents at the level $p-1$ and of coalition K_j^p to its representative at the level $p+1$ correspondingly.

Let us introduce the rules of the game 11.28.

1. The payoffs distribution between levels (compare with [GrKn 82]) $p+1$, p and p, $p-1$ is fulfilled independently in two-level two players games $\Gamma_L^2(j^{p+1}, \tilde{K}_j^p)$ and $\Gamma_L^2(j^p, \tilde{K}_j^{p-1})$. The players can change their capitals before and after the playing of the game (or its party) between this levels only.

2. The player j^{p+1} informs coalition $\tilde{K}_j^p$ about his strategy and gives to it the capital for its realizing for all the $s \in S_j^{p+1}$.

   Due to this rule player j^{p+1} knows all the set of strategies of players from $\tilde{K}_j^p$, but doesn't define the choice of any situation $\tilde{s}$ — the aggregate of strategies of the level p realizing his own strategy.

3. The players in coalitions of the level p have no information about the set of strategies of the level $p+1$ coalitions.

4. The cooperative component of the game 11.28 realizes the allocation of the capital $G_0^{p+1,p}(\tilde{K}_j^p)$ by player j^{p+1} to the level p and its distribution between coalitions and players in $\tilde{K}_j^p$. Its solution defines the complex $\mathcal{K}$ in 11.18.

5. The strategic component of the game 11.28 realizes the player's j^{p+1} strategy.

   In other words, at the beginning we make a prior estimation of the $\tilde{K}_j^p$ possibilities in fulfilling of the j^{p+1} strategies and then perform them.

6^p The payments between levels are fixed.

The game 11.29 we shall consider with rules 1–6^p, payoff functions 11.30–11.34 and denote as

$$\Gamma^p = \langle \Gamma_L^2, \Gamma_C, \Gamma_S, 1 \leq p \leq p_0 \rangle = \langle \Gamma_L, \Gamma_C, \Gamma_S, 1 \leq p \leq p_0 \rangle. \tag{11.35}$$

The game 11.35 has $\tilde{\phi}$-optimal solution.

The optimality principle $\tilde{\phi}$ in this game is based on the extension of the principle ϕ_C on the game Γ_L where we obtain the same Shapley value principle ϕ_L and connect ϕ_L and ϕ_C as one principle $\phi_{LC} = (\phi_L, \phi_C)$ in the united game $\langle \Gamma_L, \Gamma_C \rangle$. So, for the game 11.35 we have $\tilde{\phi} = (\phi_{LC}, \phi_S)$.

The coalitional games we have considered above reflect at first the quantitative estimation of the levels interactions and don't take into account description and definition of the initial coalitional interests and the value of their satisfaction as in the situations during the game playing, so as in its outcome situations, in the $\tilde{\phi}$-optimal situation as well.

Let U_I^p be the set of players interests of the level p formed as the union of the individual interests vectors $\vec{u}_i^p$ of all the players $i \in I^p$.

Let us take the simplest construction to estimate and compare $u_i^p \in \vec{u}_i^p$. We consider with u_i^p: 1) the predicates $P^p(u_i^p)$ which true values reflect player i satisfaction in realization of his interests after the game playing, and 2) the aggregate of individual players preferences (weights) γ_i^m defined on the set of their interests. So, we put the players utility functions:

$$L_i^p(\vec{u}_i^p) = \sum_{m=1}^{m_{p_i}} \gamma_i^m P(\vec{u}_i^p),$$
$$0 \leq \gamma_i^m \leq 1, \sum_{m=1}^{m_{p_i}} \gamma_i^m = 1. \tag{11.36}$$

The union

$$U_{\mathcal{K}_i}^p = \bigcup_{K \in \mathcal{K}_i} \bigcup_{i \in K} u_i^{p,K} = \bigcup_{K \in \mathcal{K}_i} U_K^p,$$

we call the set of coalitional interests.

The players $i \in \mathcal{K}_i^p$ may realize their interests and, hence, maximize the functions 11.36 only in coalitional actions: even if it is unprofitable to a certain player to join the coalitions the realization of his interests will depend upon their behavior. That's why, we include individual players into $\mathcal{K}_i^p$ also. With such assumption we can put for each coalition the following utility function

$$L_K^p(u_K^p) = \sum_{i \in K^p} L_i^p(\vec{u}_i^p). \tag{11.37}$$

By the analogy with [GrKn 82], [KkMr 84] on eqs. 11.36–11.37 we define the functions

$$v_{L_K}^p(u_K^p) = \max_{K^p} \min_{\tilde{K} \in \mathcal{K}_i^p, \tilde{K} \cap K = \emptyset} L_K^p(u_K^p), \tag{11.38}$$

reflecting the maximum of the payoff value in the case of counteraction of all other coalitions.

We can form the cooperative game on functions 11.38:

$$\Gamma_U = \langle \mathcal{K}_i^p, U_{\mathcal{K}_i}^p, \zeta_I^p, \{v_{L_K}^p\}_{K \in \mathcal{K}_i^p} \rangle.$$

This game has the Shapley value and, hence, there exists the mapping ϕ_U giving its single solution.

In the game 11.35 we estimate the "consequences" by the values $G^{p,p-1}(i^p)$ and $G^{p,p+1}(K_j^p)$ to be given to the other levels and suppose fixed payments. Further we shall do this by the functions $\{L_K^p\}_{K \in \mathcal{K}_i^p}$.

We obtain the game

$$\tilde{\Gamma}_U = \langle \{j^{p+1}\} \bigcup \mathcal{K}_i^p, U_j^{p+1} \bigcup U_{\mathcal{K}_i}^p, \{v_{L_K}^p\}_{K \in \{j^{p+1}\} \cup \mathcal{K}_i^p} \rangle. \qquad (11.39)$$

analogous to the $\langle \Gamma_L^2, \Gamma_C \rangle$ and defined on the functions 11.37 and L_j^{p+1}. It is proved that there exists mapping ϕ_{LU} giving the solution of 11.39.

6 The payments scheme between levels of the game 11.35 is defined as the game 11.39.

Further we consider the last rule instead of 6^p.

The game

$$\begin{aligned} \tilde{\Gamma}^p &= \langle \Gamma_L^2, \Gamma_C, \Gamma_S, \tilde{\Gamma}_U, 1 \le p \le p_0 \rangle \\ &= \langle \Gamma_L^2, \Gamma_C, \Gamma_S, \tilde{\Gamma}_U, 1 \le p \le p_0 \rangle, \end{aligned} \qquad (11.40)$$

satisfying 1^0–6^0 we call the multilevel coalitional general form game.

This game has Φ-optimal solution where $\Phi = (\tilde{\phi}, \phi_{LU}) = (\phi_{LC}, \phi_S, \phi_{LU})$.

11.7 The General Model

In this chapter we shall consider the concept of building up the general ecosystem model.

Above we have defined the global optimality principle in the descriptive ecosystem model as in the game 11.17 realized during the game 11.40 playing process at all levels p. As it has been already mentioned this principle reflects the stability of all the system in the conditions of different behaviours of its elements. The game 11.40 in this approach defines the most complex interactions and dependences between elements what we have described by introducing the coalitional general form game. In practice there appear more simple ways of interaction. It is natural to identify these cases with simpler models. In this sense the games 11.17 and 11.40 allow to include as the components simpler games: noncoalitional, matrix, multistage ones, etc. The classes of the game 11.17 submodels are discussed in [KPS 87]. They may be used to present different structural interactions of system elements and subsystems.

The game 11.40 structure, its optimality principle allow to introduce other principles of defining of the optimal, stable, equillibrium situations, etc on the different levels of model 11.17 together with Φ.

We suppose the structure of the ecosystem model 11.17 is fixed — its levels, elements and subsystems corresponding to them, the players, their strategies and individual payoff functions. Let $\tilde{\Phi}$ be a set of optimality principles realized in different variants of the general form game 11.40. We can connect all the model 11.17 with p_0 diminishing to 2 with the (p_0-1)-level tree ψ of optimality principles taken from $\tilde{\Phi}$ and corresponding to levels $(p, p-1)$.

When we substitute in this tree a certain principle Φ for the another one realized in this submodel, we obtain an optimality principles tree differed from ψ and, generally speaking, another game playing conditions for model levels which correspond to made substitution and are less than $p-1$. So, we can connect the model 11.17 with the set Ψ of its optimality principles trees ψ.

The aggregate $C(t) = \langle \Gamma R(t), \Psi \rangle$ we call the constructive ecosystem model (to be optimized at the game 11.40 or its subgames playing process).

Due to the described optimality principle Φ we have the following objects as the game 11.40 optimal solutions in accordance with its stages sequentially: a) a certain vector belonging to the measurable space; b) a probability measure and vectors aggregate belonging to the generalized measurable space; c) a vector belonging to the measurable space and depending on the elements of the above generalized measurable space.

According to this sequence we can define for all the levels p the tree-ordered set of functions of (in general case) random values and probability spaces defining the optimal solution for 11.17. We put $V(t)$ for all used in the model 11.17 optimality principles Ψ as the set of such functions and spaces.

The aggregate

$$N(t) = \langle C(t), V(t) \rangle \tag{11.41}$$

we call the normative ecosystem model.

Every introduced model includes the previous one. Following this rule we form the last general ecosystem model including with 11.41 the set of (trees of) optimal solutions $W(t)$ which we can get for the model 11.17 from the ecosystem research and control experience without any formal optimality principle and also the set of algorithms and their extensions $A(t)$ used to obtain descriptive, substantial nonformal interpretations of solutions $V(t)$, and, vice versa, — the formal one for $W(t)$. In other words, this procedure includes the comparing of real and formal optimality principles and choosing or synthesis of them both (one from the opposite another) due to the experience of system evolution research.

So, the aggregate $U(t) = \langle N(t), W(t), A(t) \rangle$ is called the general ecosystem model.

11.8 Conclusion

Further the model 11.17 serves as the base to build up the formal Maltsev model. Having taken as terms the players strategies and functions $\{G^p(K)\}_{K\in\mathcal{K}_i^p(t)}$, $\{H_K^p\}_{K\in\mathcal{K}_i^p(t)}$, $\{\tilde{L}_i^p\}_{i\in I^p(t)}$, the recursive formal representation of 11.17 in the class of algebraic systems is obtained. This representation instead of 11.17 forms the descriptive functional (nongame-theoretical) f_D-model of real complex system (compare with [Kon 86]).

Similarly, having defined as terms realized in f_D-model optimality principles (the separate class of algebraic systems) and found for it stable situations we form the constructive and normative f_C- and f_N-models. They are the representation of optimality principles of complex system and corresponding to them (partially) found optimal solutions also.

The union of the f_D-, f_C-, and f_N-models we call the generalized gf-model of complex system. Its recursivity in the class of ϵ-stable situations is proved. So, the gf-model includes complete or incomplete in the sense of found solutions descriptions and, that's why, may serve as the tutorial information for a certain heuristic recognizing decision making algorithm which optimizes the system (look [Kon 86]).

Using formal representation [Pol 85] of such algorithms and, more exactly, their model [Zhu 78] based on the Yanov's schemes of algorithms, we can by the analogy with gf-model of complex system define the formal gf-model of algorithms. As their union it can be obtained the generalized tutorial information for decision making problems and algorithms. Reducing these problems to the properties computation problems [Kon 86] and applying to them the apparatus of game-theoretical tutorial models and algorithms [Kon 86], we obtain the generalized algorithm which serves as a tool for complex system describing and optimizing, decision making during this procedure as well.

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Part III

Special Models

Chapter 12

Andreas Flache, Wim Liebrand, Werner Raub, Frans Stokman: Cooperation in group rewarded teams: On the effect of informal group structure on cooperation in a social dilemma

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Abstract

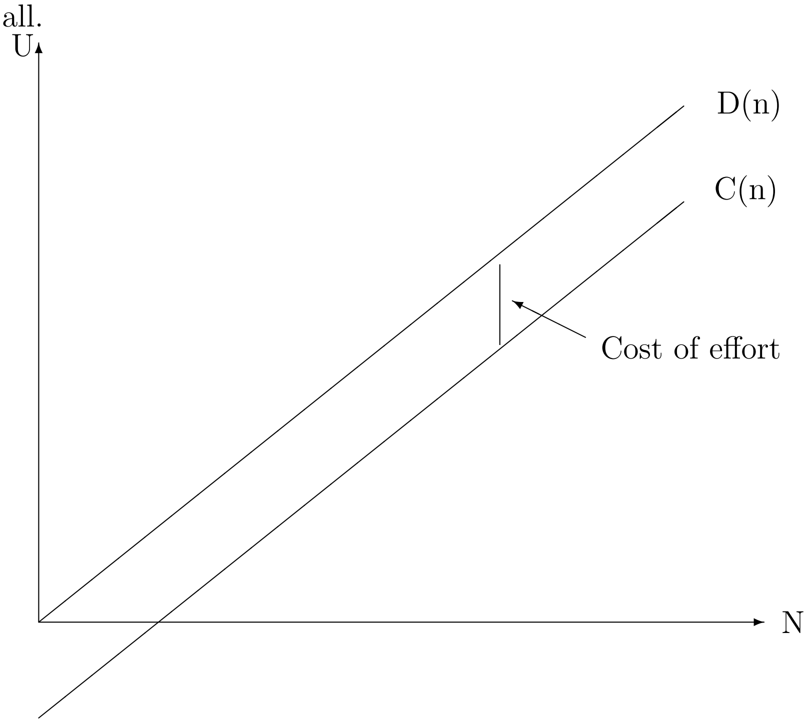
The research project presented in this paper is concerned with formal modelling of cooperation in group rewarded production teams. The social dilemma aspect of cooperation in a group rewarded team will be discussed in part 1. Furthermore the specific research problem we are focusing on will be introduced there: Modelling of the interaction between informal group structure and cooperation in a group rewarded team. Part 2 consists of a brief discussion of game theory and network theory with respect to the suitability of their theoretical concepts for our formal model. In part 3 we will outline the basic model itself. Finally we present a preliminary hypothesis and discuss possibilities to submit the model to empirical test.

12.1 Informal group structure and cooperation in a social dilemma

Alchian and Demsetz, two economists, proposed a classical example for the problem of team production [Alchian/Demsetz 1972]. Imagine that a group of workers loads cargo onto trucks. Unless a supervisor is watching the workers, it will not be possible to determine how much effort every single worker exerted to accomplish the common task. Performance of the group as a whole, however, is a result one easily can control for — for example by counting the number of trucks that have been loaded after the work day is finished.

A fair compensation scheme therefore has to be based on the output of the group as a whole. The previous example shows the conditions under which formal organizations tend to pay workers on basis of group performance. In general this will be the case if the costs of control of individual performance are considerably higher than those of measuring group performance. This often is due to non separability of individual outputs. I.e.: it is easy to observe the total group output but separation of the individual contributions requires additional effort by a supervisor. Organizations are reluctant to invest this effort, but still they want to pay a fair wage that motivates performance, i.e. a wage that is coupled to output. A known solution to this problem are group reward schemes.

But there is one well known problem: Group reward schemes are known to be susceptible to the so called "free rider effect". Every worker is tempted to take it easy and to restrict the effort to a low level, because — under a group payment scheme — the resulting loss of own income is marginal. Figure 12.1 shows the incentive structure faced by a member of a group rewarded team in schematic form. For sake of simplicity we assume that only two courses of action are possible: either to work hard or not to work at all.



- X - Axis: n: Number of others, who "work hard"
- Y - Axis: "utility" of focal worker
- D(n): "utility", if he takes it easy. Proportional to the income.
- C(n): Focal workers "utility" if he works hard. Proportional to income minus costs of effort.

Figure 12.1: Incentive structure for a worker of a group rewarded team

No matter what a worker expects the others to do: Under these conditions to take it easy always is the behavior that provides more utility. For any n, $D(n-1)$ exceeds $C(n)$.

D and C denote "Defection" and "Cooperation". Here the argument has been presented very naively, but more sophisticated game theoretic analysis of this so called N-person prisoners dilemma basically reveals the same result (e.g. [Bendor/Mookherjee 1987]). Based on the assumption of gain maximizing selfish behavior of rational actors the conclusion is: there should not be any effort invested by members of a group rewarded team - unless there are other mechanisms feasible that were not mentioned here.

Study of industrial work groups has, however, revealed that often a fair amount of output is achieved by group rewarded teams. On basis of evidence from the famous Hawthorne experiments [Roethlisberger/Dickson 1939] Homans reports that the foreman of a group rewarded relay assembly team "was proud of his boys and thought if they produced any more output they would work there fingers to the bone" [Homans 1951].

This raises the theoretical problem, to explain that there is such an amount of group performance in a group rewarded team. If we do not want to drop the assumption of goal oriented, rational behavior — and there are good reasons not to do so — we have to answer the question: What are the mechanisms by which rational actors are incited to behave cooperatively — thus: to act in their own interest and in the interest of others?

One obvious answer is external control. If the free rider effect occurs organizations can be expected to invest in supervision of the workers, although it is costly. They monitor individual performance and punish deviators. Moreover they provide selective incentives, i.e. monetary and other rewards that are given to individual workers for their compliance to the rules of the organization. But control never is perfect and there always is some room left for free riding.

Informal control works as a complementary mechanism. Workers are able to control each other. In work groups production norms establish and workers who deviate from those norms are subject to social sanctions. Social disapproval is the main mechanism that workers themselves use to influence each other. Exertion of social control is embedded in existing networks of interpersonal relations, the informal group structure. It is, for instance, less likely that a worker A sanctions a deviator B if the latter is a friend of A. That is one argument from which we can conclude that the structure of interpersonal relations matters for a groups ability to achieve cooperation. Indeed it is an intensively studied problem in research on group dynamics how the characteristics of informal group structure affect performance of a group.

To provide an example: one characteristic of the "informal group structure" that has been observed to foster cooperation is cohesiveness. Cohesiveness has been conceptualized as the degree to which in a group members provide each other with social rewards. It has been observed that in cohesive groups members stick much closer to the group norms - for our case: they will cooperate much more. Homans provides an argument for this effect: The more rewards are exchanged, the more a potential free rider has to loose in case of being subject to social sanctions. This deters him from free riding [Homans 1974].

We provide this example to back the general claim that properties of informal group structure matter for the group performance — thus cooperation. Informal group structure, however, is not a given static variable, neither is group performance one. Group structure develops in time, interpersonal relations change. Group performance undergoes dynamic changes as well. From experimental evidence it is known that if a work group is formed, slowly a production standard develops within the group and after a while the group performance settles down at the size, which is given by the production standard. Our second claim now is that one of the determinants of the dynamic development of informal group structure is group performance. This claim will, again, be backed with examples.

In the famous study "The dynamics of bureaucracy" Peter Blau reports that members of a work group try to strengthen their relations with the best performing group members [Blau 1973]. This suggests that during time interpersonal relations tend to centralize in high performing members of a group. Another, more trivial, example: if a worker behaves very friendly towards another one, but the first is working hard while the latter takes it easy, this imbalance probably will distract from the tendency for friendly behavior.

At this point we are able to outline the research problem, we are focusing at: we aim to develop a formal model for the interaction between group performance and characteristics of informal group structure, because previous research suggests that explanation of cooperation in a work group has to incorporate this interaction. The interaction between the two macro variables will be modelled as the result of goal directed behavior of individuals: our analytical primacy will be on the micro level. We will apply this model to explore the interaction.

To summarize this part, a first outline of the model is presented in figure 12.2. The macro relationship under study is the interaction of characteristics of informal group structure, such as cohesiveness, cliques or centrality of a group, with group performance. The motor of this interaction is, however, modelled as a micro level process. An informal relation of worker A with worker B affects the working behavior of B and the working behavior of B affects the informal relation of A with B. The individual decisions constitute the macro variables and their dynamics.

12.2 Theoretical background for the model

The theoretical ideas that guide the development of our model derive from the concept of the "structural individualistic approach" [Wippler 1978], well known as rational choice theory. Starting point for the model that we are going to develop therefore is the assumption that relations between macro variables of social aggregates are the result from interactions between individuals. Individual behavior is assumed to be goal directed: basically, we assume, individuals strive to maximize their physical well being and the social approval they receive from others. In the setting under study this results in specific so called "instrumental preferences" [Lindenberg 1990]. In a group rewarded team workers strive to maximize their well-being by simultaneously trying to work as little as possible, and to earn as much money as possible. They attempt to gain as much social approval from their colleagues as possible, for instance praise for their diligence. Simultaneously they want to avoid social disapproval, for instance in form of critique on their working behavior.

These assumptions, unfortunately, do not help directly to derive individual behavior and thereby hypotheses concerning the macro relationship we are interested in. Basic

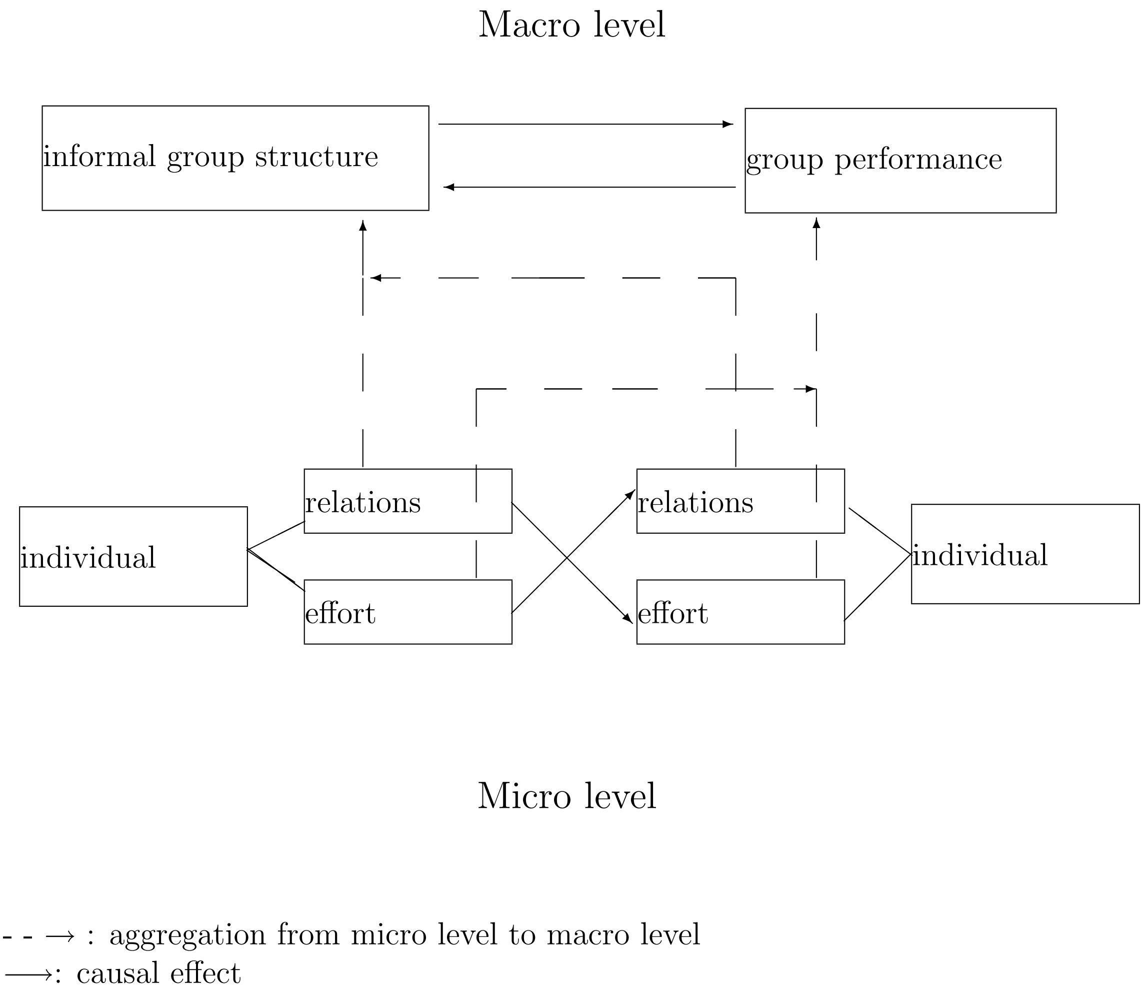


Figure 12.2: First outline of a model

problem is to determine how rational, selfish actors make use of their two basic instruments. First, how do they determine the optimal level of work effort? Second, how do they select the optimal social interaction with others? It comes down to the question: Which course of simultaneous use of these instruments maximizes the goals individuals are striving for?

12.2.1 Game theory

The decision making situation we are analyzing here has a special property. Individual outcomes not only depend on the own behavior of an actor but also on the behavior of others and vice versa. Moreover, actors do not have certain expectations of the course of action to be taken by others. They are in a situation of so called strategic interdependence. Game theory is the theoretical tool to provide solutions to the problem, how rational selfish actors behave under strategic interdependence. We do not outline the solution concepts that have been developed by game theorists, instead we will introduce one type of results that is relevant for the situation we investigate.

A well known finding of game theory is that "conditional cooperation" is rational even in a prisoners dilemma, as long as interactions are expected to last sufficiently long. Tit for Tat, the strategy that won the famous prisoners dilemma tournaments of Robert Axelrod [Axelrod 1984], is a well known version of conditional cooperation. Intuitively the argument is as follows. Suppose members of a team work together for a very long time. Let every member of the team pick the strategy: "I work hard as long as maximal output is achieved by the group. As soon as the output falls below this threshold, I stop to work."If everybody knows about the fact that all others behave according to this strategy, then there is no incentive for a rational selfish worker to free ride. The benefit from free riding, income without work, only can be enjoyed as long as others do not realize the existence of a free rider. As soon as they do, everyones income will drop drastically. Short term gains do not outweigh the losses that a free rider incurs due to the low income for the whole rest of the time.

Basically this is the consideration from which several authors conclude that cooperation in an N person prisoners dilemma can be explained by the use of strategies of conditional cooperation in an interaction of sufficient duration (e.g. [Taylor 1987]. We will borrow the notion of conditional cooperation or so called "threshold strategies" but revise it for our purposes. And we have to extend the concept, because it treats all members of the team as having the same structural position and possibilities to influence each other. The informal group structure, however, is consisting of the pattern of differences between structural positions of actors.

12.2.2 Network theory

To model informal group structure we choose for a network approach. The structure of the group is assumed to result in a graph describing the relations between each pair of workers. Classical way for the construction of such informal networks in groups is the sociometric approach. People are asked to name a list of group members they like most.

From these lists directed graphs can be constructed. A tie from A to B then has the content: "A likes B". If such a directed graph once has been found, graph theoretical methods can be applied to describe characteristics of a given group structure.

We already mentioned cohesiveness, which can be conceptualized by the density, i.e. the relative number of ties in the graph. It also is possible to compute indices for centrality, i.e. the degree to which ties of a network are centered in one specific position. The third and last network property we mention is the subgroup structure: algorithms exist to identify subgroups, i.e. subsets of team members, in which members are more intensively tied to each other than to the rest of the team.

For each of these structural characteristics we can and will explore the way they interact with the amount of cooperation. This requires that ties in the network are not treated as given edges in a graph but that we are able to describe their effect on behavior. Furthermore it has to be modelled, how behavior affects the ties. We derive ties from two basic "atoms" of interaction between two persons: social approval and social disapproval. The definition of network ties that we handle is then:

- $A \oplus \rightarrow B$ "A approves of B more than x times during n periods"
- $A \ominus \rightarrow B$ "A disapproves of B more than x times during n periods"

This definition of informal ties reveals strong similarity to the sociometric approach. Moreover, it allows to model how behavior is affected by ties and how ties are constituted as a result of behavior.

12.3 Basic model for the interaction of informal group structure and group performance in a group rewarded team

12.3.1 Phases of the interaction

Members of the model team are assumed to work together for quite a long time. Their interactions are repeated for a large number of steps. This assumption is rather plausible for most of the team work settings in formal organizations. Basically we distinguish three phases in every step. All members of the team simultaneously move through these three phases:

1. *Work phase*
   Decision: "work hard" or "take it easy".
2. *Interaction Phase*
   - Observation of decisions of all other team members from phase 1
   - Actor takes with respect to any other team member decision to "approve" or "disapprove" or "do nothing"
3. *Evaluation phase*
   - Actor receives information on "incoming" approval and disapproval.
   - Actor evaluates own outcome from current iteration:
     - wage received (group output)
     - costs of work effort
     - approval and disapproval received
     - costs of informal interaction

12.3.2 Decision making model: stochastic learning

We claim that the application of analytical decision concepts as they have been developed by game theory will fail to derive solutions for the behavior of rational actors in this game. The game just is too complex. In the basic N-person prisoners dilemma presented above, every actor has to make a decision between two options. Even this basic game generates a large number of strategies for iterated interaction. For the two person case Axelrod compared about 100 different strategies.

In our game, however, every actor has to make two types of decisions. The first, the work decision, is between two options, the second, the interaction decision, consists of selection of several actions from 3^N options, where N denotes the number of actors in the game. But the problem is relevant and we want to derive conclusions about the behavior of rational individuals in such a situation. The approach we are choosing here is stochastic learning [Bush/Mosteler 1955]. Its basic principles are rather straightforward. “Backward learning” can be characterized by the following sequence of steps:

1. start to take any of the feasible actions with a certain initial propensity (probability). Sum of the propensities for all alternatives is 1.
2. if the action was “successful”, increase propensity for this alternative and decrease propensities for the other alternatives.

To use the concept in our setting we need to specify, how it is applied to the two different types of decisions. Following Macy’s idea of stochastic trigger strategies [Macy 1991] we will borrow the notion of conditional cooperation to model the work effort decision, but we introduce a stochastic element to it. Central element of the strategy is a participation threshold that the group output has to exceed in order to trigger an actors cooperation. Adaption of propensity is mediated through adaption of this threshold. The decision making strategy can be characterized by the subsequent rules:

1. Start with an initial trigger threshold.
2. Start to “work hard” with an initial probability.
3. If the group output exceeds or equals threshold: “work hard”, otherwise “take it easy”.
4. If “work hard” was successful or “take it easy” was a failure: decrease the threshold.
5. If “take it easy” was successful or “work hard” was a failure: increase the threshold.

Adaption of the threshold results in adaption of probabilities. If the threshold is increasing, the probability drops that the group output will exceed it in the next round. This is equivalent to a drop in the propensity of “working hard”. To describe the model fully, it remains to be specified how we believe an actor to evaluate “success” and “failure”. A first simplified assumption is that any course of action is evaluated as success, if after conduction of the action the outcome increases with respect to the previous outcome. Failure is the opposite of success.

We add two comments on this basic principle of a decision algorithm for the determination of work effort. First, outcomes are the result of wage *and* social contacts. In particular it is implied that “free riding” becomes less attractive and therefore less likely, if immediate gains from free riding are compensated by the losses that result from social disapproval. Here the model goes in line with intuition. Second, a model of stochastic learning does not assume that actors cooperate per se. We do assume that they have a certain initial propensity to cooperate. This includes that every now and then "free riding" is attempted. If it turns out to be successful the tendency will be strengthened. It depends on specific assumptions about parameters of the model whether stable cooperation or low performance will be the model prediction.

Decisions for informal interaction also are assumed to be based on the principles of backward learning. Here propensities are modelled to vary directly instead of being mediated through thresholds. This seems plausible, because decisions are made with respect to any single other actor. Behavior of a single other actor towards a focal actor, however, is assumed to vary between just three alternatives: "approve", "disapprove" or "do nothing". Thresholds do not make much sense there. Every actor evaluates his behavior towards every other actor in terms of the benefit that could be derived from the other actors behavior in the past. In particular any course of action towards another actor has been successful, if he 1.) started to "work hard" while he "took it easy" before, or 2.) if he "approved" of the focal actor while not having done so before.

Further elaboration of the assumptions we briefly sketched above will provide a model that is capable to generate predictions for the interaction under study and that is based on the basic principles of goal oriented individual behavior. Figure 12.2 summarized the basic elements and relations that constitute our model. Explicit modelling of the behavioral strategies adds *two new relations* to the scheme. First, due to the conditional cooperative strategies working behavior of an actor A also is affected by previous working behavior of all other actors. Second, current social interaction of A with B also is affected by the history of interactions between A and B.

12.3.3 External conditions

The way we presented the model until here suggested that we model a closed system that always behaves in the same way, no matter in what environment it is embedded. This of course is not true. The organizational environment of a group rewarded team determines important parameters of the process described above. Some are: group size, exact shape of the payment scheme, intensity of external supervisory control. These parameters link e.g. to the costs and benefits of social interactions and thereby affect the payoff structure. Therefore we need to specify the exact organizational environment of the team group in order to determine parameters of the model and to be able to derive valid predictions for team behavior.

12.4 Plan of research and an illustrative hypothesis

Before we are able to present hypotheses derived from the basic model we will conduct three steps of research: First, assumptions of the model will be elaborated in detail. Second, the model will be implemented in form of a computer simulation model. Third, simulation runs will be performed in order to derive hypotheses from the model. Here we present a preliminary hypothesis and discuss its theoretical consequences:

> Under certain "environmental" conditions it holds that, if there are low and high performing members in a group-rewarded production team, then cliques will tend to consist of either high performing group members or low performing group members.

According to our model the emergence of the predicted team situation will proceed in three phases. In the first phase initial differences in individual performance will enhance the amount of approval that is given to high performing members of the team. In phase 2 these high performing members will tend to reciprocate the received approval mainly to other high performers. That weakens the propensities of low performing members to repeat the approving interaction, while on the other hand positive relations between high performers are likely to emerge. Finally, in phase 3, low performers will more likely engage in stable positive relations with each other, because low performers still seek for other team members from which they can receive approval. The benefits they derive from these interactions can be high enough to compensate costs of the disapproval low performers receive from high performers. Thus low performance as well as high performance will remain stable within a clique.

This hypothesis is perfectly in line with the evidence provided by Roethlisberger and Dickson in their famous Hawthorne experiments [Roethlisberger/Dickson 1939]. As Homans reports, in the so called "bank wiring room" they found two subgroups in a team of which one tended to keep much closer to the production norm of the group than the other [Homans 1951, p.72].

Moreover this finding and the hypothesis are in contradiction with an outcome suggested by equity theory [Walster e.a. 1978]. Equity theory would claim that group members distribute social rewards according to individual contributions to the group goal. The best performing group member receives most approval. Our model, instead, predicts that rewards rather are distributed according to the principle of maximization of profit from relationships.

The hypothesis is not derivable for any set of external conditions. Several parameters influence whether or not the model will predict the situation sketched above. Examples for those parameters are initial propensities for working behavior and relative value attached to monetary rewards in comparison with social rewards. We can derive empirically testable predictions by applying our model, because these parameters can be linked to empirically distinguishable sets of external conditions.

12.5 Remarks on methodology of simulation and empirical test

12.5.1 Methodology of simulation

The contributions of Klee, Möhring, Strotmann, and Troitzsch in this volume demonstrate a modelling technique based on the concept of object orientation. Apparently the model that we are going to develop meets several requirements that suggest to apply a simulation language such as MIMOSE or SMALLTALK.

- "Objects" (actors) are distinguished from each other but have the same internal structure and follow the same "rules".
- Micro level events constitute macro level variables and in turn are affected by the macro state of the system. Object oriented model description allows for an elegant formulation of this simultaneous influence process.
- Objects (individual team members) change their internal state in dependence upon their own actual state and the attributes of other objects of the same class. For the implementation of the model and its further elaboration we therefore will apply a computer language that is based on the concepts of object orientation.

12.5.2 Further developments of the model

Assumptions of the basic model necessarily have to abstract from a number of aspects that affect the process under study. We will relax these assumptions stepwise and increase the complexity of the model. To provide an example: Initially group size only has an indirect effect on costs and benefits of individual decisions. It is more realistic to assume that there is also a direct effect. The larger a group is, the more difficult it is for an actor to get access to most of the other group members. Only a small subgroup still is relatively easy to approach. Therefore, the model can be rendered more realistic if a distribution of distances between actors is introduced that is varying with group size. Distance between actors then directly relates to the costs of social interactions. In a similar way we will use the model to study effects of variations in external supervisory control, variations in payment schemes and other conditions of the organizational environment.

12.5.3 Empirical tests

Hypotheses generated in the course of our research will as much as possible be submitted to empirical tests. We have decided to reconstruct the basic game in a laboratory environment. This is due to two reasons. First, in laboratory experiment the large number of conditions that affect behavior in a group rewarded team can be controlled effectively. Second, data on the dynamic development of an informal network in a group are very hard to collect in field or survey research. The laboratory tests will allow to monitor in detail how actors interact with each other and how network structures emerge and change in the group.

12.6 Concluding remark

We conclude our contribution at this point by giving a brief assessment of the utility of our approach. As far as we are aware this will be the first approach to combine dynamic emergence of group structure with strategic behavior in a social dilemma. The game theoretic paradigm has been proved to be a useful theoretical tool for the analysis of strategic interaction. It has, however, often been criticized for the unrealism of its assumptions. Scharpf recently stated in his critique "Games real actors might play" that one way to introduce more realism into game theoretic models is consideration of structural emebeddedness of actors [Scharpf 1990]. We believe that our research will be an important step into that direction.

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Chapter 13

Vladimir B. Dubrovskiy and Alexander M. Tsvetkov, Kiev: Numerical Experiments with Chernenko's Model of Social Evolution

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Abstract

Now many investigations are devoted to modelling of complex system of social dynamics. This dynamics is formulated in terms of nonlinear equations of motion containing several control parameters which may be determinated experimentally. Such model of social evolution was considered by Chernenko in 1989.

The main object of the present article is to investigate the behaviour of this model under different control parameters values.

13.1 Introduction

In the course of the last decades the problem of global society evolution is beginning to interest more and more politicians, scientists, etc. It is no surprise that such an attention is attracted to the papers on mathematical modeling of the global evolution of society and social systems.

The first and the most important work on global society evolution was done in the beginning of the 1970s by D. Meadows's group for the Rome Club ([Med 74]).

Five main factors which define society evolution were described in this paper:

- population size;
- agrarian production;
- natural resources;

- industrial production;
- pollution.

In order to investigate the interconnection of these five factors and forecast the tendency of society evolution a simulation model was created. This model was based on the logistic type differential equations describing the exponential growth of system with restricted and nonregenerating resources.

Conclusions obtained as a result of this work gave birth to a great amount of debates, discussions and new researches in this area.

In connection with this it was quite natural to try to use synergetic approach to the modeling of the social systems evolution. The existence of changeable macro-level structures with an interaction of big amount of individuals on micro-level is the main peculiarity of these systems.

There are many interesting works on this area. We can mention P. Allen's (1986) works on the complex systems control, K. Troitzsch's (1991) works on the evolution of technologies, etc.

A very interesting model, describing a modified Eigen hypercycle, was investigated in [Che 89],[Mar 83] .

In this connection the works by Chernenko (1991), Rozinko (1990), must be mentioned as well.

13.2 Chernenko's model of the social evolution.

Let us consider the system of differential equations [ES 79]:

$$\frac{dx_i}{dt} = F_i(x_1, \dots, x_n, k_1, \dots, k_m) - \frac{x_i}{C_0} \sum F_j(x_1, \dots, x_n, k_1, \dots, k_m) \qquad (13.1)$$

where

x_i are characteristics of different technology;

F_i are Allen's production functions;

C_0 is the quasistationary sum of the x_i;

k_j are control parameters.

If one summarizes these equations he could obtain

$$\sum \frac{dx_i}{dt} = \sum F_i(x_1, \dots, x_n, k_1, \dots, k_m)(1 - \sum \frac{x_i}{C_0})$$

Let us denote $C = \Sigma x_i$. Then we obtain

$$\frac{dC}{dt} = \sum F_i(x_1, \dots, x_n, k_1, \dots, k_m)(1 - \frac{C}{C_0}) \qquad (13.2)$$

Solutions of this equation will be stable if

$$\sum F_i(x_1, \ldots, x_n, k_1, \ldots, k_m) > 0$$

that is $C(t) \to C_0$ while $t \to \infty$

Chernenko (1991) proposed an interesting modification of the equations 13.1 and 13.2

$$\frac{dx_1}{dt} = x_1(N - x_1) \tag{13.3}$$

$$\frac{dx_2}{dt} = x_2(ax_1 - x_2 + kx_2^2 - gx_2^3) \tag{13.4}$$

$$\frac{dx_3}{dt} = x_3(bx_2 - x_3) \tag{13.5}$$

where N, a, k, g, and b are control parameters.

The nonlinear right hand side in equation 13.4 (by analogy with theoretical physics) is used to account for the complexity of interacting processes.

The following balance equation is used for C_0 [Che 91b]

$$\frac{dC_0}{dt} = e(rx_1 - fC_0) \tag{13.6}$$

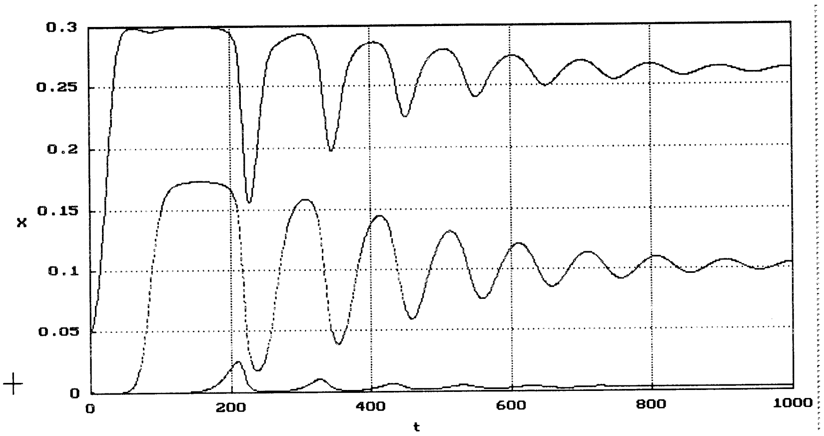
where e, r, and f are control parameters.

13.3 Numerical results

This paper is devoted to the numerical investigation of modes of behaviour of Chernenko's model under different control parameters values.

In order to fulfill this work a complex of programs for solving systems of nonlinear differential equations and for displaying trajectories of movement by means of two-dimensional graphics was created.

The following results were obtained in the course of the numerical experiments.



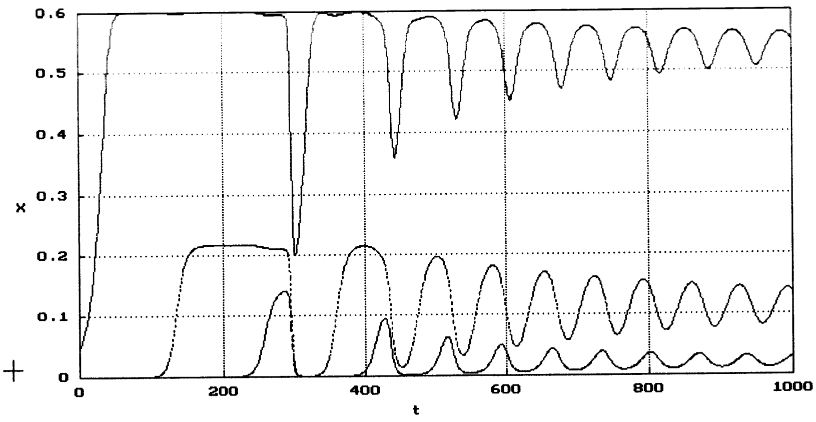


Figure 13.2: Relaxation oscillation

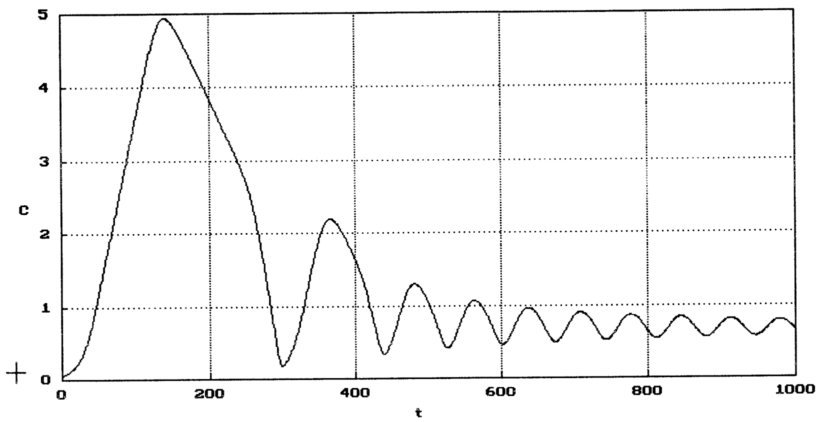


Figure 13.3: Relaxation oscillation of C

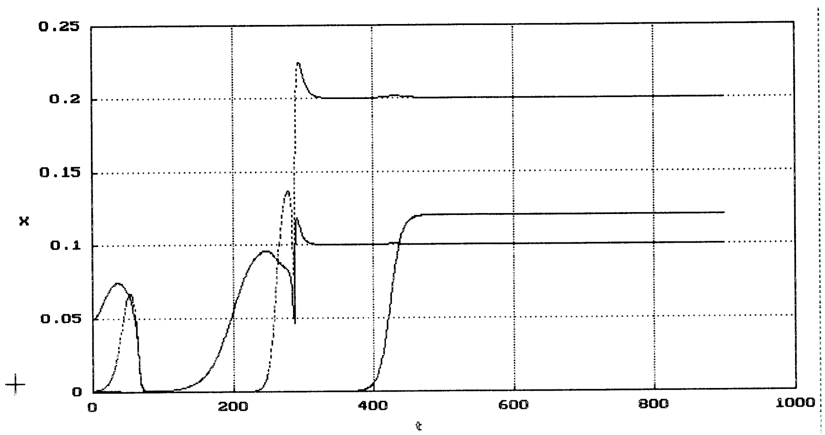


Figure 13.4: The model's behaviour depending on different control parameters

The more typical mode of model's behaviour is the mode shown on Figure 13.1 and Figure 13.2. This mode is characterized by existence of relaxation oscillations with the following stabilization during some time interval.The amplitude of fluctuations and the time of stabilization depend on the values of parameters and initial values.

The character of changes for this mode is shown on Figure 13.3.

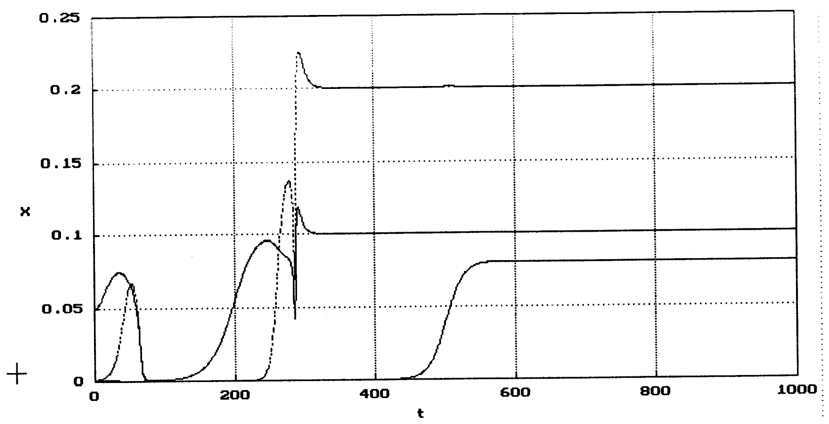


Figure 13.5: The model's behaviour depending on different control parameters

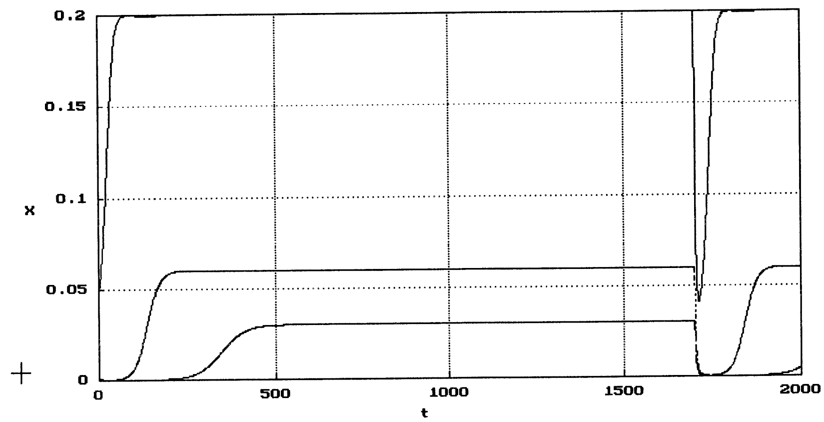


Figure 13.6: Unrelaxation oscillations

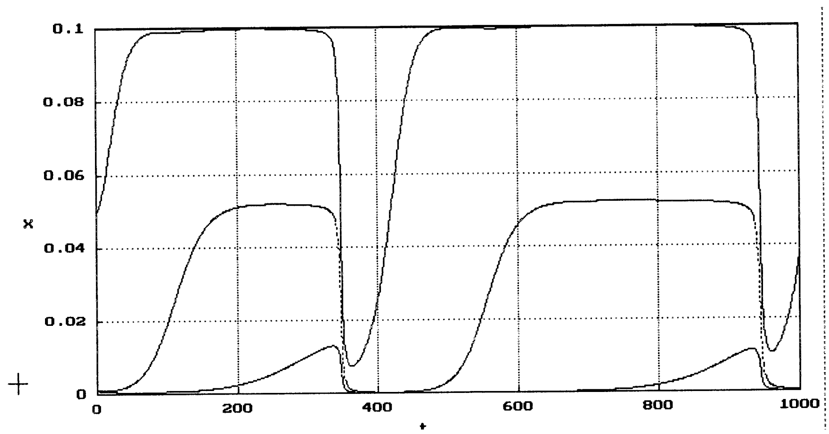


Figure 13.7: Unrelaxation oscillations

Other values of parameters give a different type of graphics which is shown on Figure 13.4 and Figure 13.5.

The next mode of the model's behaviour — the mode of unrelaxation oscillation — is shown on Figure 13.6 and Figure 13.7.

Nevertheless the most interesting mode of model's behaviour is the mode with clearly

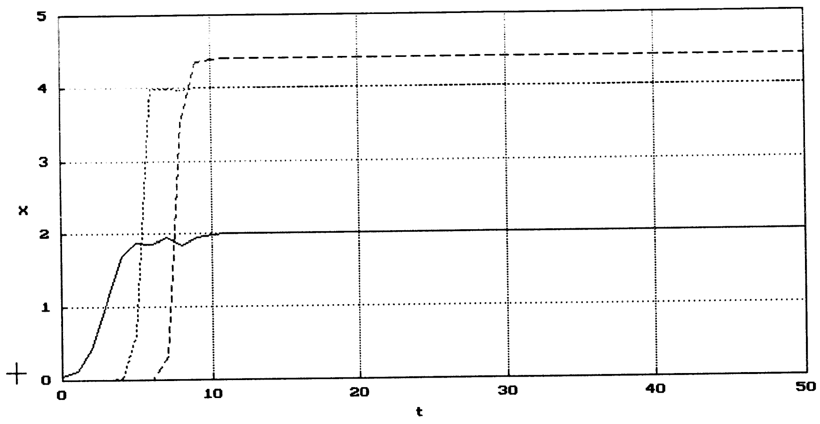


Figure 13.8: The catastrophic inversion of x_1 and x_2

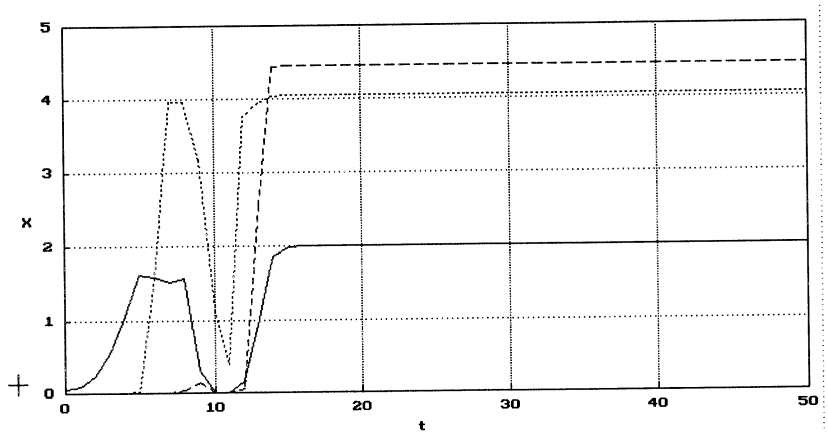


Figure 13.9: The catastrophic inversion of x_1 and x_3

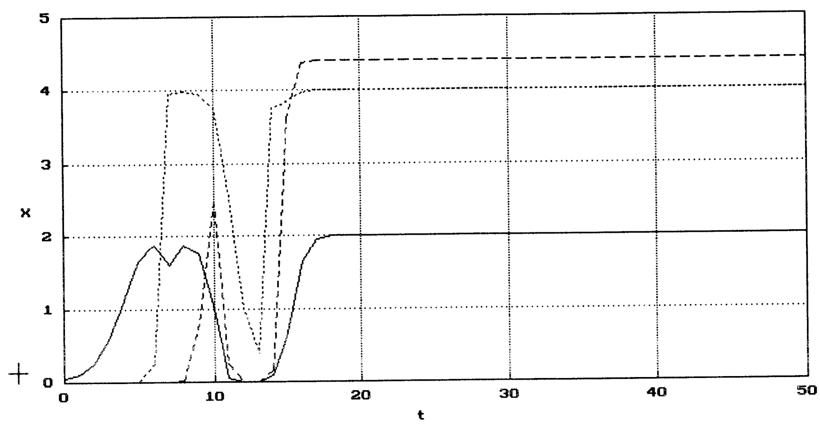


Figure 13.10: The catastrophic inversion of x_1 and x_2

expressed "catastrophic" phenomena. The examples of such behaviour is shown on Figure 13.8, Figure 13.9, Figure 13.10 and Figure 13.11.

A slightly different behaviour of the model can be obtained if we replace equation 13.6 by it's logistic analog

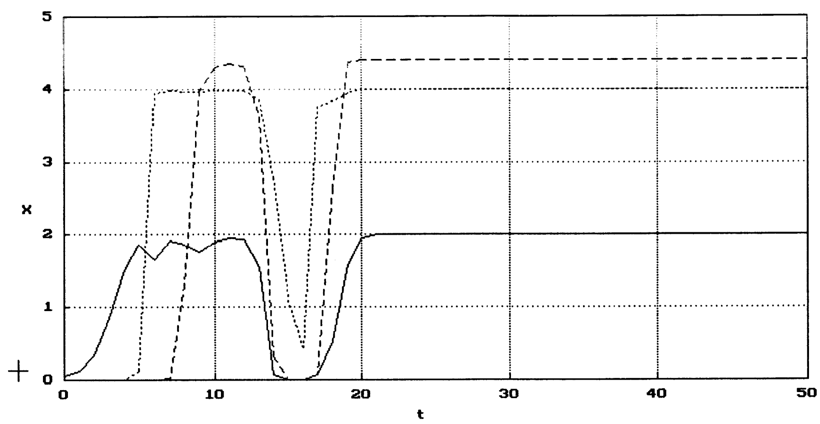


Figure 13.11: The catastrophic inversion of x_1 and x_3

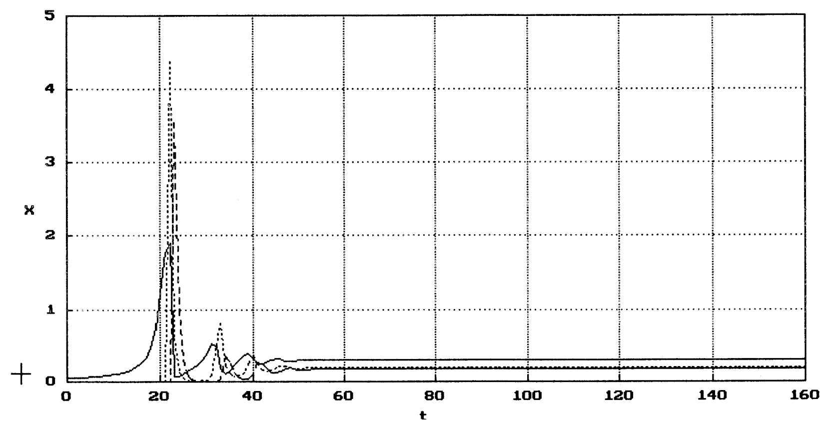


Figure 13.12: The most typical model's behaviour

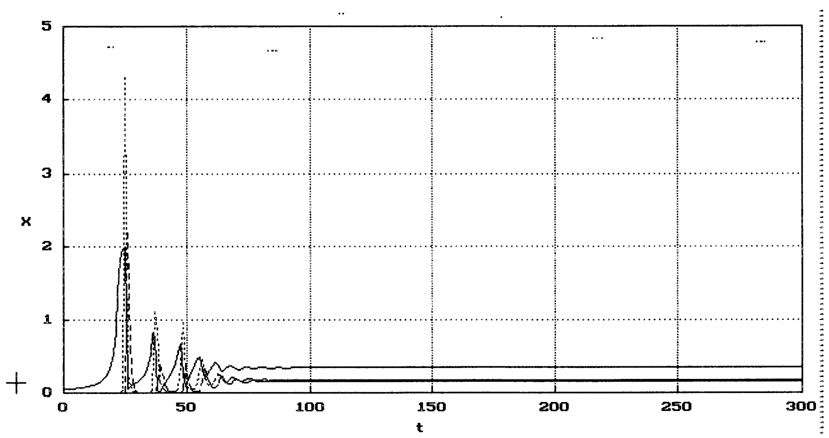


Figure 13.13: The most typical model's behaviour

$$\frac{dC_0}{dt} = eC_0(rx_1 - fC_0)$$

In this case the most typical behaviour of the model can be described by the mode shown on Figure 13.12, Figure 13.13 and Figure 13.14.

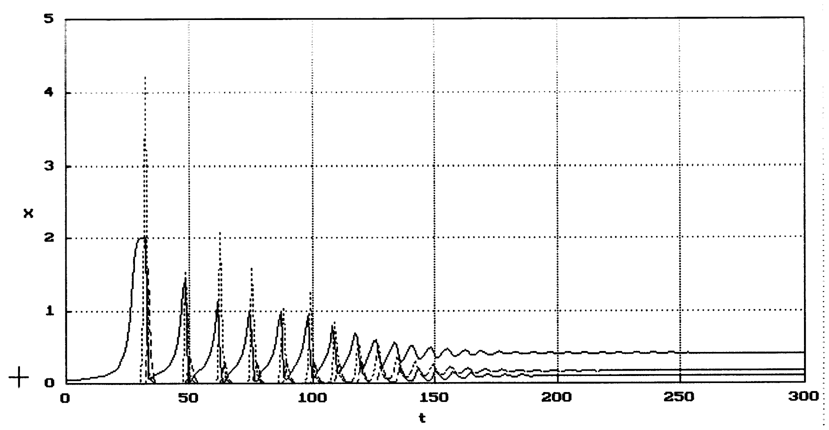


Figure 13.14: The most typical model's behaviour

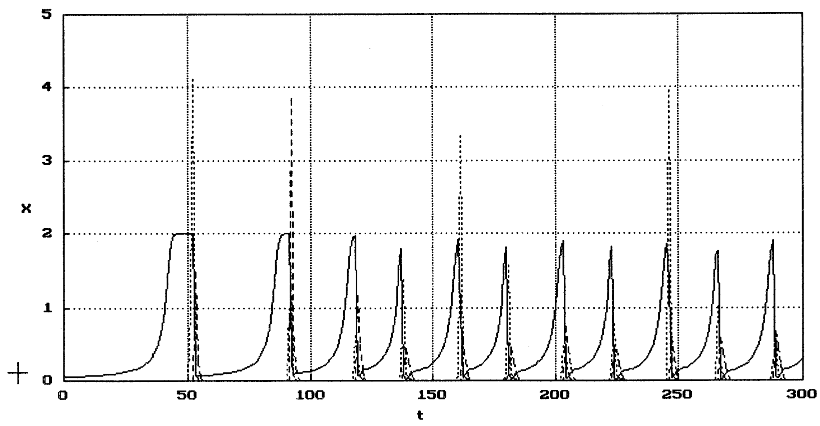


Figure 13.15: Unrelaxation oscillations

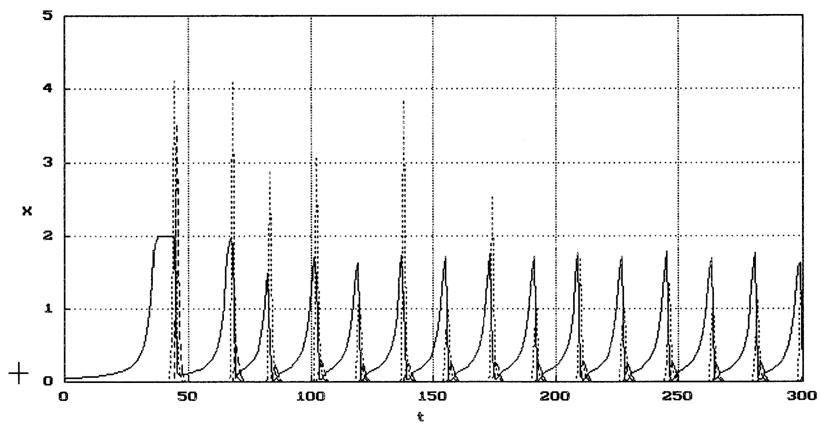


Figure 13.16: Unrelaxation oscillations

For the unrelaxation oscillations the mode is shown in Figure 13.15 and Figure 13.16. It is characterized by the existence of random peaks for the variable x_3.

It is very interesting to investigate the model's behaviour changes depending on initial conditions. Graphics of such type are shown on fig. 13.17, fig. 13.18, fig. 13.19, fig. 13.20 and fig. 13.21.

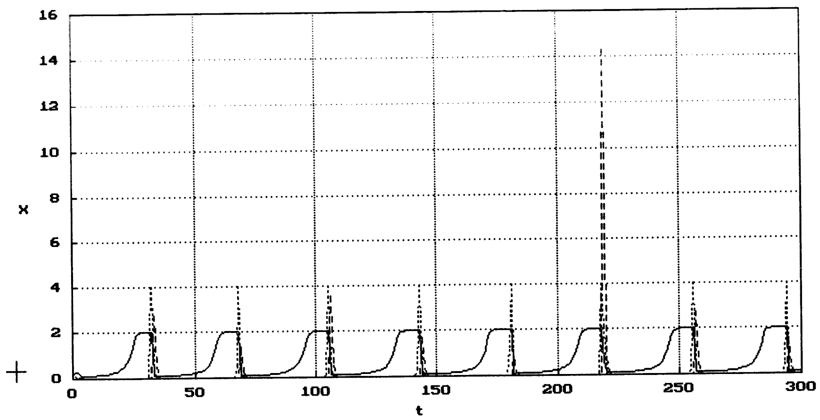


Figure 13.17: The behaviour changes depending on initial parameters

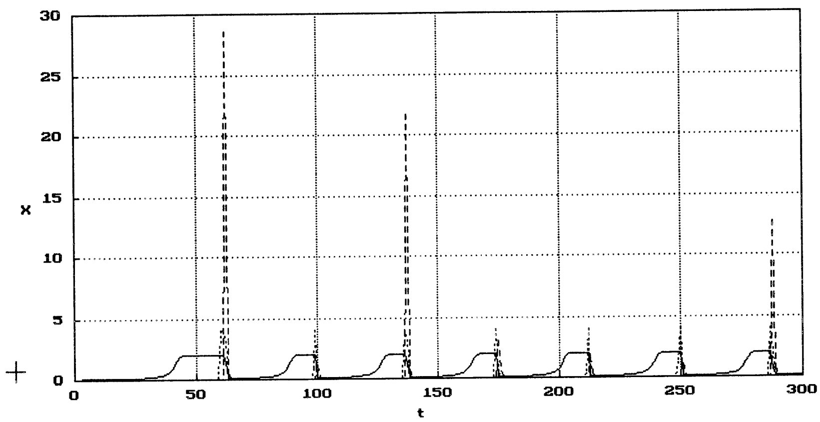


Figure 13.18: The behaviour changes depending on initial parameters

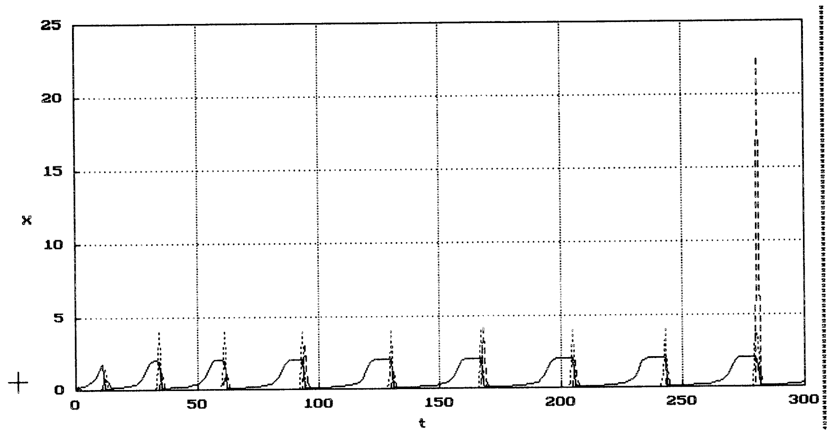


Figure 13.19: The behaviour changes depending on initial parameters

One can see changes of fluctuations frequency and the random peaks of the amplitude of variable x_3 .

But the most interesting mode is shown on Figure 13.22, Figure 13.23 and Figure 13.24.

It is characterized by qualitative changes in the model's behaviour. At the beginning

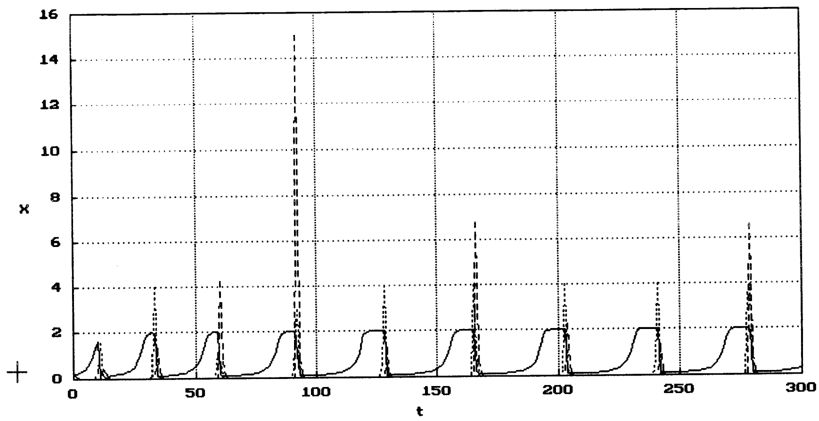


Figure 13.20: The behaviour changes depending on initial parameters

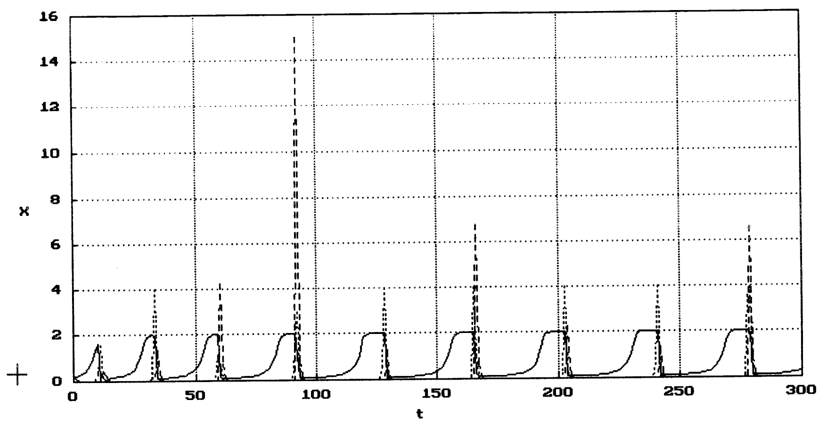


Figure 13.21: The behaviour changes depending on initial parameters

the character of the model's behaviour is the same as for the mode shown on Figure 13.15. But beginning from some moment of time x_3 becomes equal to zero and x_1 and x_2 begin to change periodically. The amplitude of these fluctuations is constant for x_1 and changes periodically for x_2 .

13.4 Conclusions

In conclusion let us compare the model developed by Meadows's group with Chernenko's model.

Meadows's model is based on the logistic equations that describe exponentional growth of a closed system under restricted and nonregenerating resources. And those catastrophic phenomena which represent one of the variants of societal evolution are determined by the speed of exhaustion of these resources.

For Chernenko's model catastrophic phenomena are determined by interior innate mechanisms and are not connected, in a general case, with the exhaustion of resources. These phenomena may take place in course of time, sometimes rather prolonged, in case of violation of proportions in evolution of these or those technologies.

Hence it follows the conclusion about the danger of planning based on immediate

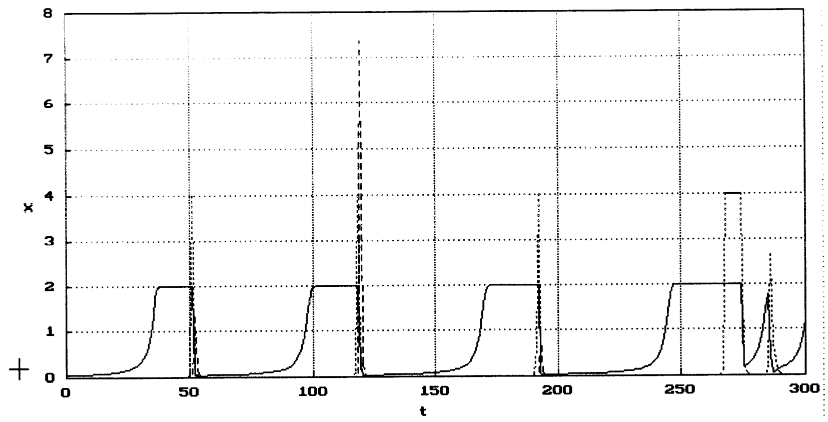


Figure 13.22: The qualitative changes in the behaviour of the model

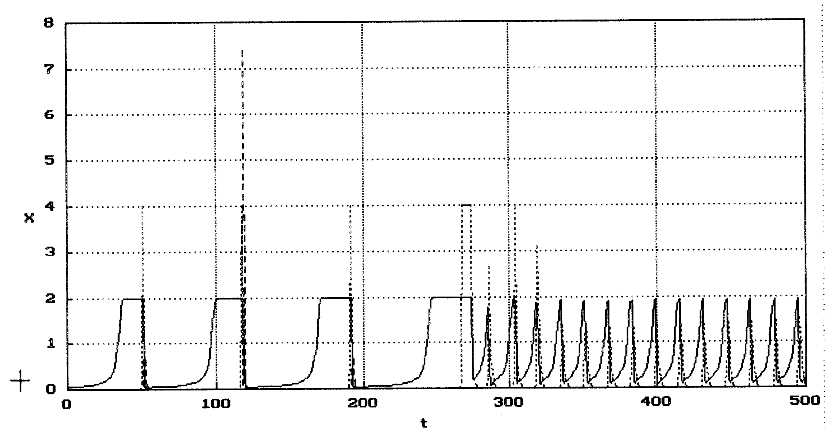


Figure 13.23: The qualitative changes in the behaviour of the model

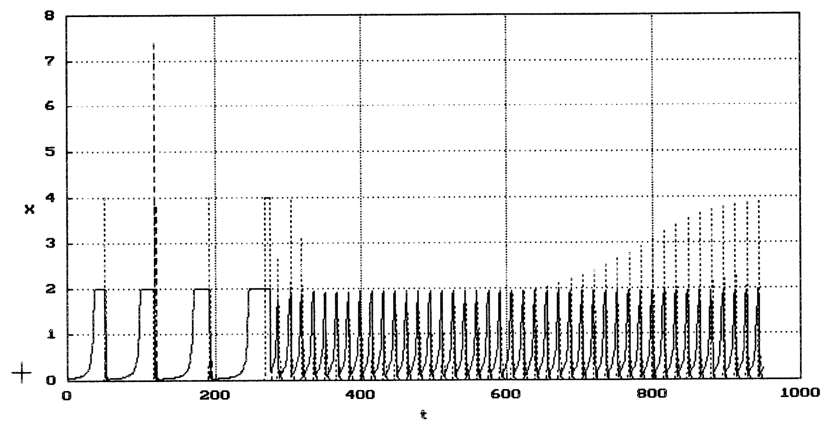


Figure 13.24: The qualitative changes in the behaviour of the model

extrapolation of previous experience. Similar static methods threaten society by stagnation and, in a course of time, by catastrophe. On the other hand, using the true strategy of behaviour, social systems may exist for a long time, renew itself and find new ways of evolution without visible falls down and shocks. It is defined by right proportions in evolution of technologies and true choice of moment for introduction of new technologies.

In case of simple modifications of Chernenko's model we can speak only about qualitative aspects of the model's behaviour and are not able to forecast it's behaviour in a real time scale.

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Chapter 14

Igor V. Chernenko and Serge V. Chernyshenko: Mathematical Script of Social Apocalypse

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Abstract

According to Ecclesiastes the social apocalypse has three main characteristics as follows: the crisis of power; the collapse of production of consumer goods; the growth of criminality.

The crisis of the ruling elite was studied by Pareto and the Weberian school. The collapse of production was investigated by Alexander N. Rozinko in his brilliant work *Economical Mechanism of the Total Power of Bureaucracy* (1990). Egalitarian criminality was analyzed by Pitirim Sorokin in his famous paper *Famine and Ideology of Society* (1922). The correlation between these phenomena was discussed in [Che 91b].

In analyzing the complex nonlinear dynamics of social systems we are shocked by the multiplicity of unique phenomena that can hardly be systematized by direct rational methods of traditional science. To reduce the immense variety of perceptive forms to a finite number of typical classes we need to postulate the existence of invariant patterns that can be described as archetypes or ideal prototypes of empirical structures. It leads, almost by necessity, to the realization that the empirical world has a functional double or a latent structure which cannot be studied in terms of common experience.

To get over the abyss between the macro and micro scales we need models that reduce macro phenomena to micro processes. Precisely these models have to be developed as procedures that aggregate nuclear micro processes into phenomenological aspects of the visible world. These procedures allow one to reconstruct the path of evolution and forecast possible futures that can be influenced by appropriate intervention to make a difference in their probabilities.

The world of artificial self-organized models that the demiurgic technology creates is based on nonlinear interactions between simple mechanisms. The key difficulty is that it is unclear precisely how these functional relations can be interpreted in terms of the cultural tradition and the "political

field" as a set of Invariant Rules that display themselves through the empirical variety of collective behaviour. But at the same time this approach can be treated as a further development of the Weberian paradigm that was initiated by the sacramental question of how a new order can arise out of disorder.

The micro level of a socio-economic system is described by the model of technological evolution based on Eigen's hypercycle and Allen's production functions.

The macro level is considered as a Cusp Catastrophe system. It should be noted that the complex system of micro processes creates a macro illusion of the existence of a potential function at the macro level. But this macro illusion can be used as an approximate map of the dynamic singularities of the system. Analyses of the micro model enable us to study hidden processes such as the strange attractor and the tunnel junction through the macro potential barrier that can be treated as a transition from socialism to post-socialism.

14.1 Introduction

> Time changes everything, and every new nagual has to incorporate new words, new ideas, to describe his seeing.
>
> *Don Juan Matus [Casta 85], p.49*

The fatalistic view that human reality has an inconceivable nature and its future is unforeseeable and inevitable appears to have been abandoned. The actual multiplicity and uncertainty of possible futures lead to a failure of rational endeavours to disenchant the visible world.

The history of humanity seems to be a palingenesis of an innate set of competing archetypes that can be described in terms of Catastrophe Theory. The implicit interaction between hidden processes induces an illusion of a stochastic force of destiny as well as a false idea of a deterministic essence of nature. Misleading interpretations create a Babel of self-consistent models and camouflage the mischievous power of ignorance [Po 63, pp. 3–7]. Latent properties of social reality display themselves in pathological phenomena such as irrational metamorphoses of the Russian Spirit. They tell us about coming cataclysms by manifestations of hardly understood omens that can be recognized by means of patterns of synergetic simulation.

The shadows from other dimensions, such as the Cusp singularities of the cubic catastrophe, reveal a hidden topology of multi-dimensional reality that includes potential forms of the empirical world as well as structures of their realization. In other words the system functional topology approach is the crux of Catastrophe Theory's sorcery. To make sense of uncertain time-variant phenomenal manifestations an invariant acupuncture map of hidden functional structures of reality could be used, i.e. to reconstruct the true reality in conformity with archetypical invariants, we need a lodestar to get over the perversity of the mind poisoned by ignorance.

Interpretations of nonlinear processes have a relative nature. Regular structures can be observed only from appropriate "positions" of view using specific tools for the arrangement of artificial realities as realizations of an investigator's intent. A deliberate organization of the event stream does not contain an objective, free-context situation but depends on objective circumstances as external factors that can be described in terms of macro parameters.

Considerations of Catastrophe manifolds and bifurcation diagrams of a modified

Eigen's hypercycle [ES 79, Mar 83, Che 89, Che 91a, Che 91b, Tro 91] enable the scholar to interpret, as far as possible, social development in terms of nonlinear dynamics of a self-organized system and to construct mathematical models of a concealed functional double of empirical reality.

The social phenomena are treated as distorted manifestations of hidden genetic prototypes of functional structures. Investigation of catastrophic functional regimes leads to the principle of the least compulsion that represents the essence of liberal society.

14.2 Morphology of Ordinary Parametric Structures

The gods did not reveal, from the beginning
All things to us; but in the course of time
Through seeking, men find that which is the better.

Xenophanes [DK]

The simplest example of a dynamic system appears to be as follows

$$\frac{dx_i}{dt} = F_i(x, k_1, \ldots, k_n)$$

Stationary solutions satisfy the algebraic equations

$$F_i(x, k_1, \ldots, k_n) = 0$$

These relations describe the spectrum of possible stable forms of empirical manifestations of the functional essence of the system. The stability of stationary solutions depends on the values of system parameters. A hidden permanent evolution of these parameters leads to abrupt changes of system behaviour at bifurcation points.

A bifurcation diagram represents a projection of an acupuncture map of a system and can be used to forecast possible ways of system morphogenesis.

14.3 The Logistic Equation

Discrimination is the demon who
Produces the ocean of transmigrations

Aryadeva

Even supposing that the logistic equation is an adequate description of a set of empirical processes, we can not be sure that it is the precise law of auto catalytic evolution that achieves a saturation regime.

It is known that the behaviour of the solutions of the logistic equation is analogous to that of the exponential equation at small scales. This makes it possible to treat both equations as approximations of the natural law. But we need rational criteria for the choice of equations. One can be sure that the equation is useful when analysis of an equation helps to realize hidden effects. By systematizing the empirical data by means of a model we can recognize the concealed nature of phenomena and forecast their future metamorphoses. The heuristic procedure of modeling is a matter of philosophy and religion but the validity of the equations must be tested by rational techniques.

Mathematical simulation is full of problems for the discursive mind. Some of these problems are easy to explain — for instance, the formation of the Malthus equation. Other problems are more tricky, and cause difficulty even to sophisticated researchers. Let us consider a simple population model from the point of view of the structural stability approach.

Taking into account a quadratic term in the Malthus equation

$$\frac{dx}{dt} = ax$$

one could obtain

$$\frac{dx}{dt} = kx(N - x)$$

or

$$\begin{aligned} \frac{dx}{dt} &= kNx - kx^2 \\ a &= kN \end{aligned}$$

where N is the maximum possible population size. This is a kinetic equation for an auto catalytic reaction

$$X + (N - X) \Rightarrow X$$

between living beings X and imaginary entities $(N - X)$, as if the population is under the influence of hidden processes that can be described by $(N - x)$ as, for example, a number of unembodied souls, i.e.potential beings. So the value N could be treated as the total number of souls.

Natural restrictions on population growth created traditional habits that have symbolic meaning as a magical control of the order of incarnations. For example, the aboriginal inhabitants of New Guinea used to restrict the number of embodied souls by killing enemies and seizing their names [Gu 90], p.221. According to the Tibetan Book of the Dead (Bardo Thödol) the soul before the incarnation looks for appropriate parents and its embodiment depends on their existence. Nevertheless, traditional Buddhism rejected the existence of a soul as a rough phenomenal interpretation of real processes. Indeed, accounting for a cubic term leads to a Cusp Catastrophe system. Thus we can treat the conception of a soul as “illusion” and its transmigrations as the devilish mystification produced by ignorance. Moreover, “ignorance is the cause of psychic construction, hence is caused consciousness, physical form, the senses, contact, sensations, craving, attachment, becoming, and so birth, old age and death with all the distraction of grief, lamentation, sorrow and despair" [Ma 88].

In any case to see the manifest truth we need to recognize concealed archetypical patterns of human reality. Eastern mystic teachings based on the reincarnation theory as a phenomenology of hidden self-organized processes can be treated as an ancient prototype of the modern mythology of synergetic theory.

14.4 Modified Eigen's Chain

> Truth is simple, it is made to appear complex by the distractive intellect. The sublimest things are always the most simple.
>
> *Sri Swami Sivananda [Si 58], p.3*

Let us consider a modified Eigen chain (see [ES 79, Che 89])

$$\frac{dx_i}{dt} = F_i(x_1, \ldots, x_n) - \frac{x_i}{C_0} \sum F_j \tag{14.1}$$

where x_i are parameters of correspondent technologies and F_i are Allen's production functions. The number of realized technologies X depends on the total volume $C_0 = \sum x_i^0$, i.e. the relation between the level of the technological complexity X and the total volume C_0 could be treated in terms of functional approximations.

For Allen's production functions [All 76]

$$\begin{aligned} F_1 &= x_1(N - x_1) \\ F_2 &= x_2(ax_1 - x_2) \\ F_3 &= x_3(bx_2 - x_3) \end{aligned}$$

there are stationary solutions as follows

$$\begin{aligned} x_1^{(1)} &= C_0 \\ x_2^{(1)} &= x_3^{(1)} = 0 \\ C_0^{(1bif)} &= 0 \end{aligned}$$

$$\begin{aligned} x_1^{(2)} &= \frac{N + C_0}{a+2} \\ x_2^{(2)} &= \frac{(a+1)C_0 - N}{a+2} \\ x_3^{(2)} &= 0 \\ C_0^{(2bif)} &= \frac{N}{a+1} \end{aligned}$$

$$
\begin{aligned}
x_1^{(3)} &= \frac{C_0 + N(b+2)}{ab+a+b+3} \\
x_2^{(3)} &= \frac{(a+1)C_0 + N(a-1)}{ab+a+b+3} \\
x_3^{(3)} &= \frac{(ab+b+1)C_0 - N(a+b+1)}{ab+a+b+3} \\
C_0^{(3bif)} &= N\frac{a+b+1}{ab+b+1}
\end{aligned}
$$

$$
\begin{aligned}
x_1^{\max} &= N \\
x_2^{\max} &= aN \\
x_3^{\max} &= abN \\
C_0^{\max} &= N(1+a+ab)
\end{aligned}
$$

where $C_0^{(1bif)}, C_0^{(2bif)}, C_0^{(3bif)}, C_0^{\max}$ are critical points. Stationary solutions are matched continuously at bifurcation points (see Figure 14.1).

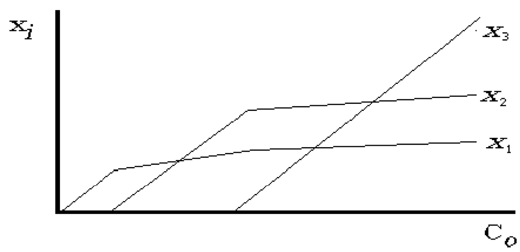


Figure 14.1: The diagram of technological evolution

When $b = 0$, we obtain the two-dimensional chain

$$
\begin{aligned}
x_1^{(3)} &= N \\
x_2^{(3)} &= aN \\
x_3^{(3)} &= 0 \\
C_0^{(3bif)} &= N(a+1)
\end{aligned}
$$

Thus in this case the system ignores the third process.

The realization of an innovation depends on the existence of the correspondent niche that can be provided by the appropriate economic policy predetermined by cultural traditions. Besides, present processes are under the influence of the potential non-existence of the particular process in the future. Our present strategy implicitly creates our future. Indeed nonessential action can predetermine cataclysms in the future. Potential catastrophes display themselves by hardly notable changes in observable processes. In the same way the manifestations of a hidden illness can be recognized by the analysis of small points on the iris of the eye. Iris ($I\rho\iota\sigma$) as the messenger of the gods reveals their fatal omens that can be systematized by means of diagnostic maps.

Using weighted variables

$$y_i = \frac{x_i}{C_0}$$

one can obtain the following modifications of Equation 14.1

$$\begin{aligned} \frac{1}{C_0}\frac{dy_i}{dt} &= F_i^* - y_i \sum F_j^* \\ \sum y_i &= 1 \end{aligned}$$

$$\begin{aligned} F_1^* &= y_1\left(\frac{N}{C_0} - y_1\right) \\ F_2^* &= y_2\left(ay_1 - y_2\right) \\ F_3^* &= y_3\left(by_2 - y_3\right) \end{aligned}$$

In this case the stationary solutions take form

$$\begin{aligned} y_1^{(1)} &= 1 \\ y_2^{(1)} &= y_3^{(1)} = 0 \\ C_0^{(1bif)} &= 0 \end{aligned}$$

$$\begin{aligned} y_1^{(2)} &= \frac{1 + \frac{N}{C_0}}{a+2} \\ y_2^{(2)} &= \frac{a + 1 - \frac{N}{C_0}}{a+2} \\ y_3^{(2)} &= 0 \\ C_0^{(2bif)} &= \frac{N}{a+1} \end{aligned}$$

$$\begin{aligned} y_1^{(3)} &= \frac{1 + (b+1)\frac{N}{C_0}}{(a+1)(b+1)+2} \\ y_2^{(3)} &= \frac{a + 1 + (a-1)\frac{N}{C_0}}{(a+1)(b+1)+2} \\ y_3^{(3)} &= \frac{ab + b + 1 - (a+b+1)\frac{N}{C_0}}{(a+1)(b+1)+2} \\ C_0^{(3bif)} &= N\frac{a+b+1}{ab+b+1} \end{aligned}$$

$$\begin{aligned} y_1^{fin} &= \frac{N}{C_0^{\max}} \\ y_2^{fin} &= \frac{aN}{C_0^{\max}} \\ y_3^{fin} &= \frac{abN}{C_0^{\max}} \\ C_0^{\max} &= N(1+a+ab). \end{aligned}$$

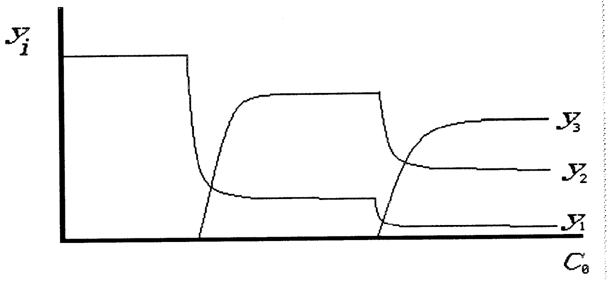


Figure 14.2: Bifurcation diagram for normalized variables

Figure 14.2 represents the singularities of connection between the stable solutions that lead to the crisis of new technologies described by Kondratiev [Ko 89]. The system has not enough time to adopt innovation and achieve the next stable solution because of the small value of the force that stabilizes it near the bifurcation point.

14.5 Catastrophe of Stalin's Industrialization

> Compared to the source of everything the most fearsome, tyrannical men are buffoons; consequently, they were classified as petty tyrants, *pinches tiranos*.
>
> *Don Juan Matus [Casta 85], p.17*

Taking into account nonlinear effects at a small level of consumption we can obtain the following modifications of the system [Che 91a, Che 91b, CCK 91]

$$\frac{dx_i}{dt} :=: F_i - \frac{x_i}{C_0} \sum F_j$$

$$\begin{aligned} F_1 &= x_1(N - x_1) \\ F_2 &= x_2(ax_1 - x_2 + kx_2^2 - gx_2^3) \\ F_3 &= x_3(bx_2 - x_3) \end{aligned}$$

where x_1, x_2 and x_3 describe the production of consumer goods, the production of industrial tools, and the production of information respectively.

The spectrum of stationary solutions is as follows

$$\begin{aligned} x_1 &= C_0 \\ x_2 &= x_3 = 0 \\ C_0^{(1bif)} &= 0 \end{aligned}$$

$$\begin{aligned} x_1 &= C_0 - x_2 \\ (1+a)C_0 &= N + (2+a)x_2 - kx_2^2 + gx_2^3 \\ x_3 &= 0 \\ C_0^{(2bif)} &= \frac{N}{a+1} \end{aligned}$$

$$\begin{aligned} 2x_1 &= -(1+b)x_2 + C_0 + N \\ 2x_3 &= (b-1)x_2 + C_0 - N \\ (1+a)C_0 &= (1-a)N + [2 + (1+a)(1+b)]x_2 - 2kx_2^2 + 2gx_2^3 \end{aligned}$$

When

$$a < \frac{k^2}{3g} - 2$$

there is the bimodal solution which is the cause of catastrophic inversions of x_1 and x_2 (see Figure 14.3 and Figure 14.4).

Stalin's "great breakdown of 1929" can be treated as an example of such inversions. Precisely the low level of productivity a of the first technology predetermines catastrophic phenomena in the future. The parameter a can be treated as an index of the future that reveals itself by means of measurable characteristics.

The graphic of the third stationary solution for x_2 and x_3 is represented in Figure 14.4.

This type of industrialization causes reverse catastrophic deindustrialization in the future (see Figure 14.4). The most probable realization of such deindustrialization as reestablishing appropriate proportion between industry and agriculture is a war that could be solve the problem of potential unemployment by killing a part of population or provide new territories to increase a number of farmers.

The pressure of economic disproportion caused the war between Nazi Germany and Soviet Russia as well as current civil war in Yugoslavia and national conflicts in the former Soviet Union. These factors would also predetermine a war between the Russia and the Ukraine in the nearest future.

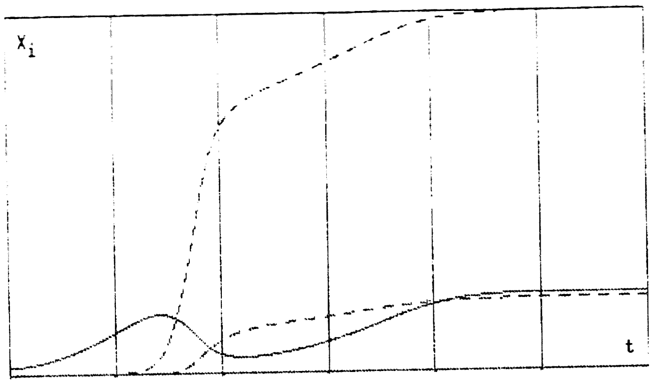


Figure 14.3: The catastrophic inversion of x_1 and x_2

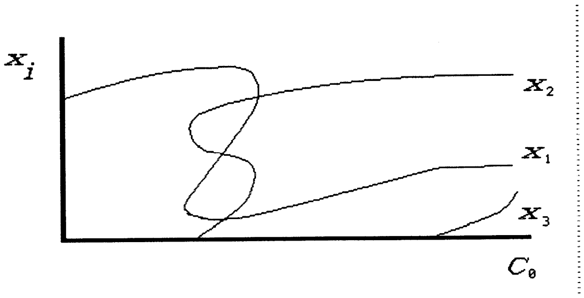


Figure 14.4: Bimodal stationary solutions

14.6 Macro Model of Socio-Economic Evolution.

Brahman is the Source and Substratum, the Basis for all the play of phenomenal relativity ... just as the dreamer is the Support of all the objects of his dream in the mind, so is Brahman the Prop or Support of the sport of Illusion as the diverse appearance of the universe.

Sri Swami Sivananda [Si 58], p.65

But in fact, nothing do we know having seen it; for the truth is hidden in the deep.

Democritus [DK]

We could suggest that time evolution of the total volume C_0 depends on the system state, i.e.

$$\frac{dC_0}{dt} = f(X, C, P)$$

where C is the level of consumption, P is the productivity of labor [RC 90], and X is the level of technological complexity that closely correlates with the level of social freedom (see Section 14.7).

Taking into account the approximate relation $C_0 = \phi(X)$ one obtains

$$\frac{dX}{dt} = F(X, C, P)$$

In the linear approximation

$$\frac{dX}{dt} = -aX + P$$

The corresponding potential function is

$$\Psi = \frac{a}{2}(X - \frac{P}{a})^2 - \frac{P^2}{2a}$$

The stable state is defined by the relation $X = \frac{P}{a}$. Thus in the first approximation the level of system development is defined by the level of production P. The second approximation gives the follows equation

$$\frac{dX}{dt} = -aX + kX^2 + P$$

or

$$\frac{dX}{dt} = -kX(C - X) + P$$

where C equals $\frac{a}{k}$. The stationary solutions of these equations are stable when $X < \frac{C}{2}$ (Figure 14.5).

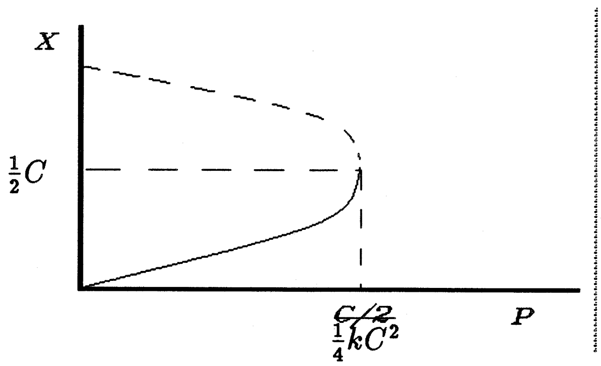


Figure 14.5: The limit to growth at the second approximation. When $P > \frac{1}{4}kC^2$ there are no stable solutions. By the same way solutions lose their stablility if $X > \frac{1}{2}C$.

In this approximation the limit of "production" P is determined by the level of "consumption" C (see Figure 14.5).

The cubic approximation leads to the Cusp Catastrophe equation [Casti 79]

$$\frac{dX}{dt} = -kX(C - X + \beta X^2) + P$$

or

$$\frac{d\zeta}{dt} = k\zeta(C_0 - C - \beta\zeta^2) + P - P_0^* \qquad (14.2)$$

where

$$\begin{aligned} \zeta &= X - \frac{1}{3\beta} \\ C_0 &= \frac{1}{3\beta} \\ P_0^* &= \frac{2k}{27\beta^2} - \frac{kC}{3\beta} \end{aligned}$$

In this approximation there are local singularities of the system development (see Figure 14.6) [RC 90, [illegible]]

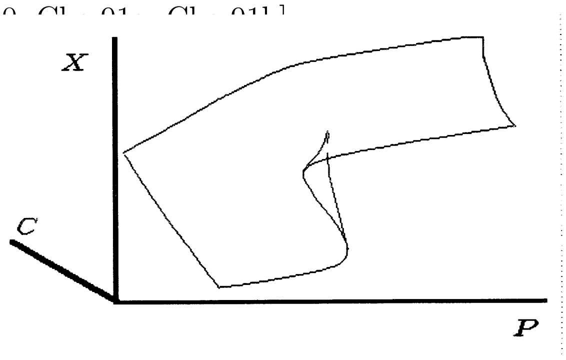


Figure 14.6: Cusp Catastrophe

The lines of constant level of freedom $X = const$ are defined by the following relation (see Equation 14.2)

$$P = kC(X_C - X_0) - k[C_0 - \beta(X_C - X_0)^2](X_C - X_0)$$

This is the family of generating lines of the surface of the Cusp Catastrophe (see Figure 14.7). The traditional system moves along these lines of fate, i.e. the lines of least resistance.

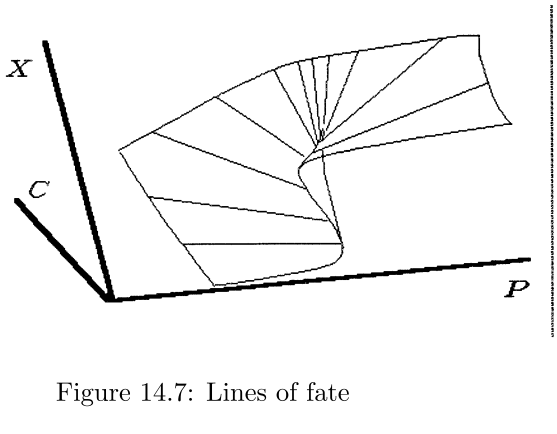


Figure 14.7: Lines of fate

To make sense of Equation 14.2 we need to investigate the specific features of the system's behaviour [RC 90]. Empirical studies of macro evolution lead to a description of system regularities and a complex manifold of irregular states that display themselves as singular phenomena and form unavoidable uncertainties of observed reality. To adopt the indefinite latent reality of the mysterious functional double of a nonlinear system, it would be reasonable to use graphic representation that systematizes evolutionary states by arranging them with an acupuncture map.

Trying to understand an immense diversity of time-variant worlds we interpret the phenomenal stream in terms of a bounded set of macro parameters and artificially construct structures of parameter correlation. To describe the hidden characteristics of system behaviour we need to introduce symbolic features such as “a level of social freedom” and “a level of technological complexity” that can be estimated by specific combinations of macro parameters, providing appropriate consideration of unavoidable singularities of these “estimations”. The same symbolic constructs could be found in different branches of human knowledge. The enigmatic Qi-energy of traditional Chinese medicine represents an example of a symbolic essence used to interpret an intricate schemata that systematizes homeoestatic states of the human organism.

14.7 Tunnel Junction and Social Apocalypse

> He who does not expect the unexpected will not detect it: for him it will remain undetectable, and unapproachable.
>
> *Heraclitus [DK]*

Social freedom used to be treated as the real ability to make a rational choice. It can be estimated as a measure of uniform distribution (entropy [Fe 72]) of social activities [Che 91b]:

$$\begin{aligned} S &= -\sum \frac{x_i}{Q} \ln \frac{x_i}{Q} \\ Q &= \sum x_i. \end{aligned}$$

Catastrophic jumps such as Stalin’s industrialization (the great breakdown of 1929) lead to sudden changes of the entropy (see Figure 14.8) as well as other macro parameters. There are short time chaos and freedom during the phase transition (see Figure 14.8).

The evolution of scale C_0 is determined by the production of consumer goods. In order to cross the barrier between the past and the future the system has to spend some of its resources [Che 91b]

$$\beta \frac{dC_0}{dt} = Ax_1 - C \sum x_i \tag{14.3}$$

The small level of consumption C allows the system to increase production at the lower part of the surface of the Cusp Catastrophe (see Figure 14.6). The leak of provisions can pull up the system development when a new process x_3 appears. The system becomes unstable and the mechanism of its life turns into the mechanism of its death (apocalypse, see Figures 14.10– 14.14). The decrease of C_0 and decomposition

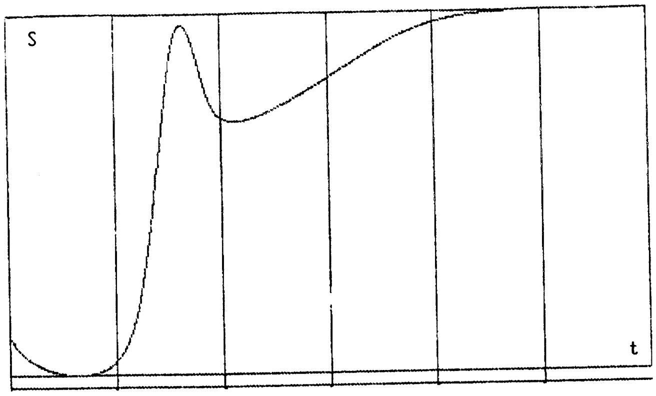


Figure 14.8: Evolution of entropy before apocalypse

of the system into a set of autonomous components leads to the inversion of x_1 and x_2 [Che 91a, Che 91b] that can be treated as an effect of the tunnel junction through the potential barrier in the macro model (see Figure 14.9). Thus the macro model allows one to describe rough effects only. To study hidden phenomena such as the tunnel junction we need to use the micro model.

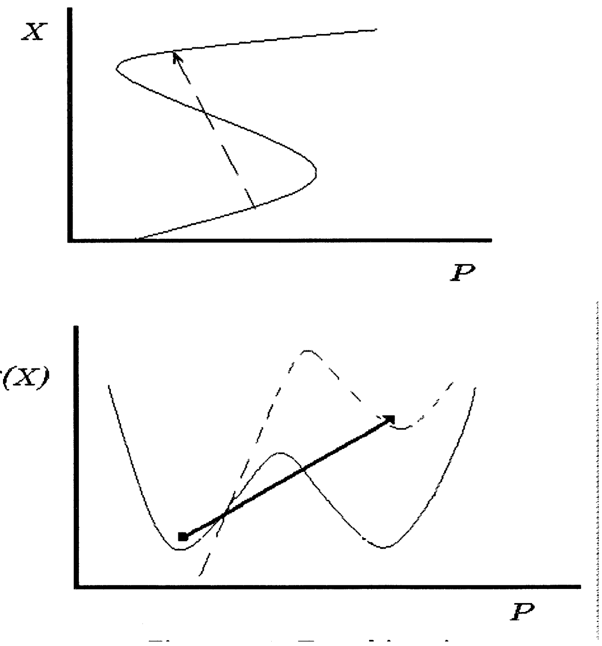


Figure 14.9: Tunnel junction

For example, let us consider the Moscow revolution in August 19-21, 1991. It was an unsuccessful endeavour of the Soviet Government to save communist power. Indeed Boris El'tsin made a coup d'état in Russia. Chaotic behaviour during the revolution corresponds to a peak of entropy on the phase transition. Really, post-socialism is a movement of the intelligentsia to rationalize socialist totalitarian society [Ha 44].

The August revolution accelerated the decay of soviet society. The low speed of disintegration at the initial stage makes a dangerous illusion of relative social stability [Che 91a]. Nevertheless apocalyptic processes exponentially increase.

14.8 Strange Attractor

> Yo-no naka-wo nani-ni tatoemu. Asaborake
> Kogi-yuki fune-no ato-no shiranami.
>
> *Man-yo-shyu*

> When Buddha lifted up his foot all could perceive upon the sole of it the appearance of a wheel of a thousand spokes.... And Shakyamuni said: "Whosoever beholds the sign upon the sole of my foot shall be purified from all his faults. Even he who beholds the sign after my death shall be delivered from all the evil results of his errors".
>
> *Buddha-dhyana-samadhi-sagara-sutra [Hea 81b], p.159*

Each historical cycle ends with annihilation of all processes in accordance with Leo Gumilyoff's conception of ethnogenesis [Gu 90], [Kuz 91]. Prehistoric evolution of the system is almost erased by Zusammenbruch (see Figure 14.10 , Figure 14.11). After a period of chaos a [illegible] 4.13).

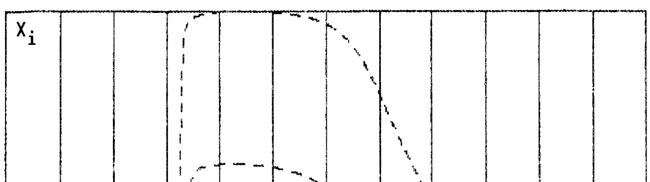


Figure 14.10: Zusammenbruch

The system contains a special strange attractor that generates initial spontaneous stimuli to development. The (x_1, x_2) — projection of a strange trajectory is shown in Figure 14.14. At the predetermined moment of historical time the C_0 trajectory achieves critical region in the phase space, and the singular points $C = Ax_1/C_0$ appear on the (x_1, x_2) — plane. The symbolic map of the strange attractor is represented in [Hea 81b], p.161.

The unstable focus of a strange attractor can be considered as an assemblage point [Casta 85] of empirical system realizations. Localization of the assemblage point is

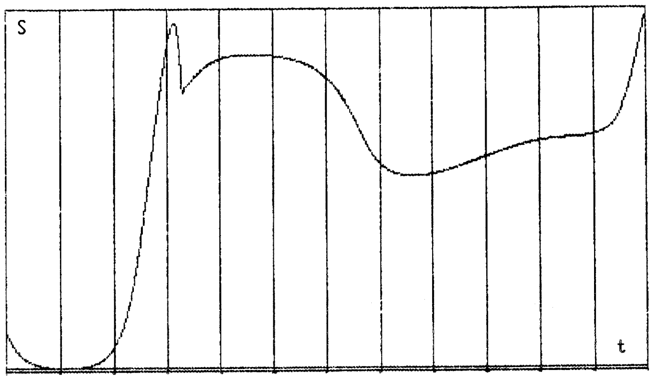


Figure 14.11: Evolution of entropy through apocalypse

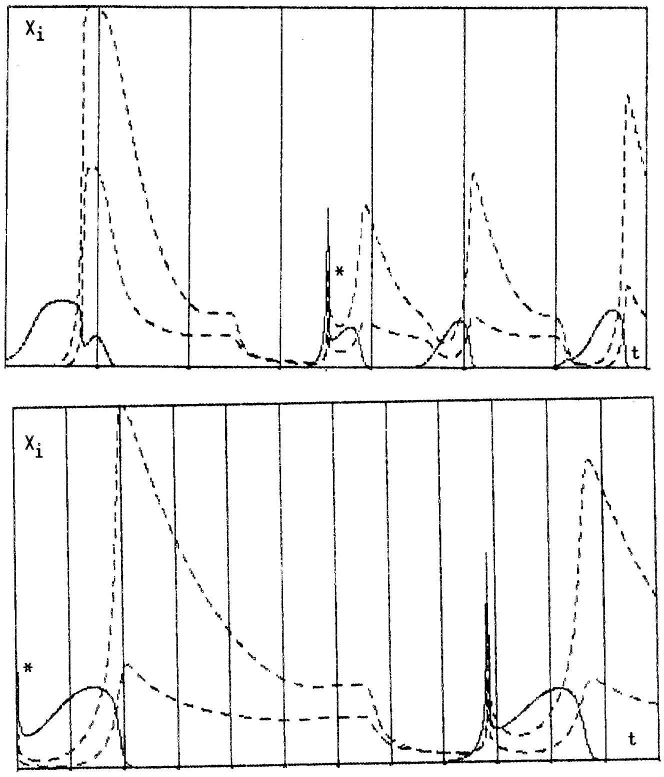


Figure 14.12: Historical cycles

determined by bifurcation parameters. Values of these parameters depend on various conditions. We have to remember that "all conditions are impermanent, and so, in the profounder sense, unreal" [Hea 81a], p.120. The Functional Double — the Supreme Buddha — is the Unconditional Reality. "To see the face of one Buddha is to see all" [Hea 81a], p.127. This doctrine appears in sotoba of the Zen sect: "All livings being have the nature of Buddha. The Nyorai [Tathagata], eternally living, is alone unchangeable" [Hea 81a], p.121.

The Buddhist Cosmology deals with the Three World, i.e. the Three Bodies of Buddha or the Three Types of Consciousness. There are the Samadhi (Intent), the Mantra

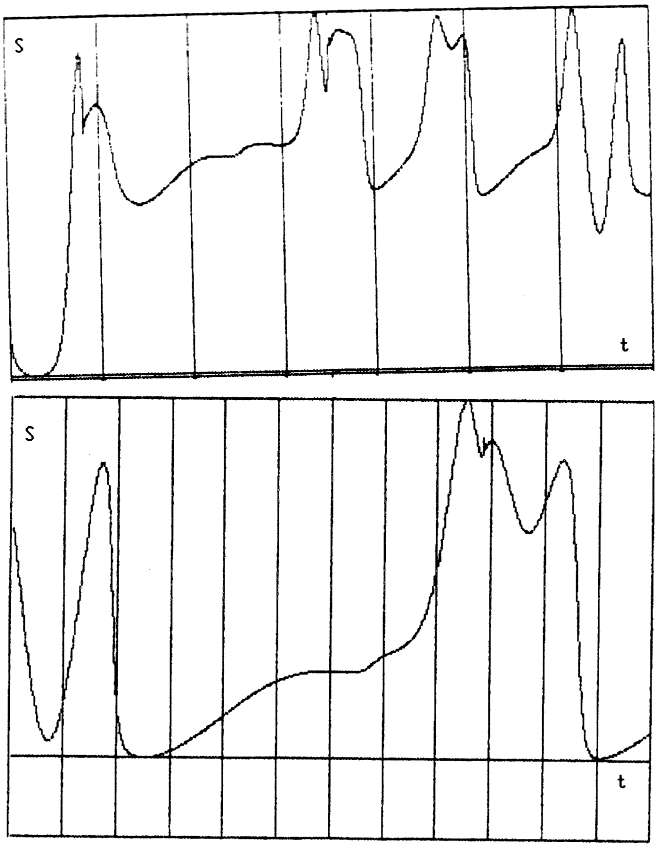


Figure 14.13: Global evolution of entropy

(Logos), the Mudra (Gesture). The deepest latent level of reality - the bifurcation diagram - remains unchangeable. The hidden structure of phase space is transformed by bifurcations. The phenomenal universe is transmuted by time. The distinguishing wisdom (Pratyavekshana-gnana) perceives the emptiness and unreality of phenomena and breaks up the cosmic illusion (Maya).

As a phenomenon, matter is unreal; but as a manifestation of the wisdom-nature of the Tathagata, matter is not different in essence from spirit (Ku-kai [Hea 81a], p.124).

14.9 Creative Chaos

> Now for the first time I perceive that all living beings have the original Buddha-nature — wherefore Birth and Death and Nirvana have become for me as a dream of the night that is gone.
>
> *Engaku-Kyo [Hea 81a], p.121*

Logistic approximation leads to the following modification of the equation that describes evolution of C_0

$$\frac{dC_0}{dt} = \xi(Ax_1 - C\sum x_i)C_0 \tag{14.4}$$

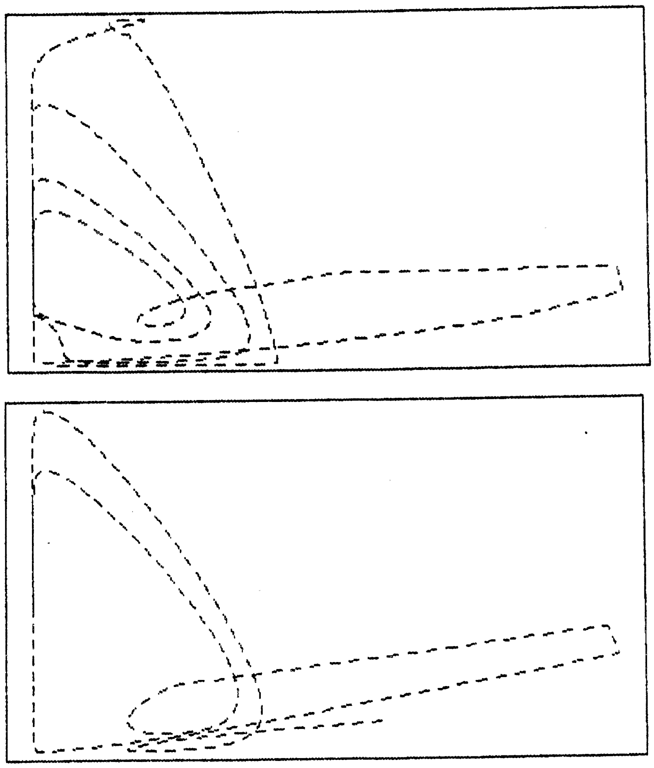


Figure 14.14: Strange attractor, (x_1, x_2) - plane.

Some scenarios of evolution are represented in Figure 14.15 – Figure 14.18. It is sure that Figure 14.15 is similar to Leo Gumilyoff's diagram [Gu 90], p.46. A limited disorder allows the system to prolong the agony of the economy. Chaotic perestroika is followed by a great crisis (see Figure 14.15, Figure 14.16) that could create a base for future development (Figure 14.16). Using energy (Shakti) of chaotic fluctuations the functional double of the system displays itself by manifestations of latent ability to generate empirical processes (see Figure 14.16) and maintain an illusion of a self-sufficient phenomenal universe.

Taoists consider that chaos is the hidden essence and creator of all things [Hal 78]. The creative power of chaos was symbolically described by Chuang-tzu in his famous legend. According to this story Hundun (Chaos), the ruler of the center of universe had two friends. The first, Shu (Rapid) was the potentate of South Sea. The second, Hu (Sudden) was the monarch of North Sea. They often took part in Chaos's mischievous escapades and wild adventures that were very joyful and light-hearted. One day grateful Shu and Hu resolved to repay Chaos's kindness. Chaos was formless and had not the seven holes like ordinary man. Considerate comrades decided to drill Chaos. But unfortunate Chaos died from perforating too many holes. After his death the phenomenal universe appeared.

In a similar way Gorbachov initiated "acceleration and perestroika" of chaotic socialist society, but the Soviet Empire died from organizing [CCK91]. It seems that the same forces are able to disintegrate the USA.

Figure 14.15: Perestroika

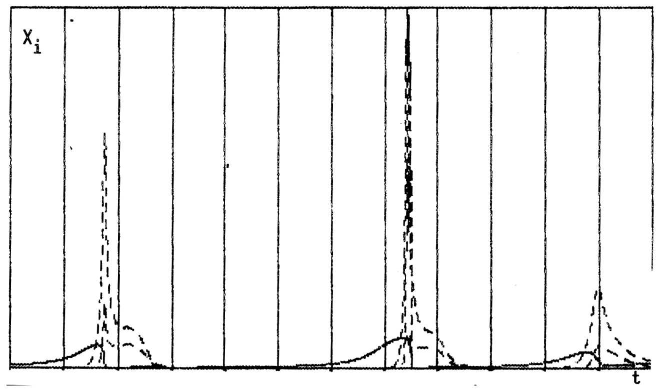


Figure 14.16: Three stages of evolution

14.10 Pseudo Reversibility of Historical Time

> We model the “genotype” to organize before its “phenotype”, or the “plan” to be conceived of before it is realized.
>
> *Klaus G. Troitzsch [Tro 91]*

Taking into account the multiplicity of interacting factors and interrelations between numerous subpopulations one could obtain the same effects as in the case of the nonlinear approximation of production functions that includes at least the fourth power of variables. This fact allows us to explain catastrophic inversions and extinctions of subpopulations that took place in Troitzsch's computer experiments. Thus the catastrophic inversion is the same thing as Troitzsch's extinction effect [Tro 91], i.e. nonlinear approximations and multi population models are dualistic descriptions of socio-economic systems. Synthesis of both dual descriptions assembles a magical world where we could be able to perceive illusions of true reality and the reality of phantoms [Cap 77]. Simulation techniques deal with the vicarious existence of empirical moments of phenomenal realizations. Vision of a hallucinating nature of “the first attention” [Casta 85] can be treated as “intrawordly mysticism” [Ye 83].

Equation 14.3 in Section 14.7 and Equation 14.4 in Section 14.9 allow us to simulate pseudo reversibility of historical time (see Figure 14.19, Figure 14.20, Figure 14.21).

14.11 Conclusions

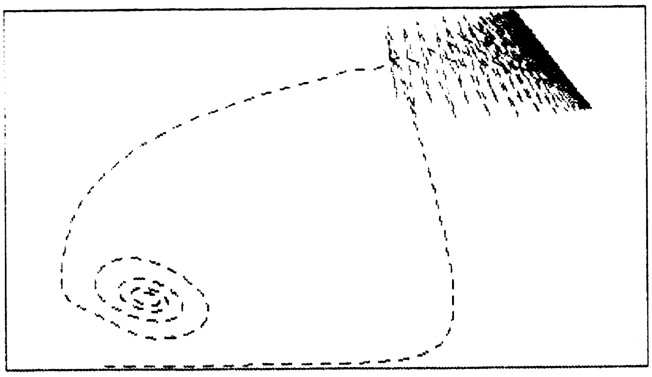


Figure 14.17: Conservative chaos, (x_1, x_2) - plane

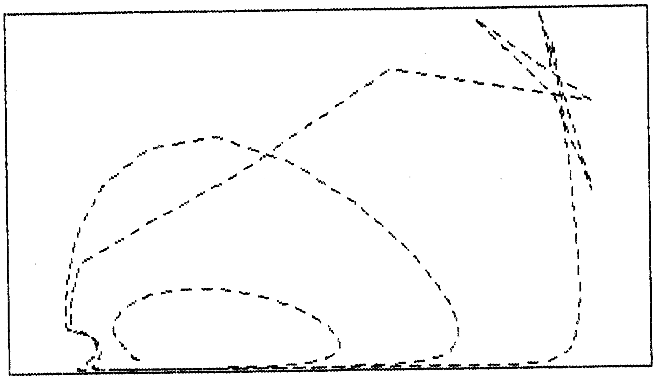


Figure 14.18: Weak chaos, (x_1, x_2) - plane.

The force of the emanations at large makes our assemblage point select certain emanations and cluster them for alignment and perception. That's the command of the Eagle, but all the meaning that we give to what we perceive is our command, our gift of magic.

Don Juan Matus [Casta 85], p.133

Hypercyclic model of self-organized evolution was proposed by Manfred Eigen and Peter Schuster [ES 79] in 1979. Synthesis of Eigen's hypercycle and René Thom's Catastrophe Theory links micro and macro levels of the complex developing system. The wealth of functional abilities of the model seems to promise a wide-spread range of its applications. The model can be considered as a good step to create a new synergetic conception to solve René Thom's "sacramental" problem of modeling system self-diversification and formation of functional structures [Tho 74].

Modified Eigen's chains are utilized to simulate a hidden catastrophic mechanism at a micro level that produces macro cataclysms [Che 91b]. Latent properties of micro processes display themselves in such manifestations as tunnel junction through a macro potential barrier that can be described in terms of micro parameters [Che 91b].

Analysis of critical regimes and singularities of parametric space is capable of re-

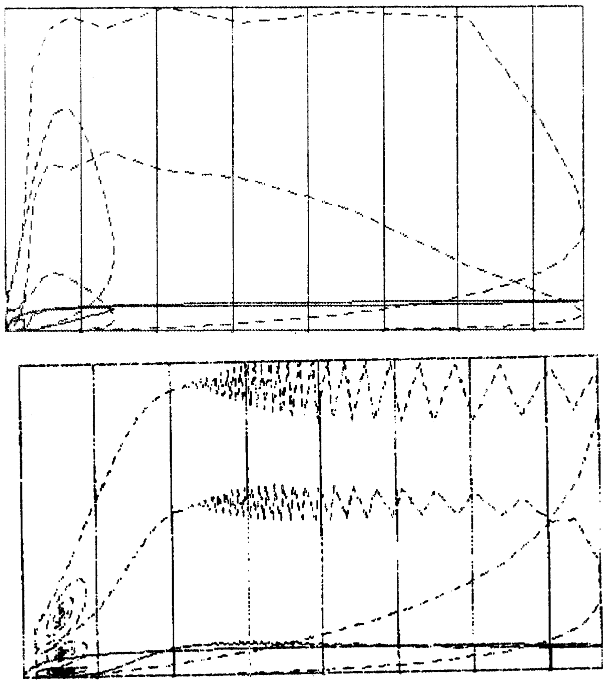


Figure 14.19: Historical vortices in (x_i, C_0) – projections (1)

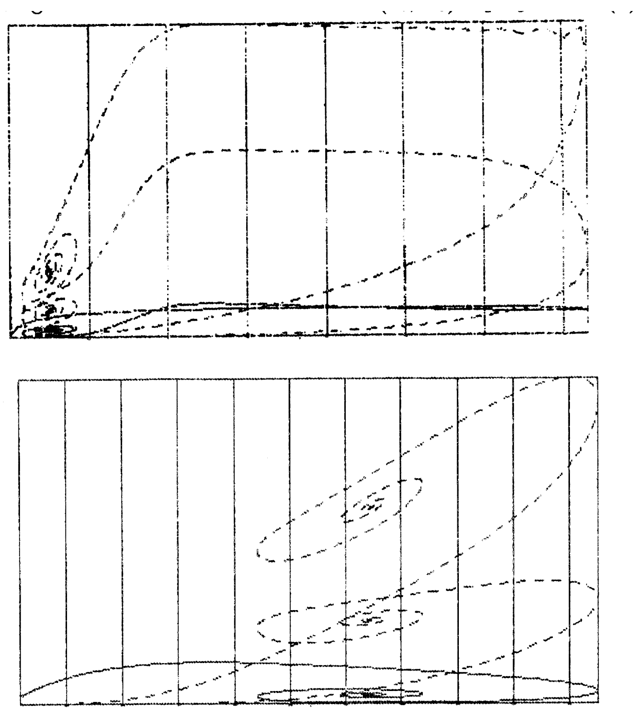
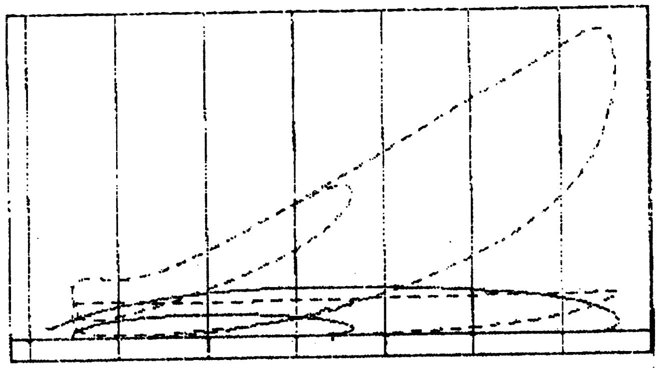


Figure 14.20: Historical vortices in (x_i, C_0) – projections (2)

vealing "acupuncture points" where small local perturbations can provoke great large scale metamorphoses of the system.

An "acupuncture map" appears to be an invariant characteristic of the system that can be used to avoid ecological catastrophes and apocalyptical environmental pollution



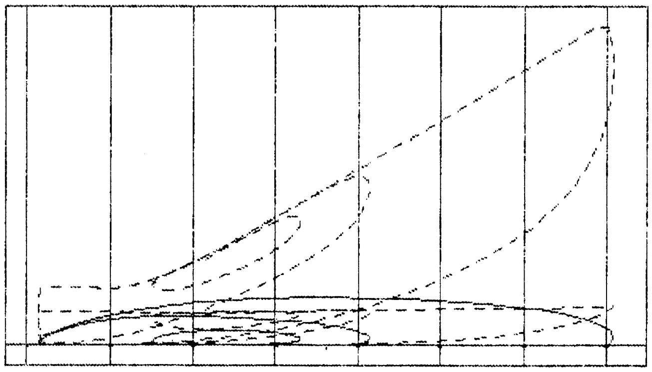


Figure 14.21: Historical vortices in (x_i, C_0) – projections (3)

as well as social cataclysms [Che 91b].

Pseudo stochastic behaviour is "predetermined" by a specific strange attractor that causes both development and decay of the system [CC 91].

A spectrum of quasi stationary solutions may be interpreted as a set of possible forms of morphogenesis. The unrealized functional structures are still within the system's reach but remain dormant, unknown to observers for the duration of some periods of evolution, i.e. the unembodied morphe belongs to the domain of Morpheus, the god of dreams.

The hierarchical model was used in computer experiments to imitate the decay of Soviet Union. Numerical calculations and analytical investigations resulted in a conditional forecast that has been corroborated by subsequent events [Che 91a].

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Chapter 15

Mikhail V. Kuz'min: Hypercyclic Models of Large-Scale System: Growth, Instability, Cyclicity

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Abstract

The paper[1] presents a few minimal models of growing socio-economic system. The models are based on a logistic multichain network that is constructed in Eigen's Hypercycle with the "constant population" constraint. The evolutionary growth is considered in the course of an integrative factor of the system. Structure reconstruction occurs in the system during its growth and inherent time appears to be an "agent of wholeness". Appearance or disappearance of new kinds of production aspects or technologies are connected with the "size" of the system and the level of "high tech". Aspects can connect themselves in couples competitive for a common resource (Model A). It is shown that these models can display long nonperiodical waves that are very much in Kondratiev's way. Nonlinear logic of investment in production aspects or economy at all is illustrated by simulations.

15.1 Introduction

> Proceeding from the similar, one judges about the different, proceeding from the solitary, one judges about the plural; the beginning is an end and the end is a beginning, this is just like a circle having no beginning, no end. Should one give it up, the skies perish.
>
> *Shun-tse (3rd century B.C.)*

The scientific mentality outlived the paradigm of classic science that the world is

[1]The author thankfully acknowledges the financial support of this work by the Ukrainian State Foundation for Fundamental Investigations

arranged simply.

Now we can see that nonequilibrium and irreversible complex systems are not so seldom in phenomenal world while stable and reversible ones are exclusions. Essentially remoteness from equilibrium reveals nonlinear properties of system makro-variables and nonlinearity forms complex space of solutions. To study unexpected bifurcations, system transitions under small variation of control parameters we need to possess "nonlinear" way of thinking.

Nonlinear seeing and moreover foreseeing of dynamic system processes is extremely useful in investigation of phenomenal reality.

The second ability our mind has to possess for an "active inquiry of nature" (I. Prigogine) is the "modelling way of thinking" that allows to recognize the particular class of models which contains the model of the observed phenomena. Such an approach is similar to the typological methodology of René Thom's catastrophe theory.

The third is integrative thinking that implies the skill to realize prolonged periods of system evolution. The phenomenal existence of this form is mutually connected with its capacity to support its own dynamic stability. Losses in the decreasing phase of the system under control could be compensated at the increasing stage. The total result can be satisfactory, and we do not destroy system stability.

Indeed the most complex systems demonstrate oscillatory dynamics. This is a form of existence of such systems which is characterized by resource accumulation in the decreasing phase and powerful metabolism in the increasing one.

The fourth important point to realize the complex nature of evolutionary processes is the necessity to study future as we study past. John Naisbitt emphasized that the East (Japan) and the New Europe (Germany) advanced in development so fast because of their capability to "look back on future" [Nai 90].

In this process, process bearing new mind, modelling plays the crucial role. There is a strategy of modelling that includes not only representation of the object and compression of information about it, but also conservation of some essential characteristics of the object, namely its uniqueness, complexity, wholeness, enclosure into a larger Whole as well.

In the case of such modelling the model is not replica, not tautology and not paraphrase. It excels the original in some extent. The model is an interpretation of originality and as the interpretation is language, a logical expression of notions about the original including empathical ones. Creating a model that reproduces ideas about the object wholeness on a rational level is true success.

The models of synergetics reveal the mentioned potencies. The language of synergetic models is tremendously interdisciplinary. In this paper well-known models such as the "logistic model" and Eigen's model of hypercycle are applied to socio-economic systems for describing evolutionary changes during the process of growth. We use "minimal models", i.e. models that are logically and structurally simple but not to such an extent that they destroy the body of presumptions and the complex character of behavior. They are general, but the general point of view gives us freedom to fill up the model with important details in specific directions.

15.2 Modelling of Evolution

> Without old chronicles, without old reference-points, markets, trees or mountains, where history had been settling, the past is curled up.
>
> *Margaret Mead, Culture and Commitment.*

In classic science the paradigm of simplicity had been living a long time jointly with the static paradigm. They aspired the fatal human tendency to order and stability. Classic mechanics could be considered as embodiment of such an aspiration. It was based on a deterministic postulate of precisely known initial conditions. Quantum mechanics refused the absolutely predetermined initial information about the system but constituted the determinism of an amplitude of probability. The general root of static inclination of different physical theories appears to be homogeneous uniform time — time as parameter. Classical and quantum physics describe empirical reality as a predetermined, reversible, static world.

A new evolutionary paradigm was elaborated by synergetics that treats evolution of complex self-organized systems as increasing in complexity, asymmetry, metabolism of processes that are accompanied by diversification and hierarchical reconstruction. The misleading illusion of absolute time was overcome, and true multidimensional historical time was realized as a universal operator, which means that complex systems are described by a spectrum of bifurcations as turning points where the selection of possible stable evolutionary trajectories is performed. Deterministic laws conserve and transfer innovative information from one bifurcation point to another. In other words, "whatever is born or done this moment of time, has the qualities of this moment of time" [Jun 31]. Thus an evolutionary process could be treated in terms of discrete and continuous, determined and stochastic, order and chaos descriptions.

The consideration of nonlinear models of complex system evolution such as Eigen's hypercycle provides a possibility to perceive the conception of the inherent time of a system as an historical indicator and individual agent as "integrative wholeness" (C. Jung) of the object.

Inherent time is not a point "now" or a parameter. It is rather the "age" of the system. The inherent time conception and the hierarchical structure of a complex system lead to a hierarchical time representation. A complex world of coherent processes on different levels weaves a pattern of macro-time that we experience as Time. Surviving of the processes that are timed with lower and higher hierarchical levels is the crucial sense of coherence, harmony of world, evolution.

The problem of historical time description for evolutionary nonlinear models will be considered below.

15.3 Logistic Way of Evolution

> Our world is like dog's tail turned up; during hundreds of years people tried to straighten it but it bent itself as soon as was let off.

Swami Vivekananda, Karma-yoga.

The logistic model is a successful and effective model that finds its usefulness in many branches of science. Its role in synergetics is similar to the role of the oscillator model in physics. It describes autocatalytical growth of a population to a limit defined by the rate of renewing of resources ("ecological niche"). In any moment of time "the population" grows with a rate that is proportional to the quantity of population and the potencies of environment:

$$\frac{dx}{dt} = rx(N - x)$$

A large number of various "populations" as multiparticle systems demonstrate such law of growth. We can find it in the study of spreading out rumours, illnesses, innovations, growth of cancer cells, birth-and-death processes of elementary particles.

N.D. Kondratiev, in his recently found paper where a model of economic dynamics is shortly represented [Kon 89], noted the logistic character of behavior of economic indicators and emphasized that it is important for understanding cyclic dynamics in economy.

Following Igor V. Chernenko [Che 89] let us now consider a model of a socio-economic system as a modified Eigen hypercycle. The modification means that growth functions are taken as logistic ones.

There are three production aspects in this "minimal model". They are described by the variables x_i. We will interpret these kinds of production as follows:

x_1 agrarian production, or labor forces in this branch;

x_2 industrial production, or labor forces in this branch;

x_3 information production, or labor forces in this branch.

We connect them in the way Figure 15.2 shows. (We make use of diagram description the elements of which are explained on Figure 15.1). The reason of such interrelations of aspects is clear: agrarian production stimulates industrial production, industrial production stimulates agrarian (using tools and machinery) and information productions, the latter stimulates industrial as well as agrarian productions.

Let us write down the equations of the model.

$$\frac{dx_i}{dt} = \Gamma_i(x_1, \ldots, x_n) - \phi_i(x_1, \ldots, x_n) \tag{15.1}$$

where

Γ_i are growth functions (Allen's production functions), and

ϕ_i are dilution fluxes,

$i = 1, 2, 3;\ n = 3.$

Taking into account the logistic character of growth and the specific interrelations between the aspects (see Figure 15.2) we write for Γ_i:

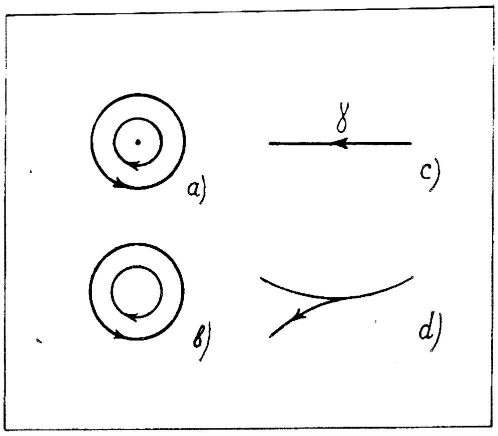

Figure 15.1: Diagram elements for logistically modified hypercycle: a) the logistic law of interaction with resource; b) the logistic law of interaction without resource; c) supporting interaction with factor γ; d) dilution fluxes outwards $\sum \Gamma_j > 0$

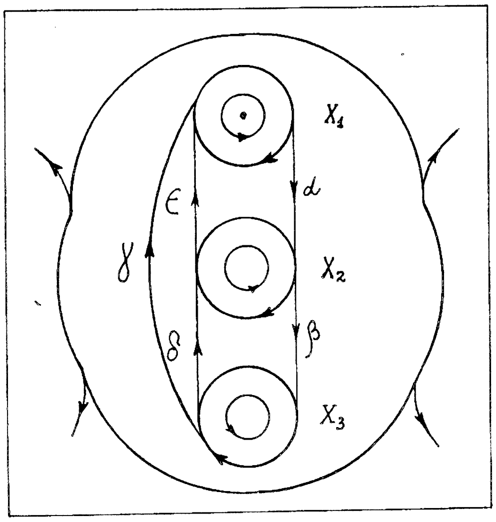

Figure 15.2: The diagram of the model A of three production aspects: x_1 : agrarian production, x_2 : industrial production, x_3 : information production

$$\begin{aligned}\Gamma_1(\mathbf{x}) &= x_1(N_1 + \gamma x_3 + \epsilon x_2 - x_1)\\ \Gamma_2(\mathbf{x}) &= x_2(\delta x_3 + \alpha x_1 - x_2)\\ \Gamma_3(\mathbf{x}) &= x_3(\beta x_2 - x_3)\end{aligned} \tag{15.2}$$

The dilution fluxes are taken in the description of "constant population" (CP), i.e. presuming that flux ϕ_i depends upon the contribution of aspect x_i to the total product C_0 and the sum of Γ_j.

$$\phi_i(\mathbf{x}) = \frac{x_i}{C_0} \sum \Gamma_j(\mathbf{x}) \tag{15.3}$$

where $C_0 = \sum\ x_j$.

Under CP constraints $C_0 = const$.

We define the value M as the sum of production functions and call it the metabolism of processes in the system:

$$M = \sum\ \Gamma_j(\mathbf{x}) \qquad (15.4)$$

15.4 Models of Growth

The Man was grown up from Lord's seeds that could yield the Gods.

Meister Eckhart, Collection of preaches.

Usually system evolution is connected with its growth which can be characterized by increasing integrative indicators. Nature has several scenarios of growth.

The first type means space expansion of system, e.g. crystallization when outer components are used or reproduction of individuals in a population when resources of the external area are consumed. The second type is related with space restriction that cause replication of system parts and metabolic acceleration, e.g. the growth of coacervates[Rud 69]. Nature, by means of series of dividing, finds a possibility of expansion of increasing dimension in space (fractal one). We will call this type "Cantor's growth". The third corresponds to the case of a high degree of freedom given by the environment and rich functional potentialities of the system. The growth realizes as reproduction with structural diversification and hierarchical development. An example of such way of growth is represented by the model of prebiotic Eigen's evolution[ES 79].

Growing systems seem to be described by a "system scale" as a control or bifurcation parameter which represents geometrical size as well as size of population. It could be another integrative factor such as system energy, "Gross National Product", etc.

Achieving its definite size, the system bifurcationally transforms itself, and a new system state arises. The law of growth is predetermined by its innate potential ability, therefore the system size could be treated as inherent time of system that parameterizes a cascade of bifurcations.

The model of pattern formation during the growth of an embryo (see e.g. [Mur 77]) as developing Turing's idea of morphogenesis (1952) represents a good example of such bifurcation growth. The final result of the color pattern of mammal skin depends on size and form of the embryo in the moment when the chemical reactions switch on. The chain of diffusion instabilities forms a pre-pattern structure of substrate concentrations that yield spots and stripes in the end. "From an evolutionary point of view the common simple mechanism producing a large variety of mammal skin coloration is more effective than genetic one that has to be complex" [Mur 77].

We can evaluate the similar approach to the growth of Universe. On the very first second of Universe's life the speed of growth is extremely fast and diffusion instabilities can not occur. However in that moment smallest violation of PC-conservation law leads to death of antibarions and to the survive of barions based on logistic way of interaction [Ten 85]. At the time of slower growth (approximately a week after Big Bang) the clasterization of substance is possible for Embrion of Universe.

Let us turn back to our abstract model of evolution of products. The growth of the system can be presented by increasing of integrative factor C_0. In a real socio-economic system there are a large number of causes for the growth namely the growth of population, the expansion of new territory, the obtaining of external financial assistance or credits. In any case the causes are *external in reference to our model* however can be *internal according to the system.* Therefore we will change C_0 artificially keeping in our mind endogenous character of the growth.

According to Equations 15.1 and 15.3:

$$\frac{dC(t)}{dt} = (1 - \frac{C(t)}{C_0}) \sum \Gamma_j(\mathbf{x}) \tag{15.5}$$

where $C(t) = \sum x_i(t)$

It is clear that until $\sum \Gamma_j(\mathbf{x}) > 0$ the point $C = C_0$ is stable stationary state.

When the system grows, i.e. $C(t) = \sum x_i(t)$ is increased, the system loses a former stationary state and goes to a new one.

Such growth is the pursuit for a new stable stationary state. What is why the speed of growth must be slow that in any moment of time we can consider system in quasistationary state. In real system it may be not the case and than the evolutionary path of system in space of stationary states will be distinguished from possible stationary states and with different supply of stability.

Let us find out the stable stationary states depending on different C_0.

The stationary states of the system (Equations 15.1, 15.2, 15.3) are:

Zone 1: from $C_0^{*(1)} = 0$ to $C_0^{*(2)} = \frac{N_1}{1+\alpha}$

Case 1:

$$\begin{aligned} x_1 &= C_0 \\ x_2 &= 0 \\ x_3 &= 0 \end{aligned} \tag{15.6}$$

Zone 2: from $C_0^{*(2)} = \frac{N_1}{1+\alpha}$ to $C_0^{*(3)} = \frac{N1(1+\alpha+\beta)}{1+\alpha\beta+\beta-\alpha\epsilon}$

Case 2:

$$\begin{aligned} x_1 &= \frac{C_0(1+\epsilon)+N_1}{2+\alpha+\epsilon} \\ x_2 &= \frac{C_0(1+\alpha)-N_1}{2+\alpha+\epsilon} \\ x_3 &= 0 \end{aligned} \tag{15.7}$$

Zone 3: from $C_0^{*(3)} = \frac{N1(1+\alpha+\beta)}{1+\alpha\beta+\beta-\alpha\epsilon}$

Case 3:

$$
\begin{aligned}
x_1 &= \frac{C_0(1+\epsilon+\delta\epsilon+\gamma-\beta\delta)+N_1(2+\beta+\delta)}{Z} \\
x_2 &= \frac{C_0(1+\alpha+\gamma\alpha+\delta)-N_1(1-\alpha+\delta)}{Z} \\
x_3 &= \frac{C_0(1+\alpha\beta+\beta-\alpha\epsilon)-N_1(1+\alpha+\beta)}{Z}
\end{aligned} \tag{15.8}
$$

here $Z = 3+\alpha+\beta+\gamma+\delta+\epsilon+\alpha\beta+\beta\gamma+\gamma\alpha+\delta\epsilon-\alpha\epsilon-\beta\delta$.

Z represents "convolution factor" of the cognitive level of the system.

$C_0^{*(1)}, C_0^{*(2)}, C_0^{*(3)}$ are critical points of C_0, where structure stability loses: new kind of production can appear only after specific value of C_0 as the "size" of the system.

Some conclusions follow from solutions for stationary states:

- In specific case the third aspect could not be appeared for any large C_0. It is realized when

$$\alpha\epsilon \geq 1+\beta+\alpha\beta \tag{15.9}$$

  while $Z > 0$.
  The meaning of this inequality is that for large values of α and ϵ and/or small value of β the couple of aspects x_1 and x_2 forms the independent complex competitive to x_3 aspects:
  if technology of a society is not "high tech", i.e. not sufficient to produce information base, or industry works to obtain more food (produces agrarian machines mostly) than "third wave" does not rise up.

- Sometimes first aspect can disappear in Zone 3. This is the case when the slope of the line $x_1 = q_1C_0 + q_2$ is negative, i.e. $q_1 < 0$ or:

$$\beta\delta > 1+\epsilon+\delta\epsilon+\gamma \tag{15.10}$$

  and $Z > 0$.
  We see it is realized when the couple $(x_2 - x_3)$ is isolated from x_1 by large values of β (mostly) and δ.
  Therefore we have one additional case, when Equation 15.10 is satisfied and

$$C_0 \geq \frac{N_1(2+\beta+\delta)}{\mid 1+\gamma+\epsilon+\delta\epsilon-\delta\beta \mid}$$

  .

Case 4:

$$x_1 = 0$$

$$
\begin{aligned}
x_2 &= \frac{C_0(1+\delta)}{2+\beta+\delta} \\
x_3 &= \frac{C_0(1+\beta)}{2+\beta+\delta}
\end{aligned} \tag{15.11}
$$

- There are the limits to growth connected with the rate of renewing of resource N_1. The limit is reached when $\sum \Gamma_j = 0$, i.e. full exploitation of the "ecological niche".

Case 5:

$$
\begin{aligned}
x_1^{\max} &= \frac{N_1(1-\beta\delta)}{1-\alpha\beta\gamma-\beta\delta-\alpha\epsilon} \\
x_2^{\max} &= \frac{\alpha N_1}{1-\alpha\beta\gamma-\beta\delta-\alpha\epsilon} \\
x_3^{\max} &= \frac{\alpha\beta N_1}{1-\alpha\beta\gamma-\beta\delta-\alpha\epsilon} \qquad (15.12) \\
C_0^{\max} &= \frac{N_1(1+\alpha+\alpha\beta-\beta\delta)}{1-\alpha\beta\gamma-\beta\delta-\alpha\epsilon} \qquad (15.13)
\end{aligned}
$$

We can state the fact: x_1 can be reborn after its death in Zone 3.

15.5 Mapping of Evolution

He had bought a large map representing the sea
Without the least vestige of land
And the crew were much pleased when they found it to be
A map they could all understand.

Lewis Carrol, The Hunting of the Snark.

We postulated the growth of C_0 in the course of time t. The law of C_0 growth depends on various factors: as internal (natural growth of quantity of people) as well external (migration, territorial expansion, credits). However, we saw, the moment of bifurcations and structure reconstructions depend upon technological level of society (parameters $\alpha, \beta, \gamma, \ldots$). In this sense C_0 represents inherent time of the system.

Therefore it is important to have the map of stationary states of the system stepping in the course of C_0. It is understood from stationary solutions of latest section that functional character of

$$x_i^* = f(C_0),$$

where x_i^* is stationary state.

The function will be peacewise linear (see Figure 15.3).

However during real growth or computer simulation of this process the system will run from a stationary state to another stationary state while $C(t)$ is changing. Thus

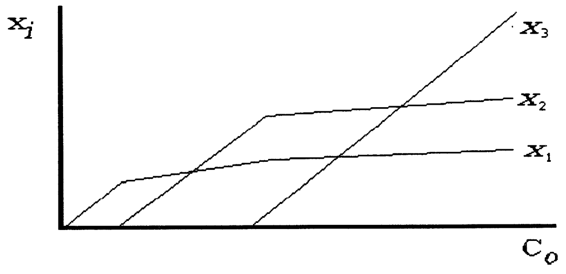

Figure 15.3: The theoretical map of stationary states of the system

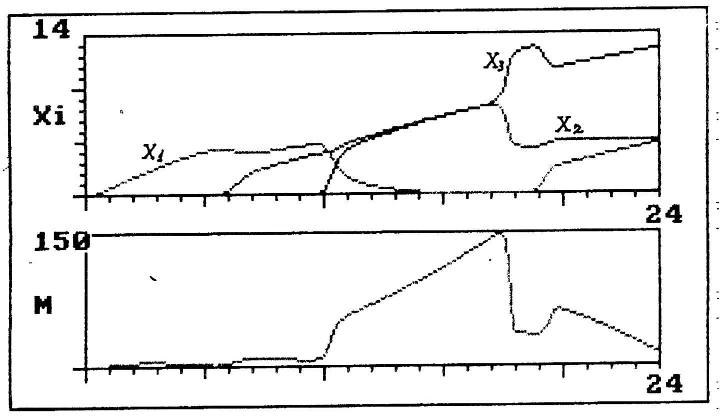

Figure 15.4: The map of evolution of the system (Eq. 15.1–15.3) stepping in the course of growing C_0. Simulation result. Here the x_i are stationary states of production aspects; M is the metabolism of the system (see Equation 15.4)

we obtain a real map of evolution of the system during its growth ($C(t)$ is increases) as sequence of stationary states with continuously changing integrative value C_0.

It is important that intrinsic processes finding of equilibrium go faster than the rate of changing $C(t)$. Then the real map will be similar in some extent to theoretical one.

An example of such map is shown on Figure 15.4. We observe on it all above mentioned Zones and Cases. x_1 was born, developed, stagnated, dead and was reborn again like famous bird of Phoenix.

$C(t)$ was changed linearly as

$C(t) = k\,t$,

where k is small (it is easy to regulate by small step of simulation procedure).

Generally speaking for quasistationary growing system we need to modify the right side of Equation 15.1 as follows:

$$\frac{dx_i}{dt} = \Gamma_i(\mathbf{x}) - \frac{x_i}{C(t)}\,(\sum \Gamma_j - \frac{d\,C(t)}{dt}) \tag{15.14}$$

It is simplified when we pass to concentration variables and new time parameter.

$$\frac{d\,\zeta_i}{d\tau} = \Gamma_i(N_i^{(*)}, \zeta) - \zeta_i \sum \Gamma_j \tag{15.15}$$

here $\zeta_i = x_i/C(t)$, $d\tau = C(t)\,dt$, $N_i^{(*)} = N_i/C(t)$, $\sum \zeta_j = 1$

Let the law of $C(t)$ growth be exponential (the growth of population):

$$C(t) = c_0 \exp \lambda t$$

with c_0 the initial value of $C(t)$ and λ the rate of population growth.

Then the dynamic equation will be the same as in Equation 15.15 but with

$$N_i^{(*)} = \frac{N_i/c_0}{1 + \tau\lambda/c_0}$$

The evolution of concentrations is shown on Figure 15.5.

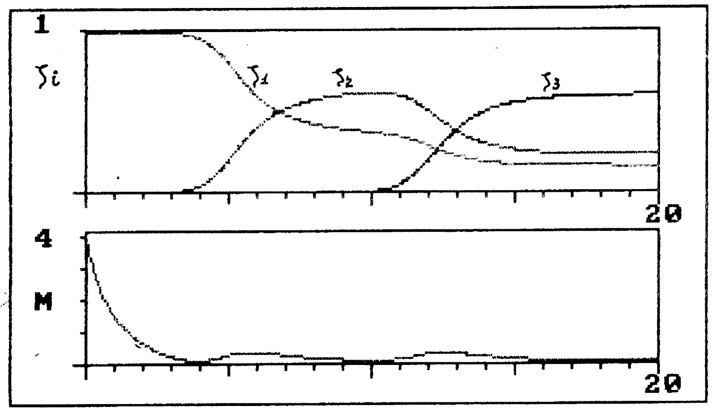

Figure 15.5: Evolution mapping of concentrations: $N_1 = 5, \alpha = 2, \beta = 3, \gamma = \delta = \epsilon = 0$

Now let us consider one specific simplified model of our original model A. Its interaction diagram is presented on Figure 15.6. It takes place when δ and ϵ equal to 0 in original model, i.e the feedback of x_2 and x_3 aspects is vanish in accordance to supporting aspects, x_1 and x_2, respectively. Obviously in this case we can interpret the meanings of x_2 and x_3 aspects in another way. For instance x_3 could be treated as ecological activity directed upon rising "ecological niche" N_1 up.

Below we present the way of evolutionary growth of this model B (as particular case of solutions placed in Equations 15.6, 15.7, and 15.8) and corresponding map of quasistationary evolution is shown on Figure 15.7.

(Bifurcation points are stressed by •).

$$\bullet C_0^{*(1)} = 0$$

$$\begin{aligned} x_1^{(1)} &= C_0 \\ x_2^{(1)} &= 0 \\ x_3^{(1)} &= 0 \end{aligned}$$

$$\bullet C_0^{*(2)} = \frac{N_1}{1+\alpha}$$

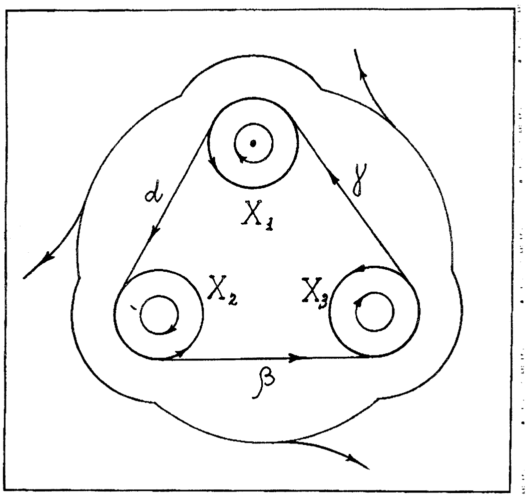

Figure 15.6: The diagram of the model B of three production aspects: x_1 : agrarian production, x_2 : industrial production, x_3 : ecological activities

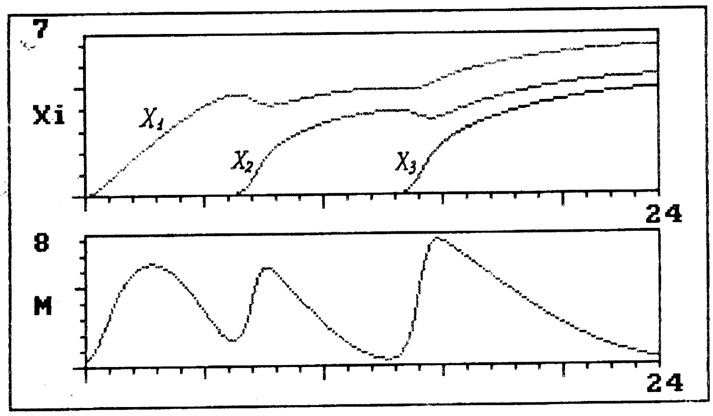

Figure 15.7: The evolutionary mapping of growth for model B: $N_1 = 5, \alpha = 0.4, \beta = 0.8, \gamma = 0.9$

$$
\begin{aligned}
x_1^{(2)} &= \frac{C_0 + N_1}{2 + \alpha} \\
x_2^{(2)} &= \frac{C_0(1 + \alpha) - N_1}{2 + \alpha} \\
x_3^{(2)} &= 0
\end{aligned}
$$

$$\bullet C_0^{*(3)} = N_1 \frac{1 + \alpha + \beta}{1 + \beta + \alpha\beta}$$

$$x_1^{(3)} = \frac{C_0 + N_1(2 + \beta)}{z}$$

$$\begin{aligned} x_2^{(3)} &= \frac{(\alpha+1)C_0 - N_1(1-\alpha)}{z} \\ x_3^{(3)} &= \frac{(\alpha\beta+\beta+1)C_0 - N_1(1+\alpha+\beta)}{z} \\ z &= 3+\alpha+\beta+\gamma+\alpha\beta+\beta\gamma+\gamma\alpha \end{aligned}$$

$$\bullet C_0^{\max} = \frac{N_1(1+\alpha+\alpha\beta)}{1-\alpha\beta\gamma}$$

$$\begin{aligned} x_1^{\max} &= N_1/(1-\alpha\beta\gamma) \\ x_2^{\max} &= \alpha N_1/(1-\alpha\beta\gamma) \\ x_3^{\max} &= \alpha\beta N_1/(1-\alpha\beta\gamma) \end{aligned} \quad (15.16)$$

Considering value $C_0^{\max}$ we notice that maximum of C_0 is reached under following condition for this simple model B:

$$\alpha\beta\gamma < 1 \quad (15.17)$$

We can interpret this condition as the condition of "technological noneffectiveness" or "low technological level" of society.

We can observe this case of the limit of growth on Figure 15.7 ($N_1 = 5$, $\alpha = 0.4$, $\beta = 0.8$, $\gamma = 0.9$).

One can note that $x_1^{\max} > N_1$. It is result of hypercyclic support of aspects. x_1 gives birth to $x_2, while x_2$ gives birth to $x_3, and x_3$ stimulates x_1 by means of intensive ecological actions.

In a case when $\gamma = 0$, i.e. in the case of modified Eigen's chain, no hypercycle, C_0 reaches its maximum without fail:

$$C_0^{\max} = N_1(1+\alpha+\alpha\beta)$$

$$\begin{aligned} x_1^{\max} &= N_1 \\ x_2^{\max} &= \alpha N_1 \\ x_3^{\max} &= \alpha\beta N_1 \end{aligned}$$

We see that x_1 can not be greater than N_1 now.

In the case of logistic multi-chain interaction there is guarantee limit of growth and productivity in the system.

15.6 Swings of Hypercyclic Economy

If somebody asks you: what is the sign of your Father ? Reply: Motion and rest.

Bible, Thomas, Apocrypha, 55.

Economic statics and dynamics are only another names for concepts "state of equilibrium" and "economical forces".

Nikolas Kondratiev, [Kon 91]

Looking at Figure 15.5 or Figure 15.7 one can find that metabolism in the system varies its magnitude while system grows. We can not state this changes is periodical in general because M's minima coincide with critical points that depend on parameters of interaction (technological level or kind of usage resources given by corresponding aspects) such as α, β, γ and so on. However character of curves (sharp rising up and slow downing) looks like waves, inherent causes of oscillation (system growth), possible long time between bifurcation points (depend on rate of growth) demand to analyze the problem of waves from Kondratiev's point of view.

Let us find the analytical notion for M. We need to take into account Equations 15.1–15.4, and 15.6–15.8 that obtain general presentation of M as follows:

$$M = \sum \Gamma_j = p_1 C_0^2 + p_2 C_0 \tag{15.18}$$

where p_1 and p_2 are expressed in terms of model parameters and depend on zones.

For example,

- in zone 1: $M = -C_0^2 + N_1 C_0$,
- in zone 2: $M = -C_0^2/(2 + \alpha + \epsilon) + \frac{N_1(1+\alpha)}{2+\alpha+\epsilon} C_0$,
- in zone 3: $M = -C_0^2 \frac{1-\alpha\beta\gamma-\alpha\epsilon-\beta\delta}{z} + \frac{N_1(1+\alpha+\alpha\beta-\beta\delta}{z} C_0$.

and so on.

The depth of M's minimum can be varied by parameters of model. For instance first minimum is as deeper as α smaller.

We see that the character of the curves is parabolic. The same character was also noted between some "copulative economic variables" by N.D. Kondratiev [Kon 89]. Kondratiev also related the quantity of population to them.

N.D. Kondratiev also mentioned that deviation from equilibrium state plays most role in cyclicity of economic variables and considered causes of oscillation are endogenous mechanism.

Our model demonstrates all these factors:

- endogenous mechanism is the growth of system because of arising population and/or territorial expansion (Kondratiev was a witness of it), accumulation of capital and so on.

- during growth the system loses a stable stationary state and seek another one but supply of stability different in different moments of evolution and depend on rate of growth also.
- the level of technology plays one of principal roles as well as "size" of the system. New kind of products, technology can not appear until the system become sufficiently "large" corresponding to technological effectiveness that society possess. Probably what is why the time delay (from 10 to 120 years) exists between an invention and its innovation [Men 79]. The cause is not so psychological as rather functionally endogenous.
- it is very likely that economical long waves are not absolutely periodical but under some conditions our model can demonstrate quasiperiodical ones even in the case of three products or technologies (Figure 15.7).

15.7 Nonlinear Logic of Investments

There is most important that is invisible.

Antoine de Saint-Exupery: The Little Prince.

In socio-economic system as our model represents it there is the problem of envestment into specific aspect of production and into all economy as well.

It is connected with nonlinear interaction of all aspects by means of nonspecific dilution fluxes namely by means of C_0 in the right side of dynamical equation (Eq. 15.1).

Let for simplicity's sake model B reached a stable stationary state and we try to invest into one of the aspects.

Let the value of investment be $0.1C_0$.

Simulation results for logistic multi-chain are shown on Figures 15.8, 15.9, and 15.10 for different invested production aspects.

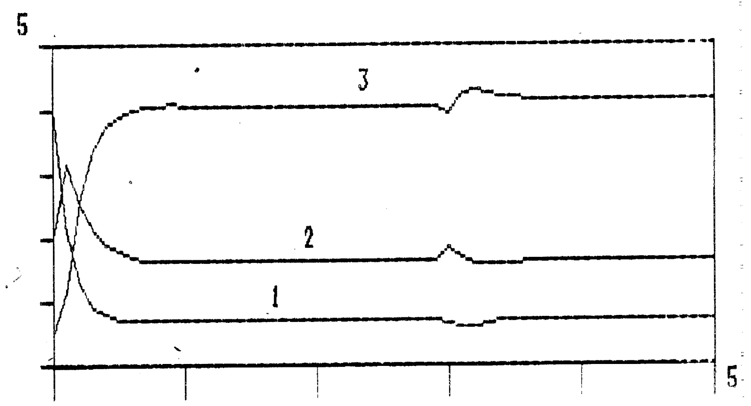

Figure 15.8: Nonlinear logic of investment. Model B($\gamma = 0, C_0 = 6.5$): x_1 was invested

We note that:

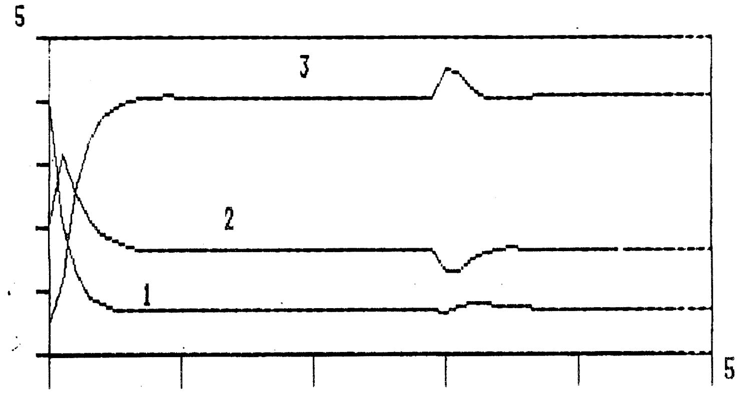

Figure 15.9: Nonlinear logic of investment. Model B($\gamma = 0, C_0 = 6.5$): x_2 was invested

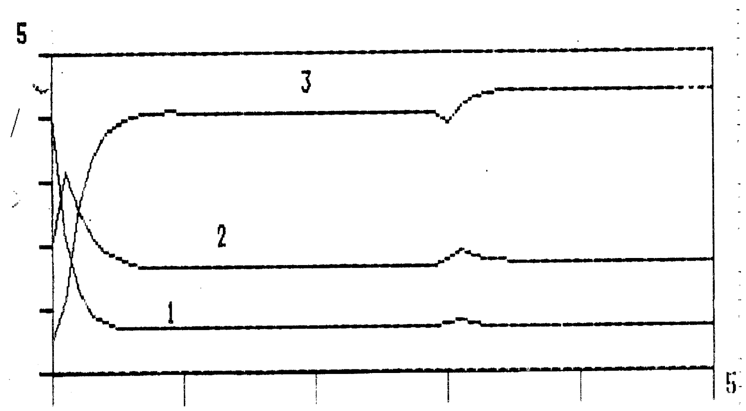

Figure 15.10: Nonlinear logic of investment. Model B($\gamma = 0, C_0 = 6.5$): x_3 was invested

- Figure 15.8: in response to investing x_1 we obtain decreasing of x_1 with forthcoming seting up of the same value. x_2 survives real increasing but only on a while. x_3's behavior is opposite to x_2. It gets investment.

- Figure 15.9: in response to investment of x_2 the sharp decreasing of x_2 is occurred. In the end after sharp process in x_3 all values goes to the initial state.

- Figure 15.10: in response to investment of x_3 all aspects rise up but good deal of investment is taken by x_3 indeed.

Obviously this is only illustration of nonlogical character of investment policy because later will depend on the point on bifurcation map. Thus for model A investments to x_1 in Zone 3 can lead to decreasing x_1 in such extent as investment are powerful.

Investments into the whole of economy, i.e increasing C_0 (external financial assistance or internal monetary issues (inflation)) lead to instabilities and in the result to economic chaos (see Figure 15.11). We saw it in China when Mao was in power and we can understand now period of stagnation in former USSR. Attempts to jump went to instabilities that were dangerous for leadership. Wider discussions of this theme on macrolevel were carried out in [CCK 91] and [RC 90].

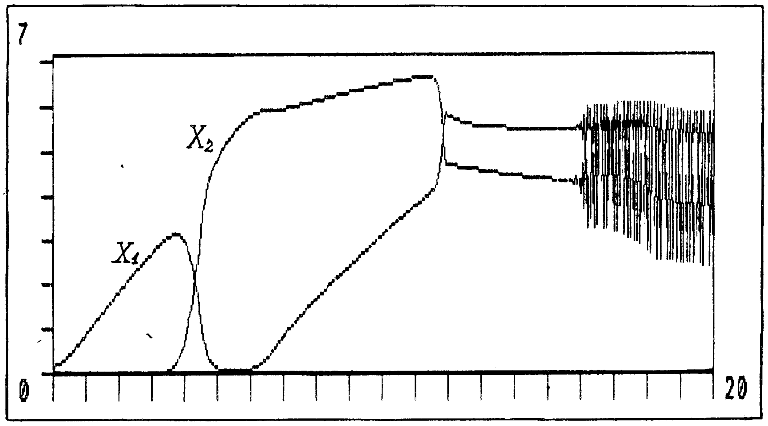

Figure 15.11: Economic chaos as result of economic jumping. C_0 is varied rapidly

15.8 Conclusion. Are there the Limits to Growth?

We have a proverb: "Think globally, act locally".

Dennis Meadows. An Interview to "Chemistry and Life".

Nevertheless we evaluate the minimal model of hypercyclically connected logistic aspects the model occurs rich on solutions and nontrivial evolution during the growth of the system.

C_0, the most important integrative factor in the model, plays the role of a bifurcation parameter and can be treated as the inherent time of the system.

In the course of varying C_0 structure reconstructions occur. A new aspect or technology can be adopted only in very specific moments of system evolution.

Under consideration of model A it became clear that inside the system the competition is for resources. Two aspects are connected in the cooperative couple and can be relatively independent. In the case $x_1 - x_2$ close interactions inside this couple the third can be born. As to x_1 it can be dead if interaction is strong within the $x_2 - x_3$ couple, but it is reborn again.

Metabolism in the system varies and has the character of oscillation with nonconstant period, but this is defined by the "state of technology" in the society.

Some arguments give reason to assume that the model describes long economic waves including their endogenous mechanism by means of the inherent natural growth of the system.

As is known, the most global and most pessimistic model of world development was "Meadows' model" [MMRB 89], which forecasts an unlimited growth of population and capital with forthcoming ecological catastrophe, if mankind would not change its system of values.

Our model is not such a global, but an optimistic one in respect of the limits to growth.

First, the model states that for a system which is organized like a multichain and "low tech" the limits to growth exist.

Second, for hypercyclically organized systems with "high tech" there are no limits to growth. This is the case we called "Cantor's way of growth": expansion of dimensions of space. Computerization and fibre optics are a good example. Moreover ecological activities rise up the "ecological niche".

Third, when we reach the limits to growth, then dilution fluxes can become negative; that means an expansion of the traditional living area in different directions, e.g. higher and lower lithosphere.

Fourth, the human system of values can really be changed as positive feedback changes on negative one as we can see in the logistic way of evolution.

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Chapter 16

Eugene Platon: Synergetic Effects in Interacting Models

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Abstract

Interacting effects in systems of mathematical models and results of numerical experiments are presented. Synergetic behaviour and self-organization in algorithm systems designed according to Bohr's principle and catastrophe theory methodology is discussed.

In attempting to construct an adequate model of a complex system we have inevitably to use Bohr's additional principle: there is a set of describing models; each model enables to investigate one feature of the system and each one has its own mathematical peculiarities.

According to Thom's catastrophe theory [Tho 75] when the internal functioning mechanism of a complex system is unknown we can assume that a potential function exists, which characterizes the system behaviour and local minima of the potential function correlate with equilibrium states of the system. The potential function could be regarded as an external descriptive model. Hence, if the system input is fixed we can observe the system state, which corresponds to a local minimum of the potential function. Using the potential functions formalism for describing the well investigated physical systems was found as a good alternative to internal models. For instance, in classical physics there are the variational principles of Hamilton and Fermat. In many cases the internal description could be obtained from the external, potential one according to Hamilton-Jacoby equations.

The catastrophe theory methodology perfectly works in another kind of systems as well. For instance, this approach was used for solving the traditional load flow problem in electric power networks (EPN) [Pl 89a]. The nonlinear equation of EPN is based on Ohm's and Kirchhoff's current laws. Implicitly it is assumed that local (in other words: internal) specification, obtained from two classical laws and transferred to upper (system) level is enough for understanding the global behaviour of electric power systems. Usually the solution of EPN equation depends on the initial starting point of the numeric algorithm. Thus, it is natural to assume that the EPN classic model is not comprehensive.

One possible way of solving the numeric algorithm stability problem, proposed the first time in [Pl 89a], is using a criterion for selecting the adequate solution as additional, external description of EPN. A concave potential function, the minimum of which corresponds to a real state of EPN, could be used as such criterion. The power losses in EPN are defined as system potential function [Pl 89a]. Similar to classical mechanics and field theory the reaction of EPN to an external influence is regarded as dynamic change of system state to one which minimize the power losses scalar potential function.

The mixed model, in which internal and external description interact is more adequate: the numeric solution obtained from that model is single and does not depend on numeric algorithm starting point. More details are presented in section 16.1.

Another kind of interactive effects was obtained in parallel numeric algorithms. The attempt to use very simple parallel adaptive asynchronously interacting algorithms for solving integer programming problems (IPP) demonstrated that nonlinear interaction effects could be detected even in that kind of systems [KP 84], [Pl 86], [PS 88a], [PS 89]. It was shown that there are particular IPP, for which the time T_p of problem solving on a parallel computer system with p processors is less than $T_p < \frac{T_1}{p}$, where T_1 is the time of solving the same problem on an one processor computer [KP 84], [Pl 86], [PS 89]. The time of IPP solving nonlinearly depended of processors number. In a such case each working in its own processor and generating new better solutions algorithm could be regarded as an open system which transmit information to other algorithms. That additional information generated during the process of problem solving enabled in some cases to reduce nonlinearly the problem solving time [PS 89]. Details in section 16.2.

While in the first example the interaction effect resulted in numeric stability in the second one it concluded in more than processors amount times diminishing of problem solving time.

More interesting interactive effects are presented in [RC 90a], [RC 90b]. It was shown that two interacting internal models are equivalent to the external system description based on potential function.

It is a very representative example of interaction effects, when the system behaviour obtained from an agregated external description is similar to system trajectory received from two interacting internal models. This approach is opposite to catastrophe theory methodology and could be regarded as further development of Bor's principle — not only the global system behaviour mechanism is obtained, but details of internal system functioning are received as well. Models involved in an interacting through parameters modelling system could be based even on different mathematical formalism. The procedure when two interacting models are used for system simulation is similar to obtaining of fractional numbers by dividing (interaction) two integers. For details see section 16.3.

The more detailed studies of hypercycle self-organized models were presented in [ES 79], [Mar 83], [Che 89], [CC 91], [CCK 91], [Che 91b], [Che 91a], [Kuz 91].

Particular attention should be paid to stochastic models in which fluctuation phone automatically ensure stable solutions.

It should be mentioned that parallel computations could substantially diminish the computation complexity of interacting models.

16.1 An Electric Power Network Model and Catastrophe Theory

Abstract

An electrical network model based on using of potential functions and catastrophe theory methodology is discussed. Software for IBM PC XT/AT and numerical experiences are presented. An approach for constructing complex nonlinear models of electrical power systems is proposed.

In this section we describe an Electric Power Network (EPN) model based on using of potential function and Bohr's additional principle.

16.1.1 Theoretical Background

Let us consider the standard electrical load flow problem: find unknown node's voltages in an electrical network when the node's powers are known. According to [GO 81] the load flow equation of an EPN is as follows:

$$g(U) = U_d Y(U_0 - U) = S \tag{16.1}$$

where Y is node's conductivities matrix, U is node's voltages vector, U_d is diagonal matrix (each component of vector U is on diagonal of matrix U_d ; all other elements of U_d are equal to zero), U_0 is fixed voltage at balancing (reference) node (usually, in an $(n+1)$ nodes network the first node is considered as reference one and the voltage U_0 at this node is prescribed (fixed), i.e. unknown voltages at n rest nodes should be calculated), S is node's powers vector. Positive sign of component $s_i \in S$ means that at node i electric power is taken from the network while negative sign means that power is injected to the network.

For the sake of presentation simplicity let us begin with considering direct current network: $U, S, U_0 \in R^n$; $U_d, Y \in R^n \times R^n$; i,i-th diagonal element of matrix Y is equal to sum of line's conductivities connected with node i; i,j-th nondiagonal element of Y is negative conductivity of line connecting nodes i,j; $g(U) \in R^n$ is a vector function.

It is well known [GO 81] that the solution U^* of Equation 16.1 obtained by a numerical algorithm depends on initial starting point $U^{(0)}$ of procedure: $U^* = U^*(U^{(0)})$. One of explanations of that fact consists in: the mapping $g^{-1} : S \Rightarrow U$ is point to set.

From other point of view Equation 16.1 is an internal, local description of EPN based on Ohm's and Kirchhoff's current laws. Implicitly it is assumed that local description transferred to upper (system) level is enough for understanding the global behaviour of electric power system.

According to René Thom's catastrophe theory [Tho 75] when the internal functioning mechanism of a complex system is unknown we can assume that exists a potential function describing the system behaviour and local minimums of this function are equilibrium states of the system. Hence, if the system input is fixed we can observe the system state corresponding to local minimum of the potential function.

Taking into account that the solution U^* of Equation 16.1 depends on initial starting point of numerical algorithm it is naturally to assume that the model of EPN given by Equation 16.1 is not comprehensive. One possible way of solving this problem proposed the first time in [Pl 89a] is using a criterion for selecting the adequate solution as additional, external description of EPN. A concave potential function $f = f(U)$ minimum of which corresponds to real state of EPN could be used as such criterion. In our case the power losses in EPN could be regarded as system potential function [Pl 89a]. For an EPN described by Equation 16.1 the power loss $f(U)$ is given by formula:

$$f(U) = (U - U)'Y(U - U). \tag{16.2}$$

The positive definition of matrix Y ensure existence of single solution of next nonlinear programming problem:

$$f(U) = (U_0 - U)'Y(U_0 - U) \Rightarrow \min \tag{16.3}$$

$$U_d Y(U_0 - U) = S$$

if the Equation 16.1 is solvable.

Without considering the balancing node sum power P in EPN is equal to

$$P = \sum_{i=1}^{n} s_i + f(U). \tag{16.4}$$

If exists a set $\overline{U} = \{U_i^* : i \in [1 : M]\}$ of solutions of Equation 16.1 and taking into account that fluctuations are inevitable in an EPN, we are postulating that the "true" solution coresponds to minimum of P or according to Equation 16.4 to minimum of $f(U)$. As in classic mechanics and field theory we regard the reaction of EPN to an external influence as dynamic change of system state to one which minimize the scalar potential function in Equation 16.2.

Considering the sum power P_s as function of voltages U

$$P_s = P_s(U) = \sum_{i=1}^{n} s_i = U'Y(U_0 - U)$$

it is very easy to see that at point $U = \frac{U_0}{2}$ the function P_s achieves its maximum (since the matrix Y is positive defined).

Let us regard in more detail the Tailor's presentation of vector function $g = g(U)$ near an solution U^* of Equation 16.1:

$$g(U^* + \delta U) = g(U^*) + J(U^*)\delta U + o(\|\delta U\|)$$

where $J(U^*)$ is Jacobian of vector function $g = g(U)$. $J(U^*)$ is equal to

$$J(U^*) = U_d^* Y + (Y(U^* - U_0))_d.$$

For some U^* the rank r of matrix $J(U^*)$ could be less then n: $r < n$ and

$$\det J(U^*) = 0 \tag{16.5}$$

in this case and the next equation

$$J(U^*)\delta U = 0 \tag{16.6}$$

has a set of solutions:

$$\delta U^* = \sum_{i=1}^{n-r} c_i \delta U_i^*$$

where $\delta U_i^*, i \in [1 : n - r]$ are linear independent solutions of Equation 16.6 and c_i are free constants.

Hyperspace defined by Equation 16.5 separates the voltage space and as a result numerical algorithms used for solving Equation 16.1 and based on linearization technique often fail near particular points where $\det J(U^*) = 0$. A single solution $U^* = \frac{U_0}{2}$ of Problem 16.3 exists where $\det J(U^*) = 0$. Therefore, for convergence ensuring the power's vector S should be nonequal to (at least one component $s_i \in S$)

$$S \neq \frac{1}{4} U_{0d} Y U_0.$$

For $S > \frac{1}{4} U_{0d} Y U_0$ the problem 16.3 has complex solutions only. It should be mentioned that if $S = \frac{1}{4} U_{0d} Y U_0$ the power loss $f(\frac{U_0}{2})$ is equal to sum total power taken from the EPN.

Let us regard the power flow equation for an alternating current EN. In this case the Equation 16.1 is considered in the complex field. In real field 16.1 is equivalent to two equations:

$$V_d(A(U_0 - V) + RW) - W_d(AW - R(U_0 - V)) = P \tag{16.7}$$

$$-V_d(R(U_0 - V) - AW) + W_d(A(U_0 - V) + RW)) = Q$$

where $U = V + iW, Y = A + iR, S = P + iQ; A, R \in R^n \times R^n$ are active and reactive node's conductivities matrixes; $P, Q \in R^n$ are active and reactive node's powers vectors. U_0 is considered real so voltage's angles are calculated in comparison with balancing node. Active ΔP and reactive ΔQ losses are given by formulas:

$$\begin{aligned}
\Delta P &= (U_0 - V)(A(U_0 - V) + RW) + W(R(U_0 - V) - AW) \\
\Delta Q &= (U_0 - V)(R(U_0 - V) - AW) - W(A(U_0 - V) + RW).
\end{aligned}$$

According to our postulate for voltage state identification of EPN we have to solve the next nonlinear programming problem:

$$f(V, W) = (\Delta P(V, W))^2 + (\Delta Q(V, W))^2 \Rightarrow \min \tag{16.8}$$

under Constraints 16.7.

Usually in the complex field many problems are more simple. One complex equation is equivalent to two real. Sets defined by two equations do not separate the space. Therefore, it is possible to achieve any point of space starting from any other one without crossing these sets. It means that set defined by Equation 16.5. is avoidable in case of alternating current EPN and numeric algorithms are more reliable. Nevertheless, if for a solution V^*, W^* of Problem 16.8.

$$\det J(V^*, W^*) = 0$$

the numerical algorithm based on linearization will work more longer time . It is very easy to obtain that similar to direct current network a single such solution $V^* = \frac{U_0}{2}, W^* = 0$ exists. Hence, if vectors P and Q are nonequal to particular power vectors (at least one component $p_i \in P$ and $q_i \in Q$)

$$\begin{aligned} P &\neq \frac{1}{4} U_{0d} A U_0 \\ Q &\neq \frac{1}{4} U_{0d} R U_0 \end{aligned} \tag{16.9}$$

the convergence time practically will not depend on EPN data.

16.1.2 Software and Numerical Experiences

The theory presented in previous section is realized in program system "ELECTRO". "ELECTRO" was developed for modeling EPN associated with the strong nonlinearities both for direct and alternative current networks.

It is well known that for power system flow analysis Newton's like algorithms possess very good convergence and stability properties. Some of ill conditioned cases where the Jacobian is approximately zero can be solved by using the special versions of the same basic algorithm. But Newton's like algorithms falure very often when the EPN is tested on big power flows. "ELECTRO" algorithms converge in all ill cases.

The "ELECTRO" software consists of two program packages: "ELECTRO-1" and "ELECTRO-2" designed for direct and alternative current networks modeling respectively.

The "ELECTRO" software has the main features of a Decision Support System. The basic program parts are:

- user interface;
- database management subsystem;
- problem solvers.

Using "ELECTRO" next problems could be solved:

- voltages calculation when powers are known;
- powers calculation when voltages are known;
- currents calculation;
- maximum current in EPN calculation;
- needed power at the balancing node including active and reactive losses in EPN;

both for direct and alternative current networks.

User interface is based on an professional spreadsheet. “ELECTRO” is designed as a menu-driven interactive system with user oriented dialog. Dialog structure enables to organize the EPN modelling process as dynamic and interactive.

Database is designed in spreadsheet environment and includes:

- EPN structure description;
- electrical line’s conductivities;
- EPN powers;
- EPN voltages;
- EPN currents.

EPN is described by collections of nodes and connecting arcs corresponding to electrical lines. EPN structure is represented by conductivity matrix. Powers and voltages are associated with nodes of EPN and are shown as table values. The arcs associated parameters are line’s conductivities and currents.

The main calculator in presented program system is based on MINOS — the solver of nonlinear programming problems. The solver’s database interface is supported by special program converters. According to our experiences the algorithm produced very rapidly (8–10 iterations) the solution with an accuracy of 10^{-5}. The behaviour of algorithm did not depend on EPN structure and data values and depended practically of nodes number only (see testing results for IBM PC XT without coprocessor in Table 16.1). But when power vectors were equal to right side of Equation 16.9 the solution generation time was about 2.5–3 times greater than average time in Table 16.1 for all generated tasks. From our point of view the convergence time independence on data (excluding one particular point in power space) is the main good feature of algorithm. Therefore, the software could be used for “big power flows” modelling in overloaded EPN. Different accident situations and their consequences can be imitated as well and as a result EPN current protection could be estimated.

Using special graphic tools of “ELECTRO” EPN power and voltage levels could be visualized.

The “ELECTRO” programs provide for changing all aspects of the model without requiring extensive reentry of data.

It is possible to save and retrieve EPN data using floppy or hard disks.

An extensive error checking with getting computer error messages is provided.

Task number	Network nodes number	Number of generated tasks	Average calculation time $\pm$ max. deviation (sec.)
1	5	22	72.57 ± 27.57
2	10	18	211.37 ± 65.43
3	15	15	547.53 ± 136.77
4	20	14	1181.57 ± 315.26
5	25	12	2378.23 ± 477.62

Table 16.1: Testing results for IBM PC XT

One of important feature of "ELECTRO" is using ASCII files for interfacing with other program systems. In these files both the model input and solution data are presented.

16.1.3 Conclusions

The presented formalism enables to obtain the solution of enumerated higher tasks in not too long time. For instance, the solving of power flow problem for an 45 nodes EPN needed less than 20 min. on IBM PC XT. Hence, these tasks could be involved as submodels in any EPN modelling program system pretending to be more or less exhaustive. In many linear economical models nonlinear nature of EPN is not taken into account and Kirhhoff rules only are considered. Using of nonlinear models could make the description more adequate and found solutions more reliable. In our opinion particular attention should be paid to stochastic nonlinear models in which fluctuations phone automatically will ensure stable solutions.

For diminishing modeling time parallel systems of interacting nonlinear models could be used. In this kind of models we can expect some qualitative effects like in systems of nonlinear differential equations. Models involved in a parallel interacting modelling system could be based even on different mathematical formalism. That set of interacting models could be investigated by program tools designed as Decision Support System [PS 88b], [PS 88c], [PS 90], [PS 91]. In our opinion this approach could be regarded as further development of Bohr's principle and, we hope, will be very fruitful.

16.2 Interacting Effects in Self-Organized Algorithms for Solving Integer Programming Problems on Traditional and Parallel Computers

Abstract

A model of computation process in self-organized consecutive and parallel algorithms for solving integer programming problems based on using of automate theory is described. Syn-

ergic effects in self-organized optimization algorithms and results of numerical experiences on two-processor computer are presented.

16.2.1 Introduction

Integer programming problems (IPP) are the most difficult optimization tasks. Practically interesting tasks lead to great size IPP. Besides that, usually, IPP are NP-hard — there are no algorithms with polynomial complexity for their solving.

Well known methods (for instance, branch and bound) based on consecutive search generate sets of algorithms, which have a common drawback: the IPP solving time is very sensible to IPP definition information (matrix values and structure, and object function coeffitients). Therefore, choise of the best algorithm for each IPP could substantially diminish problem solving time. One possible solution of this problem is using adaptive self-organized algorithms for IPP solving [Pl 88], [Pl 89b], [PS 88a], [PS 88e], [PT 88].

Another perspective way of IPP solving time diminishing is parallel computations. Some branch and bound parallel algorithms designed for MIMD (Multilpe Instructions — Multiple Data) parallel computers are described in enumerated above papers and in [KP 84], [KP 86], [MB 85], [PPT 86], [PS 89]. All these algorithms are based on decomposition principle: the initial IPP is divided into several independent IPP of lower dimention and each new IPP is solving in its own processor. If a new, better solution is obtained in a processor — that solution asynchronously is transmited to other processors. That externally generated information (for all processors exept new solution generator) is able to diminish the solving time in all processors. The system property of the parallel algorithm consists in the possibility of each processor to obtain important for calculation speed information from other processors during the solving process.

If in each processor the algorithm is self-organized in sense of the best algorithm selection the initial IPP solving time could be substantially decreased.

In [PS 89] are presented results of numeric experiences. For some IPP the solving time on a 2-processor MIMD computer was more than two times less than solving time by the same algorithm on a single proceccor. The interactive effects were observed for an enough wide class of practical IPP. But there are patological IPP for which the solving time cannot be decreased by parallel self-organized algorithms. Some examples are given in this paper.

In [Ha 69], [ES 79], [PN 77], [RC 90a], [RC 90b] are described self-organization effects in chemical, biological, physical, social and economical systems. In this paper is presented a nonlinear interactive effect in an information system.

16.2.2 Theoretical Background

Let us regard next IPP:

$$\begin{array}{rcl} f(x) & \rightarrow & extr \\ x & \in & X \end{array} \tag{16.10}$$

where $f(x)$ is a scalar function, $X \subset R^m$ is finite set of feasible solutions. $V = \{v_j | j \in [1:n]\}$ is a set of algorithms for solving IPP of Equation 16.10. Let us assume, that algorithms $v_j \in V$ are able to work a time quantum $\bar{t}$, save the current state of calculation and any other algorithm $v_i \in V$ may continue the extremum search. Let's denote by f_{i-1} object function value before algorithm v_i starting and by f_i value of $f(x)$ after v_i has been working the time quantum $\bar{t}$.

Let us define a function Q on the set $V - Q : V \to R^1$ as: $Q(v_i) = a_i$ where $a_i = |f_i - f_{i-1}|$ The function $Q(v_i)$ characterize the speed of algorithm v_i. Let us define Q_{ij} as

$$Q_{ij} = Q(v_i) - Q(v_j) = a_i - a_j$$

and let $\operatorname{sigm} Q_{ij}$ be

$$\operatorname{sigm} Q_{ij} = \begin{cases} 1 & \text{if } Q_{ij} = a_i - a_j > 0 \\ 0 & \text{if } Q_{ij} \leq 0 \end{cases}$$

Taking into account the fact that the function $Q(v)$ cannot be expressed analyticaly the single way of detecting the algorithm with the highest speed is $Q(v)$ calculation and maximization simultaneously.

Let us regard the random choice algorithms. For object function $Q(v)$ identification at step N are made additional substeps, i.e. the function $Q(v)$ is calculated in points:

$$\Omega_N = v_N^{(1)}, v_N^{(2)}, \ldots, v_N^{(l)} \tag{16.11}$$

chosen by chance.

Let w_{N-1} be the memory state, which characterize the history of best algorithm selection and define the influence of previous steps at step N. The choice of best algorithm is made according to a probability distribution law:

$$\Pi = p\,(v_N^i | w_{N-1}, v_N^{(1)}, v_N^{(2)}, \ldots, v_N^{(l)}) \tag{16.12}$$

which is recalculated during the search process. Information about function $Q(v)$ values in points Ω_N (see Equation 16.11) enables to select another algorithm v_N according to a selection function:

$$v_N = \Phi(w_{N-1}, v_N^{(1)}, v_N^{(2)}, \ldots, v_N^{(l)}) \tag{16.13}$$

Memory state w_{N-1} is corrected after each step in accordance with a self-learning function:

$$w_N = \Psi(w_{N-1}, v_{N-1}, v_N^{(1)}, v_N^{(2)}, \ldots, v_N^{(l)}). \tag{16.14}$$

In random choice algorithms substeps Ω_N (see Equation 16.11) are selected according to distribution Π (see Equation 16.12). Let us regard algorithms without self-learning ($w = 0$) with linear and nonlinear tactics.

In algorithms with linear tactic in case of unsuccessful step
($\operatorname{sigm} Q_{ij} = 0$) additional supsteps are made according to Equation 16.12 and v_N is

selected according to Equation 16.13. In case of successful step no substeps are made and next step is equal to the previous one. An example of such algorithm $(\overline{A}_1)$ is:

$$v_N = \begin{cases} v_{(N-1)} & \text{if } Q_{N-1} = Q(v_{N-1}) - Q(v_{N-2}) > 0 \\ \sum_{i=1}^{n} v_i o_i & \text{if } Q_(N-1) \leq 0 \end{cases} \tag{16.15}$$

where $o = (o_1, o_2, ..., o_n)$ is a n-dimensional boolean vector one component of which o_i is equal to 1 with probability $p = \frac{1}{n}$ while all other are equal to zero.

In algorithms with nonlinear tactic in case of successful step (sigm $Q_{ij} = 1$) additional supsteps are made according to Equation 16.12 and v_N is selected according to Equation 16.13. In case of unsuccessful step no substeps are made and next step is equal to the last one. An example of such algorithm $(\overline{A}_2)$ is:

$$v_N = \begin{cases} v_{N-2} & \text{if } Q_{N-1} = Q(v_{N-1}) - Q(v_{N-2}) < 0 \\ \sum_{i=1}^{n} v_i o_i & \text{if } Q_{N-1} \geq 0 \end{cases} \tag{16.16}$$

where o_i are defined above. In described algorithms one substeb Ω_N (see Equation 16.11) coinciding with work step is made.

In algorithms with self-learning probability characteristics of choice are changing according to Equation 16.14 in dependance on memory state w_{N-1} and last step results. An example of such algorithm $(\overline{A}_3)$ with one substep coinciding with work step is:

$$v_N = \begin{cases} v_{j+1} & \text{neifhbouring to } v_{N-1} = v_j \text{ with probability } (1 - p_N) \\ v_{N-1} & \text{with probality } p_N \end{cases} \tag{16.17}$$

First v_1 and last v_n elements of set V are considered neighbours. Probability p_N depends on w_N:

$$p_N = \begin{cases} 0 & \text{if } w_N < -1 \\ \frac{1+w_N}{2} & \text{if } |w_N| \leq 1 \\ 1 & \text{if } w_N > 1 \end{cases} \tag{16.18}$$

Selection function is defined by Equation 16.17 and Equation 16.18 while self-learning function is given by:

$$w_{N+1} = w_N - 2d\,(\frac{1}{2} - \text{sigm}Q_N) \tag{16.19}$$

where $Q_N = Q(v_N) - Q(v_{N-1})$ and d is parameter $0 < d < 1$.

Algorithms for solving IPP 16.10 based on adaptive rules for best algorithm choice similar to $(\overline{A}_i)$, $i \in [1:3]$ are self-organized.

The solving process of Problem 16.10 could be represented as a stochastic environment (IPP) interacting with an probability automate (best algorithm selection rule). Let us regard Mur's type automate described by equations:

$$v(t+1) = \phi(v(t), c(t+1)), \tag{16.20}$$

$$a(t) = Q(v(t)) \tag{16.21}$$

where $c(t+1) = 1 - \text{sigm}\{Q(v(t+1) - Q(v(t))\}$, $t = 0, 1, \ldots, N, \ldots$ are automate steps. Input variable $c(t+1)$ is boolean: $c = 0$ means automate success while $c = 1$ is falure. Output variable $a(t)$ is defined as: $a(t) \in \{a_i | i \in [1 : n]\}$. Automate could be in one of n states $v(t) \in V$. Equation 16.20 defines two mapping of set of automate states V into V. Both mapping could be writen as transition matrixes of automate:

$$A_0 = \|a_{ij}^0\| \quad for \quad c = 0$$

and

$$A_1 = \|a_{ij}^1\| \quad for \quad c = 1, i, j \in [1 : n]$$

Let us regard the Automate 16.20– 16.21 behaviour in stationary stochastic environment $C = (a_1, a_2, \ldots, a_n)$. Let us assume that action a_i, $i \in [1 : n]$ made by automate at step t induce at step $(t+1)$ value $c = 1$ with probability s_i equal to $s_i = \frac{1-Y(a_i)}{2}$ and value $c = 0$ with probability $1 - s_i = \frac{1+Y(a_i)}{2}$ where $Y = Y(a_i)$ is a normed function: $|Y| < 1$. Trasition probability a_{ij} from state v_i into state v_j is given by:

$$a_{ij} = s_i a_{ij}^1 + (1 - s_i) a_{ij}^0 \tag{16.22}$$

Since matrix $A_C = \|a_{ij}\|$ is a stochastic automaton 16.20–16.21 behaviour in stationary stochastic environment is described by Markov's chain. So far as X is a finite set and the function $Q(v_i) = a_i$ could have a finite set of values $a_i \in A^i$, $|A^i| = m_i, i \in [1 : n]$ it is necessary to regard the best choice algorithm task as an adaptive control problem.

Let us denote by $W[P^t(\overline{A}_i), Q(v)]$ the mean of function $Q(v)$ with probability distribution $P^t(\overline{A}_i)$ at step t. Looking at the solving process of IPP 16.10 by self-organized algorithm $\overline{A}_i$ as infinite (or repeatable) we can regard the limit distribution

$$P^*(\overline{A}_i) = \lim_{t \to \infty} P^t(\overline{A}_i)$$

If the distribution $P^*(\overline{A}_i)$ exists it characterizes the interaction between algorithm $\overline{A}_i$ and stochastic environment — i.e. IPP.

The best choice algorithm problem could be formulated as: find distributions $P^t(\overline{A}_i)$, $t = 0, 1, 2, \ldots, N, \ldots$, relatively to which the

expectation $W[P^t(\overline{A}_i), Q(v)]$ for $t \to \infty$ achieves its maximum:

$$\lim_{t \to \infty} W[P^t(\overline{A}_i), Q(v)] \;\to\; \max, \qquad v \in V. \tag{16.23}$$

Definition 1 *Adaptive algorithm $\overline{A}_i$ achieves the control object in stochastic environment C defined at each step t by distribution $P_C^t(\overline{A}_i)$ if the limit*

$$\lim_{t \to \infty} W[P_C^t(\overline{A}_i), Q(v)] = W[P_C^*(\overline{A}_i), Q(v)] \qquad exists$$

where

$$P_C^*(\overline{A}_i) = \lim_{t \to \infty} P_C^t(\overline{A}_i)$$

is a limit distribution.

Definition 2 *Algorithm $\overline{A}_i$ solves the Problem 16.23 if distribution $P_C^t(\overline{A}_i)$ exists for any t and a single limit distribution*

$$P_C^*(\overline{A}_i) = \lim_{t\to\infty} P_C^t(\overline{A}_i) \qquad \textit{exists.}$$

Let us regard the case when the function $Q = Q(v)$ is stochastic. Automate 16.20– 16.21 interact with a complex stochastic environment consisting of stationary environments $C^{(\alpha)} = (a_1^{(\alpha)}, a_2^{(\alpha)}, \ldots, a_n^{(\alpha)})$. Transition from one environment to other is made according to a Markov chain. Complex stochastic environment $K = (C^{(1)}, C^{(2)}, \ldots, C(M))$ described by Markov chain with $M = \prod_{i=1}^n m_i$ states and transition matrix $B = \|b_{ij}\|;\ i, j \in [1 : M]$.

At each step t the automate interacte with one of environmentes $C^{(\alpha)}$. Transition probability from state v_i of environment $C^{(i_1)}$ into state v_j of environment $C^{(i_2)}$ is given by:

$$a_{ij}^{(i_1)(i_2)} = b_{i_1 i_2}((1 - s_i^{i_1})a_{ij}^0 + s_i^{i_1} a_{ij}^1) = b_{i_1 i_2} a_{ij}^{i_1} \tag{16.24}$$

Theorem 1 *If*

$$\begin{array}{llll} a) & a_{ij} & > 0; & i, j \in [1 : n]; \\ b) & b_{i_1 i_2} & > 0; & i_1, i_2 \in [1 : M]; \\ c) & a_{ij}^{i_1} & > 0; & i_1 \in [1 : M]; i, j \in [1 : n] \end{array}$$

then adaptive algorithms $\overline{A}_i$ described by Matrixes 16.22 , 16.24 achieve the control object.

Proof. According to a) the Marcov Chain 16.22 is ergodic and hence, the limit distribution $P_C^* = (p_1^*, p_2^*, \ldots, p_n^*)$ exists, does not depend on initial distribution $P_C^0 = (p_1^0, p_2^0, \ldots, p_n^0)$, and could be found by solving the linear equation $P = PA_C$ under constraint $\sum_{i=1}^n p_i = 1$. According to b), c) the Marcov chain described by Equation 16.24 is ergodic too and limit probability vector P_K^* exists. This vector could be find from equation $P = PA_K$ where $A_K = \|a_{ij}^{(a_1)(a_2)}\|$. Vector P_K^* does not depend on initial distribution P_K^0

Theorem 2 *Adaptive algorithm with linear tactic $\overline{A}_1$ solves the Problem 16.23 in stationary $C = (a_1, a_2, \ldots, a_n)$ and complex $K = (C^{(1)}, C^{(2)}, \ldots, C^{(M)})$ environments if $Q(v) > 0, \quad v \in V$.*

Theorem 3 *Adaptive algorithm with nonlinear tactic $\overline{A}_2$ solves the Problem 16.23 in stationary $C = (a_1, a_2, \ldots, a_n)$ and complex $K = (C^{(1)}, C^{(2)}, \ldots, C^{(M)})$ environments for any values of $Q(v), \quad v \in V$.*

Theorems 2, 3 proof could be easy obtained writing matrix Equations 16.22 , 16.24 for algorithms $\overline{A}_1$, $\overline{A}_2$ and using Theorem 16.2.2.

Thus adaptive algorithms $\overline{A}_1$, $\overline{A}_2$ could be used as organizing rules in self-organized algorithms for solving IPP 16.10.

Let $\overline{A}$ be a finite set of organizing rules for self-organized algorithms $\overline{A} = \{\overline{A}_i | i \in [1 : n_A]\}$. Let us assume that for all that rules distributions $P^t_K(\overline{A}_i)$, $t = 1, 2, \ldots, N$, exist. The average speed of Problem 16.10 extremum search process at step t is $W[P^t_K(\overline{A}_i), Q(v)]$. Regarding adaptive rules which solve the Problem 16.23 only let us define the average speed $W(\overline{A}_i)$ of rule $\overline{A}_i$ as $W(\overline{A}_i) = W[P^*_K(\overline{A}_i), Q(v)]$ where $P^*_K(\overline{A}_i)$ is limit distribution. Let $\overline{W}$ be the average defined as:

$$\overline{W} = \max_{\overline{A}_i \in \overline{A}} \lim_{t \to \infty} W[P^t_K(\overline{A}_i), Q(v)] \tag{16.25}$$

The optimal control problem could be stated as: find the adaptive rule $\overline{A}_i \in \overline{A}$ which maximize the right side of Equation 16.25. Since the function $Q = Q(v)$ is stochastic and distributions $P^*_K(\overline{A}_i)$ depend on environment K (in other words of IPP 16.10) parameters, for Problem 16.25 solving one of algorithms $\overline{A}_i \in \overline{A}$ could be used. Solving Problem 16.25 we obtain the second level of the algorithm of self-organization, on which the rules of first level self-organization are choosen. Since the nonlinear tactic rule does not depend of environment characteristics, the algorithm $\overline{A}_2$ could be used as second level organization rule.

16.2.3 Algorithms and Numerical Experiences

One of the most investigated method for solving IPP is branch and bound. Branch and bound algorithms are defined by search rules (such as First In — First Out, Last In — First Out) and a set of parameters. Variating rules and parameters the set V of algorithms could be generated. All these algorithms are very sensible to solution generation speed.

Using the set $\overline{A}$ of adaptive rules for first and second self-organization levels next algorithm for solving IPP on a single processor computer is obtained:

Step 1. *SELECT* a first-level self-organization rule $\overline{A}_j \in \overline{A}$ according to the rule $\overline{A}_2 \in \overline{A}$; $k = 0$; The first-level self-organization rule is choosen from the set $\overline{A}$ according to the rule $\overline{A}_2$ after L quantums of time $\bar{t}$.
GO TO Step 2;

Step 2. *SELECT* an algorithm $v_i \in V$ according to current first-level self-organization rule $\overline{A}_j \in \overline{A}$ and solve the IPP during the time quantum $\bar{t}$; $k = k + 1$;
GO TO Step3;

Step 3. *SAVE* the IPP search state; If $k = L$ *GO TO* Step1; *GO TO* Step2;

One of basic feature of IPP is the possibility to divide the problem into a set of independent IPP of lower dimention. This feature enables to use effectively parallel computations for solving IPP. An example of a parallel self-organised algorithm is:

Step 1. *DIVIDE* the initial IPP into p subproblems where p is processors amount;
SEND all subproblems to their own processors.
GO TO Step2;

Step 2. *SOLVE* in each processor the subproblem by the self-organized algorithm described higher.
IF in a processor a new better solution is generated
THEN TRANSMIT asynchronously through a global variable the object function value on this solution to all other processors.

For the sake of simplicity let us regard in more detailes a parallel branch and bound algorithm without self-organization for boolean programming problems. If values of k variables are fixed the problem is divided into a set of $p = 2^k$ subproblems $\{(P_1), (P_2), \ldots, (P_n)\}$ equivalent to the initial task.

The boolean programming problem solving process by a branch and bound algorithm could be represented as motion on a full binary tree $G = (U, R)$, where U is a set of tops and R is a set af arcs. Each top u_i corresponds to a subproblem in which some variables values are fixed. The search rule (LIFO, FIFO, "best estimation" search) defines the order (or strategy) in which the tops $u_i \in U$ are analysed. For each analysed top u_i the upper value estimation f_i^* of object function is calculated and either the top u_i is excluded from consideration itself or the top u_i is excluded together with all descendent tops.

If the problem is divided into p subproblems the tree G is divided into p subtrees $G_1 = (U_1, R_1), G_2 = (U_2, R_2), \ldots, G_p = (U_p, R_p)$ where $U_1, U_2, \ldots, U_p$ are sets of tops and $R_1, R_2, \ldots, R_n$ are sets of arcs.

Definition 3 *The supbroblems $\{(P_1), (P_2), \ldots, (P_p)\}$ solving process on p-processor computer system by the same strategy in all processors is called p-strategy.*

In a p-strategy a set of tops $\{u_i^j\}, u_i^j \in U_j, j \in [1 : p]$ is analysed simultaneously. If after the estimation problem solving in tops u_i^j a feasible solution of integer problem is obtained and the function value on this solution is better than the current known solution (record) the new solution is considered as record. The function value on record solution is remembered in a global variable z. All processors asynchronously are reading/writing z after each top u_i^j analysis.

For comparing the problem solving time by consecutive and parallel branch and bound algorithms let us denote by:

$Z^0 = \{z_0^0, z_1^0, \ldots, z_l^0, \ldots, z_r^0\}$ is set of record object function values obtained by 1-strategy (consecutive algorithm);

$Z = \{z_0^p, z_1^p, \ldots, z_l^p, \ldots, z_q^p\}$ is set of record object function values obtained by p-strategy (parallel algorithm);

U^0 is set of tops excluded in 1-strategy after comparing their object function estimation with current record value;

U^j is set of tops excluded in processor j after comparing their object function estimation with current record value in p-strategy.

Let us define the complexity T_1 of a boolean n variables problem solving process by 1-strategy as:

$$T_1 = 2^{n+1} - 1 - |U^0|$$

and the complexity T_p of the same problem solving process by p-strategy as:

$$T_p = \max_{j \in [1:p]} (2^{n-k+1} - 1 - |U^j|),$$

and assume that the top analysis time is the same for all tops and for all processors both in consecutive and parallel algorithms.

Theorem 4 *Under assumptions made higher the p-strategy with fixed order of top analysis (LIFO,FIFO) is more than p times effective than coresponding consecutive algorithm*

$$\frac{T_1}{T_p} > p$$

with probability $P = \frac{1}{p}$.

A proof of this theorem could be easily obtained regarding sets Z^0 and Z^p and taking into account that all processors in p-strategy analyse top sets which are not excluded in consecutive algorithm with probability $P = \frac{1}{p}$.

In other words, according to Theorem 4 the interactive effect resulting in more than p times problem solving time diminising on a p-processor computer could be detected with probability $P = \frac{1}{p}$ (for fixed order strategies).

In [PS 89] results are presented of numerical experiences made on a two-processor computer ES-1045 (analogous to IBM-370). Self-organized algorithms described above were realized in a parallel program system "ILPP ADAPT" designed for solving integer linear programming problems on ES compatible computers. In "ILPP ADAPT" were used three branch and bound algorithms $v_i, i \in [1:3]$ with LIFO, FIFO and "mixed" strategies respectively. As self-organization rule was used adaptive algorithm with linear tactic $\overline{A}_1$ and for the sake of simplicity only one level of self-organization was tested.

For testing were generated some practical problems of differend dimentions (tasks 1–4 in Table 16.2):

$$\begin{aligned}
\sum_{i,j=1}^{n} (2c_{ij}x_{ij} - c_{ij}y_{ij} \quad &+ \quad 2d_{ij}z_{ij} - d_{ij}y_{ij}) \rightarrow \min \\
\sum_{i,j=1}^{n} c_{ij}y_{ij} \quad &- \quad bU < 0 \\
- \sum_{i,j=1}^{n} c_{ij}y_{ij} \quad &+ \quad bU < U \\
\sum_{i,j=1}^{n} d_{ij}y_{ij} \quad &- \quad cU < 0 \\
- \sum_{i,j=1}^{n} d_{ij}y_{ij} \quad &+ \quad cU < U
\end{aligned}$$

Task number	Number of constraints	Number of linear variables	Number of boolean variables
1	163	235	38
2	279	307	66
3	427	627	102
4	513	755	123
5	20	0	20
6	98	0	98

Table 16.2: Tasks dimentions

$$\begin{aligned}
\sum_{i=1}^{n} y_{ij} &= 1, \quad j \in [1:n] \\
\sum_{j=1}^{n} y_{ij} &= 1, \quad i \in [1:n] \\
y_{ij} + b - x_{ij} &\leq 1, \quad i,j \in [1:n] \\
y_{ij} + c - z_{ij} &\leq 1, \quad i,j \in [1:n] \\
y_{ij} &\geq x_{ij}, \quad i,j \in [1:n] \\
y_{ij} &\geq z_{ij}, \quad i,j \in [1:n] \\
\sum_{i,j=1}^{n} x_{ij} &\leq n^2 b \\
\sum_{i,j=1}^{n} z_{ij} &\leq n^2 b \\
y_{ij} = 0 \vee 1, \quad b = 0 \vee 1, \quad c &= 0 \vee 1, \quad x_{ij} \geq 0, \quad z_{ij} \geq 0, \quad i,j \in [1:n]
\end{aligned}$$

Tasks 5,6 in Table 16.2 were tested for illustration the fact that exist patological problems for which prallel algorithms cannot diminish solving time:

$$\begin{aligned}
\sum_{i=1}^{n} \frac{k}{k+1} y_i &\rightarrow \max \\
y_i - \frac{k+1}{k} x_i &\leq 1, \quad i \in [1:n] \\
y_i - (\frac{k+1}{k})^2 x_i &\leq (\frac{k+1}{k})^2, i \in [1:n] \\
y_i = 0 \vee 1, \quad x_i = 0 \vee 1, \quad i &\in [1:n], \quad (n-2) \leq k < (n-1)
\end{aligned}$$

All tested tasks were solved by algorithms $v_i, i \in [1:3]$ and by self organized algorithm $\overline{A}_1$ on one and two processors. Results of experiments are presented in Table 16.3.

Task number	One processor (T_1)				Two processors (T_2)				Diminishing rate T_1/T_2			
	v_1	v_2	v_3	A_1	v_1	v_2	v_3	A_1	v_1	v_2	v_3	A_1
1	27.5	1.18	14.1	19.2	13.2	0.59	6.39	8.57	2.07	2.00	2.21	2.24
2	51.0	29.0	3.26	12.2	15.3	7.07	1.21	3.07	3.33	4.01	2.69	3.97
3	60.1	15.3	46.5	38.6	29.2	6.03	21.5	16.3	2.08	2.54	2.16	2.35
4	21.5	63.0	58.3	43.2	10.2	30.1	27.4	19.1	2.1	2.08	2.12	2.25
5	0.03	0.04	0.03	0.03	0.03	0.04	0.03	0.03	1.0	1.0	1.0	1.0
6	0.15	0.17	0.16	0.16	0.15	0.17	0.16	0.16	1.0	1.0	1.0	1.0

Table 16.3: Testing results. Optimal solution obtaining time (min) T_1 and T_2

16.2.4 Conclusions

Results obtained in numeric experiences enable to conclude:

- there are IPP for which the solving time on a parallel computer could be decreased by more than number of used processors times (tasks 1-4);
- particular patological IPP exist for which parralel computations cannot decrease optimal solution finding time (tasks 5,6);
- during the IPP solving process interactive effects could be detected and processors interactions could diminish problem solving time (tasks 1-4);
- self-organized algorithms could be successfuly used for solving the best algorithm search problem.

Each self-organised algorithm working in a processor is an open system, which can get and generate/transmin important information to other processors. Positive reverse links in that kind of open systems could lead to considerable nonlinear effects of problem solving time decreasing. Interactions in self-organized algorithms remind the famous lithograph “Drawing hands” by M.C. Escher(1948).

16.3 Interactive Effects in Systems of Models

Abstract

Two interactive hypercycles like Eigen’s describing economical processes are regarded. It is shown that two interactive models on microlevel are equivalemt to a macro-model description based on potential function.

This section gives some details of results obtained by Chernenko [RC 90a], [RC 90b] in models of interaction effects.

16.3.1 Theoretical Background

Chernenko's macro-economic model presented in [RC 90a], [RC 90b] deals with relations between economical freedom, technological complexity, consumption and production. The basic idea of both papers consists in constructing and using for further investigations a potential function depending on these three macro variables and having a tipical catastrophic jumping behaviour.

The model variables are:

X: the degree of economic freedom, measured by the number of different aspects of economical activities;

P: productivity of labour;

C: level of consumption.

According to [RC 90a], [RC 90b] a model in these variables aproaches the equilibrium surface defined by the equation:

$$\frac{dX}{dt} = A(\aleph_0 - C - KX^2)X + \Phi(P) = 0 \tag{16.26}$$

The projection of equilibrium surface on (P, C) subspace gives the interpretation of jumping economic behaviour. The system could be kept within the region 1 (at lower X equilibrium value) by either increasing consumption C by slow decrease of productivity P or increasing productivity P by latent decrease of consumption C. It is supposed that increasing both consumption and productivity beyond the semicubic parabola should make the system to jump to the upper equilibrium value of X.

Chernenko's micro-economic production model regards three differential equations:

$$\frac{dx_i}{dt} = F_i - \frac{x_i}{C_0}\sum F_i, \quad i \in [1:3] \tag{16.27}$$

$$\begin{aligned} F_1 &= x_1(N - x_1) \\ F_2 &= x_2(a_x x_1 - x_2) \\ F_3 &= x_3(b_x x_2 - x_3) \end{aligned} \tag{16.28}$$

where variables and parameters mean:

x_1: the quantity of goods prodused in the agrarian sector,

N: the total of natural resources,

x_2: the quantity of industrially produced goods,

$a_x x_1$: the relative increase of industrial production due to the agrarian sector,

x_3: production of high informational technologies,

$b_x x_2$: the relative increase of high tech production due to industrial products supplied to the high tech sector,

C_0: the total production in the stationary state.

The Chernenko's system of differential Equations 16.27 has four stationary states:

1.

$$\begin{aligned} x_1 &= C_0 \\ x_2 &= 0 \\ x_3 &= 0 \\ C_0^{(1)} &= 0 \end{aligned}$$

2.

$$\begin{aligned} x_1 &= \frac{N + C_0}{a_x + 2} \\ x_2 &= \frac{(a_x + 1)C_0 - N}{a_x + 2} \\ x_3 &= 0 \\ C_0^{(2)} &= \frac{N}{a_x + 1} \end{aligned}$$

3.

$$\begin{aligned} x_1 &= \frac{C_0 + N(b_x + 2)}{a_x b_x + a_x + b_x + 3} \\ x_2 &= \frac{(a_x + 1)C_0 + (a_x - 1)N}{a_x b_x + a_x + b_x + 3} \\ x_3 &= \frac{(a_x b_x + b_x + 1)C_0 - N(a_x + b_x + 1)}{a_x b_x + a_x + b_x + 3} \\ C_0^{(3)} &= N\frac{a_x + b_x + 1}{a_x b_x + b_x + 1} \end{aligned}$$

4.

$$\begin{aligned} x_1 &= N \\ x_2 &= a_x N \\ x_3 &= a_x b_x N \\ C_0^{(4)} &= (1 + a_x + a_x b_x)N \end{aligned}$$

and C_0 is supposed to be a monotonously increasing time-dependent function $C_0 = C_0^* + \alpha t$. Depending on time only one of these stationary states is stable.

Let us regard a second production model interacting with the first one:

$$\frac{dy_i}{dt} = G_i - \frac{y_i}{P_0}\sum G_i, \quad i \in [1:3] \tag{16.29}$$

$$\begin{aligned} G_1 &= y_1(K - y_1) \\ G_2 &= y_2(a_y y_1 - y_2) \\ G_3 &= y_3(b_y y_2 - y_3) \end{aligned} \tag{16.30}$$

where

$$\begin{aligned} P_0 &= P_0^* + x_1 + x_2 \\ C_0^* &= P_0^* \end{aligned}$$

All variables in the first model are restricted by:

$$\begin{aligned} x_1 &= N \\ x_2 &= a_x N \\ x_3 &= a_x b_x N \end{aligned}$$

and if

$$P_0 = P_0^* + N + a_x N < P_0^{(3)}$$

y_3 will not arise.

If $a_y < a_x y_3$ will not appear and it means that the second model has less economic freedom while the consumption level is lower than in the first model. Regarding $P_0^{(3)}$ as function of a_y the same hyperbole like in macro-model could be obtained.

16.3.2 Conclusions

Some effects and behaviour obtained from an agregated model could be detected in a system of interacting models.

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Chapter 17

Klaus G. Troitzsch, Koblenz: Processes of Opinion Formation with Externally Changing Parameters

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Abstract

The purpose of this paper is to present some simulation experiments concerning a process of opinion formation as it was modeled by [WH83, pp. 49–53]. In this simple model individuals make up their decision in favour of or against one of two alternatives according to the majority they find in their population in the following manner: The probability of a supporter to become an opponent is the higher the greater the majority of opponents is (and the other way round), and this probability is modeled by the function

$$\mu_{-\leftarrow+} = \nu \exp\left[-(\delta + \kappa x)\right]$$

where ν is a flexibility parameter, δ is a preference parameter, and κ is a cohesion parameter, while $x \in [-1, +1]$ is the macro state of the population, where -1 stands for "all opponents" and $+1$ stands for "all supporters".

An analytical solution was given for the case that δ and κ are not constant, but periodically changing by [WH83]. This analytical solution does neither depend on the frequency of the δ-κ change nor on the speed with which the individuals change their minds (ν). Our stochastical simulation experiment show that it makes a difference whether the change of the external parameters δ and κ is rapid or slow compared with the individual speed of changing minds.

In most papers on catastrophic phenomena the microdynamics is believed to be so fast that catastrophic jumps occur whenever macrodynamics impose them on the system. This paper shows that delayed microdynamics may prevent catastrophic jumps.

17.1 Weidlich's original model

In Weidlich's simple model of opinion formation in a homogeneous population individuals can make a decision against or in favour of a certain proposal, i.e. they can be either supporters or opponents. The transition probability between the two possible individual states is given by

$$\mu_{+\leftarrow -} = \nu \exp(\delta + \kappa x) \quad (17.1)$$
$$\mu_{-\leftarrow +} = \nu \exp\left[-(\delta + \kappa x)\right] \quad (17.2)$$

where x is the macrostate of the population given by

$$x = \frac{n_+ - n_-}{n_+ + n_-} \quad (17.3)$$

with n_+ the number of supporters and n_- the number of opponents ($n_+ + n_- = 2N =$ const).

The parameters have the following meaning:

ν is a *flexibility parameter*, the greater ν is, the more probable is any individual change of mind,

δ is a *preference parameter*, the greater δ is, the more probable it is for any individual to become a supporter,

κ is a *cohesion parameter*, the greater κ is, the greater is the influence of the collective on the individual; for $\kappa > 0$ we have the case that the individuals adapt to the majority's opinion (we shall consider only this case).

After a master equation has been derived from our model, a function yielding the probability of finding a population in a given macro state may be calculated. The stationary state of this function is given by

$$p_{st}(Nx) = p_{st}(0) \exp\left[NU(x)\right] \quad (17.4)$$
$$U(x) = 2\delta x + \kappa x^2 - \left[(1+x)\ln(1+x) + (1-x)\ln(1-x)\right] \quad (17.5)$$
$$\sum_{Nx=-N}^{N} p_{st}(Nx) = 1 \quad (17.6)$$

which is an approximation which is sufficiently correct for large N. The probability function has its extrema for

$$U'(x_m) = 2(\delta + \kappa x_m - \text{ar tanh } x_m) = 0 \quad (17.7)$$

For $\kappa \leq 1$ there is only one maximum, while for $\kappa > 1$ depending on whether the expression

$$\sqrt{\kappa(\kappa - 1)} - \text{ar tanh } \sqrt{\frac{\kappa - 1}{\kappa}} - \delta \quad (17.8)$$

is negative, zero or positive, we have only one maximum, one maximum and one saddle, and two maxima and one minimum, respectively. Figure 17.1 shows the equilibrium surface of the macrostate of the population which is the set of all representative points of the population (δ, κ, x) corresponding to states of either maximum or minimum probability. The bifurcation set is the triangular curve on the bottom of figure 17.1 for which the expression 17.8 evaluates to zero. Obviously, we have the case of a cusp catastrophe.

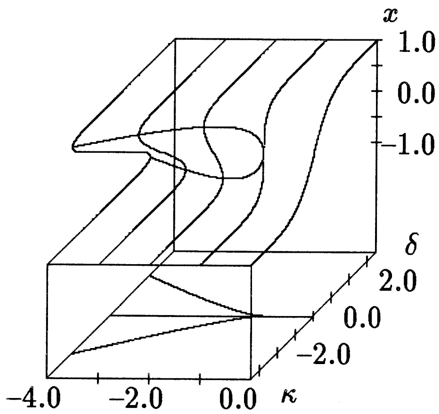

Figure 17.1: Equilibrium surface of a two-level system described by equations 17.1 to 17.7

17.2 Varying Parameters

We now direct our attention to the case when δ and κ are not constants but change in time. We refer to our simulation experiments in which we model both control parameters as periodically changing, i.e.

$$\delta(t) = \delta_0 + \delta_A \sin 2\pi\delta_f t \quad (17.9)$$
$$\kappa(t) = \kappa_0 + \kappa_A \sin 2\pi\kappa_f t \quad (17.10)$$

The results of our simulation experiments match the approximate analytic results reported by Weidlich and Haag [WH83, pp. 49–53], the details of which we may omit here. Instead, we give an interpretation of our simulation results in terms of catastrophe theory [Tho75] [Rap80, 88–99] [Arn86] [Sau86].

Now, the state space of our system is three-dimensional, it is $I\!R \times I\!R^+ \times [-1, 1]$.

Disregarding for a while the inherent stochasticity of our system, we should expect that any single realization of our process will always stay in the immediate neighbourhood of the equilibrium surface of figure 17.1. As long as we suppose that the probability function always approaches its stationary state much faster than the control parameters are changed, this is even realistic: we have to expect that our system stays in the immediate neighbourhood of the projection of the control curve onto the equilibrium surface. In the case that the control curve described by equations 17.9 and 17.10 crosses the bifurcation set, the system performs a jump from the upper part of the equilibrium surface to the lower or vice versa will occur.

In the figures to follow our system is exposed to control parameter changes of different frequency. We show the results for the following cases: we leave $\kappa = 1.5$ constant and ν constant, too, such that neither $\mu_{-\leftarrow+}$ nor $\mu_{+\leftarrow-}$ ever exceed 1.0, and vary δ between -0.5 and 0.5, consider a population of 100 individuals which is simulated over 100,000 steps where in every step we ask exactly one individual whether it changes its mind, i.e. each individual has 1,000 occasions to decide. The frequency of δ is varied in a manner that it performs 5 cycles in the slowest example of figure 17.2 and 40 cycles in the fastest example of figure 17.5. Catastrophic jumps would occur from the point with coordinates $(-0.2075\ldots, 1.5, 0.5773\ldots)$ to $(-0.2075\ldots, 1.5, -0.9197\ldots)$ on the left hand side and from $(0.2075\ldots, 1.5, 0.5773\ldots)$ to $(0.2075\ldots, 1.5, -0.9197\ldots)$ on the other side.

In the case of slow change of the control parameters we see that the internal variable x follows the "deterministic trajectory" with catastrophic jumps as expected.

Doubling the frequency of δ leads to the result that at least one catastrophic jump — the one at the beginning of the third cycle — is not completely performed.

Another doubling of the δ frequency leads to the effect that only few catastrophic jumps are performed completely.

Yet another doubling of the δ frequency — δ now changes eight times as fast as in figure 17.2 — yields a stochastic trajectory during which catastrophic jumps never occur any longer.

In most presentations of catastrophic phenomena the microdynamics is believed to be so fast that systems reach their equilibrium states defined by the actual control parameters without any delay and that consequently catastrophic jumps occur whenever macrodynamics impose them on the system. In real systems the difference between macro and micro flexibility need not be so great that the effects of relatively slow microdynamics may be neglected. This paper shows that slow microdynamics may even prevent catastrophic jumps.

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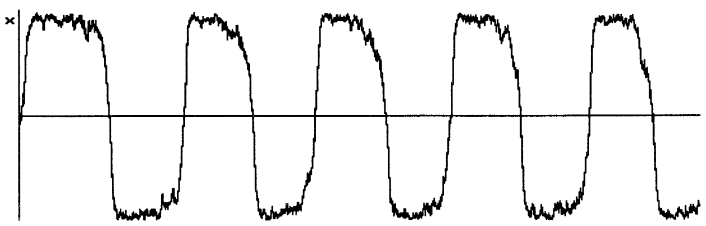

Figure 17.2: Internal variable x under the influence of the control variable δ varying with a frequency such that it performs 5 cycles during 100,000 simulation steps (horizontal axis is time, vertical axis is $x(t)$ as defined in equation 17.3

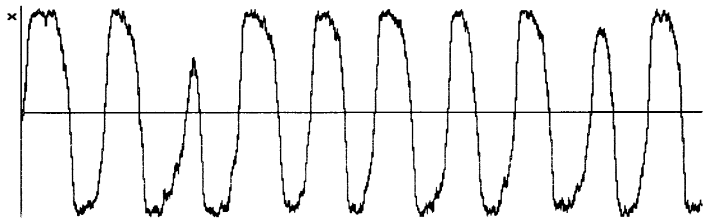

Figure 17.3: Same as figure 17.2, but with 10 cycles during 100,000 simulation steps

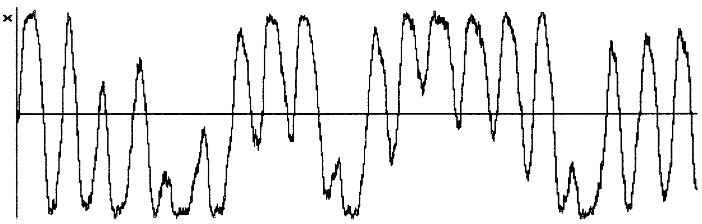

Figure 17.4: Same as figure 17.2, but with 20 cycles during 100,000 simulation steps

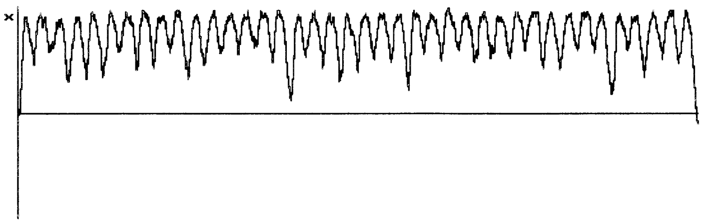

Figure 17.5: Same as figure 17.2, but with 40 cycles during 100,000 simulation steps

Chapter 18

Eugene V. Chesnokov and Igor V. Chernenko: Sunyata and Emptiness of Nonlinear Structures

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Abstract

Consideration of the skew-symmetric tensor field allows one to specify a type of nonlinearity and obtain evolution equations using the principle of least action and the space-time metric [CC 92].

The nonlinear field interpretation implies that neither quanta of energy nor substance particles can be treated as existence elements. Both phenomenal and functional elements are manifestations of nonlinear field local effects that used to be called synergies or interacting factors. The same effects display themselves as compact coherent structures with particle-like properties as well as solitons or solitary vortices peculiar to nonlinear hydrodynamics equations [CCC 88], [CCY 88].

The brave new world of magic nonlinear phenomena appears to be a transcendental reality of original relativity (sunyata) that spiritualizes the lifeless world of separate substantial entities.

18.1 Introduction

> Considering causes and conditions we call this world a phenomenal one. But when causes and conditions are thrown away the same world is called absolute.
>
> *Nagarjuna*

A mathematic formalization of self-organized processes leads to a parametric set of nonlinear equations. An evolution of the parametric nonlinear system displays actualization of an innate set of homoeostatic states. Appearance of new structures (groups of phenomenal existences) is due to appropriate external conditions that can be described by macro parameters. The essence of the system (i.e. the spectrum of possible states) seems to remain unchangeable in spite of time-variant behaviour.

The parametric manifold of stationary states forms a multi-dimensional functional double of the nonlinear system. Projections into empirical subspace transform equations, decompose the single-valued regular solution into a set of solutions of different equations, and cause an illusion of stochastic behaviour of empiric processes at the vicinity of singularity.

In this interpretation multi-dimensional reality seems to break down into interacted subspaces. Singular phenomena in subspaces appear to be caused by hidden factors belonging to multi-dimensional space. A local influence on a nonlinear media at the neighborhood of singularity can provoke latent interactions.

According to Catastrophe Theory these singularities can be formed by projecting multi-dimensional space into its subspaces observed in experiments.

While accounting for hidden dimensions it is possible to describe micro structures of singular phenomena. The nature of these transcendent properties is analogous to concealed symmetries of Korteweg de Vries and nonlinear Schrödinger equations attributed to correspondent invariants and local conservation laws that are utilized to generate solitary particle-like solutions (solitons) and investigate their stability as well as transform a vacuum state into a multi-soliton solution. Nevertheless, these ghost-like symmetries can hardly be realizable in physical experience [New 85].

Latent parameters could be only measured by indirect methods on the base of a priori models. Procedures of empirical measurement can be valid only locally and have unavoidable singularities.

The principle of self-consistent description implies that the nature of mutually connected structural elements is conditioned by relations between these elements, and these relations themselves are determined by the nature of structural elements. Functional connections submit to invariant relations that represent the eternal essence of self-organized processes. System invariants can be treated in terms of general field interpretation.

18.2 Sunyata and System-Field Dualism

> [Phenomenal] elements which do not exist there, in the Absolute, they really do not exist in all; they are like kind of terror which is experienced when, in the dark, a rope is mistaken for a snake and which dissipates as soon as a light [is brought in] . . .
> Obsessed by the unreal devil of their "Ego" and the "Mine" the obtuse men and common worldlings imagine that they really perceive separate entities which in reality do not exist, just as the ophtalmic sees before himself hair, flies and other [objects which never did exist] . . .
>
> *Chandrakirti*

"A field" and "a system" appear to be the two complementary descriptions of a complex reality. Consideration of a self-organized system which is generated by some nonlinear interaction that creates homoeostatic structures as well as illusion of phenomenal manifold.

Rejecting the existence of separate system elements allows one to consider system media as interacted nonlinear processes. For example, stable structural units can be treated as incoherent autonomous pseudo entities that are called hypnons because of their independent somnambulist behaviour in equilibrium states and hidden structural complexity.

At the vicinity of system bifurcation points, independent hypnons can be awaken and converted into coherent structures by concealed agencies that metamorphose the nature of separate elements and their functional relations according to a possible way of system evolution.

In traditional physics, the conception of elementary particles such as free electrons or photons implies their autonomy which can hardly be explained using the common substantial interpretation. This difficulty can be avoided by treating elementary particles as solitary particle-like solutions of nonlinear equations.

The same approach appears to be fruitful for studying social morphogenesis processes. For example, a "social field" could be treated as a hidden factor that creates conditions for the generations of social institutions. Interaction between hypnon-like units could generate specific types of collective behaviour under appropriate conditions. For instance, initial consortia interacts with the environment and turns into corpuscular systems opposed to their surroundings, i.e. historical evolution begins with creation of rigid system of functional relations.

A mutual dependence of structural elements means their relative existence. According to the sunyata principle a dependent existence is not real, as well as borrowed money and can barely be considered as real. A formed matter (dharma-svabhava) is a distorted projection of multi-dimensional eternal reality. The phenomenal world (duhkha) is a manifestation of the fundamental invariant reality that remains unchangeable during all visual transformations.

The permanent essence can be described in terms of general field invariants e.g., morphogenesis could be studied using Leo Gumilyov's energetic approach that leads to system invariant interpretation [Gu 90].

The principle of fundamental invariance is the opposite site of the Hegel's principle of universal negativity (Negativität ist die Seele die Welt), i.e. energy conversions restricted by conservation laws are simultaneous annihilations and generations of distinctions.

Thus the principle of negativity (see *sunyata* [Le 86]) or universal relativity (pratitya-samutpada-sunya) a priori implies a finite empirical space measure (see Equation 18.1)

According to Leibniz's approach the functional double of a complex system could be based on *une parfaite Equation* between the whole cause and the whole action that could be obtained from the least action principle. This treatment provides a possibility to use geometric archetypes to reconstruct the acupuncture map of complex system being investigated.

Let us consider a vector field $A^\gamma(x_\mu)$ in Minkovsky's space with action

$$S = \int d^4A = \int d^4x det\, A^\gamma_\mu \tag{18.1}$$

It is known that each tensor can be represented in the form of superposition of its symmetric and antisymmetric parts

$$\begin{aligned} A^\gamma_\mu &= g^{\gamma\nu}A_{\mu\nu} = \frac{1}{2}g^{\gamma\nu}(F_{\mu\nu} + S_{\mu\nu}) \\ F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu \\ S_{\mu\nu} &= \partial_\mu A_\nu + \partial_\nu A_\mu \end{aligned}$$

Term $S_{\mu\nu}$ could be written in the form $S_{\mu\nu} = \Lambda g_{\mu\nu} + (S_{\mu\nu} - \Lambda g_{\mu\nu}) = \Lambda g_{\mu\nu} + D_{\mu\nu}$ where term Λ obeys the following condition $Sp\,(g^{\gamma\nu}(S_{\mu\nu} - \Lambda g_{\mu\nu}) = 0$ i.e. $\partial_\mu A^\mu + \partial^\mu A_\mu = 4\Lambda$ or $\Lambda = \frac{1}{2}\partial_\mu A^\mu$.

The equations of motion are obtained from the least action principle $\delta S = 0$:

$$\partial_\mu(\frac{\delta}{\delta A_{\mu\nu}} det\, A_{\alpha\beta}) = 0$$

on the surface of zero deviator

$$D_{\mu\nu} = 0 \tag{18.2}$$

In three-dimensional terms $A^\nu = (\varphi, A)$, $E = -\nabla\varphi - \partial_t A$, $B = rot A$ we have the following system:

$$\begin{aligned} div\, D &= -div\,(\Lambda[E \times B]) - \partial_t(\Lambda^3 + \Lambda B^2) = 4\pi\rho \\ rot\, H - \partial_t D &= -\partial_t(\Lambda[E \times B]) + \nabla(\Lambda^3 - \Lambda E^2) \\ &+ \partial_j(EE_j + BB_j) = 4\pi j \end{aligned}$$

where

$$\begin{aligned} D &= \Lambda^2 E + (B \cdot E)B \\ H &= \Lambda^2 B - (B \cdot E)E \end{aligned}$$

It is obvious that the equation of continuity is true, i.e.

$$\partial_t \rho + div\, j = 0$$

Equation 18.2 means that the observed forms do not have solid patterns indeed. This approach seems to be in a good agreement with Hegel's conception of formless matter which can not be perceived either by means of feelings or experimental tools. This also gives a hint to explain the eastern esoteric doctrines of the functional double of phenomenal reality. Its hidden properties reveal themselves to us in variable forms of empirical matter which is no more than a set of distorted manifestations of fundamental formless matter or self-consistent substance (causa sui).

Moslem mystics (Sufi) postulate that only one substance really exists, i.e. Universal Mind. It is a basis of "vahdati vujud" doctrine that treats nature as a divine emanation.

Bhagavati-Prajnyaparamita-hridaya-sutra proclaimed that a form is empty and emptiness is the form. Moreover feelings, discursive thoughts, energies, and consciousness are empty (i.e. formless). That is creative emptiness (vacuum) does not contain forms, definite matter, energies etc.

In other words to get over an illusion of empirical pseudo solid forms we have to get essence of Reality-Tathata as raw and rough qualities of things. These fundamental qualities represent formless nature of the form. In this way one could achieve the conception of shunyata-emptiness (nothing) which is the basic idea of Buddhism.

Hinayana prophets realized the great mystery of existence trough instability of appearing forms. This doctrine was revised by the great Buddhist mystic Nagarjuna. After the second turn of the Wheel of Dharma Nagarjuna developed madhyamika teaching to reject the false conception of existent things and forms. He revived a fundamental mysticism that aspires to perceive the invariable functional double of reality as formless essence of concrete world (Asis Nasafi).

The western scientific interpretation of this doctrine could be formulated as follows:

The functional essence of reality is an invariant that remains unchanged when specified transformation is applied.

Let us consider the condition $D_{\mu\nu} = 0$ in detail. According to elastity theory a deviator $D_{\mu\nu}$ (with zero spur) describes a shearing strain of a continuous medium. Those strains do not change the volume of that medium. We could say, in terms of elastity theory, that the shear modulus in this system is equal to zero. Thus the initial form is not conserved.

18.3 Particles of *Sunyata*

> That [undefinable essence] which can neither be extinguished . . . ; which neither can be annihilated, as e.g., all the [active] elements of our life, nor is it everlasting, as a non-relative [absolute principle]; which cannot really disappear nor can it be created; [that something] which consists in the Quiescence of all Plurality, that is Nirvana . . . This phenomenal world is imagined as existing in the sense that [its separate entities] are dependent upon a complex of causes and conditions, [they are relatively real] as, e.g. the long [is real] as far as there is something short . . .
>
> *Chandrakirti*

Let us assume that Λ is a nonzero constant. The new variables could be introduced as follows $E = \Lambda e, B = \Lambda b$. For these variables the system looks like

$$\begin{aligned} div\, d &= -div\,[e \times b] - \partial_t(1 + b^2) \\ rot\, h - \partial_t d &= -\partial_t[e \times b] + \nabla(1 - e^2) \\ &+ \partial_j(ee_j + bb_j) \end{aligned}$$

where

$$\begin{aligned} d &= e + (b \cdot e)b \\ h &= b - (b \cdot e)e \end{aligned}$$

the energy-momentum conservation equation is the following

$$\partial_t(\frac{e^2 + b^2 + (b \cdot e)^2}{2}) + div\,(d + [e \times b]) = 0$$

Let us consider for further simplicity the axial-symmetry case, when all variables do not depend on ϕ:

$$\begin{aligned} -\partial_t(b_z^2 + b_\rho^2 + b_\phi^2) &= \partial_z(e_z + (b \cdot e)b_z + [e \times b]_z) + \partial_\rho(e_\rho + (b \cdot e)b_\rho + [e \times b]_\rho) \\ &+ \frac{e_\rho + (b \cdot e)b_\rho + [e \times b]_\rho}{\rho} \\ \partial_t(d_z - [e \times b]_z) &= \partial_z(e_\rho^2 + e_\phi^2 - b_z^2) + \partial_\rho(h_\phi - (e_\rho e_z + b_\rho b_z)) \\ &+ \frac{h_\phi - (e_\rho e_z + b_\rho b_z)}{\rho} \\ \partial_t(d_\rho - [e \times b]_\rho) &= -\partial_z(h_\phi + (e_\rho e_z + b_\rho b_z)) + \partial_\rho(e_\rho^2 + e_z^2 + e_\phi^2) - \partial_\rho(e_\rho^2 + b_\rho^2) \\ &- \frac{e_\rho^2 + b_\rho^2}{\rho} + \frac{e_\phi^2 + b_\phi^2}{\rho} \end{aligned}$$

$$\partial_t(d_\phi - [e\times b]_\phi) = \partial_z h_\rho - \partial_\rho h_z - \partial_z(e_\phi e_z + b_\phi b_z) - \partial_\rho(e_\rho e_\phi + b_\rho b_\phi) - \frac{e_\rho e_\phi + b_\rho b_\phi}{\rho}$$

where

$$\begin{aligned}
e_z &= -\Phi_z - \partial_t A_z\\
e_\rho &= -\Phi_\rho - \partial_t A_\rho\\
e_\phi &= -\partial_t A_\phi\\
b_z &= \partial_\rho A_\phi + \frac{A_\phi}{\rho}\\
b_\rho &= -\partial_z A_\phi\\
b_\phi &= \partial_z A_\rho - \partial_\rho A_z\\
[e\times b]_z &= e_\rho b_\phi - e_\phi b_\rho\\
[e\times b]_\rho &= e_\phi b_z - e_z b_\phi\\
[e\times b]_\phi &= e_z b_\rho - e_\rho b_z\\
(b\cdot e) &= e_z b_z + e_\rho b_\rho + e_\phi b_\phi
\end{aligned}$$

Note that if $A_\phi = 0$ then $b_z, b_\rho, e_\phi = 0$ as well as $(b\cdot e) = 0$.
Let us accept the condition $A_\phi = 0$ and mean $\beta = b_\phi$. Thus we get simplified system

$$\begin{aligned}
-\partial_t\beta^2 &= \partial_z(e_z + \beta e_\rho) + \partial_\rho(e_\rho - \beta e_z) + \frac{(e_\rho - \beta e_z)}{\rho}\\
\partial_t(e_z - \beta e_\rho) &= +\partial_z e_\rho^2 + \partial_\rho(\beta - e_z e_\rho) + \frac{\beta - e_z e_\rho}{\rho}\\
\partial_t(e_\rho + \beta e_z) &= -\partial_z(\beta + e_z e_\rho) + \partial_\rho(e_z^2) + \frac{\beta^2 - e_\rho^2}{\rho}
\end{aligned} \tag{18.3}$$

The two last equation of the system above could be rewritten as follows

$$\begin{aligned}
\partial_t e_z - \beta\partial_t e_\rho &= \partial_\rho\beta + \frac{\beta - e_z e_\rho}{\rho} - (e_z\partial_\rho - e_\rho\partial_z)e_\rho\\
\partial_t e_\rho + \beta\partial_t e_z &= -\partial_z\beta + \frac{\beta^2 - e_\rho^2}{\rho} + (e_z\partial_\rho - e_\rho\partial_z)e_z
\end{aligned}$$

The system could be further simplified if one suppose the sphere symmetry with zero angle components. In this way the system can be reduced as follows:

$$\begin{aligned}
\partial_t(\frac{E^2}{2} - B^2) &= \partial_r E + \frac{E}{r}(2 + B^2 - 2E^2)\\
0 &= \partial_r B + \frac{B}{r}(1 + B^2 - 2E^2)
\end{aligned}$$

The static possible solutions of the last system are

$$
\begin{aligned}
E &= \frac{1}{\sqrt{1-(k\cdot r)^2+\sigma(k\cdot r)^4}} \\
B &= \frac{(k\cdot r)}{\sqrt{1-(k\cdot r)^2+\sigma(k\cdot r)^4}}
\end{aligned}
\tag{18.4}
$$

where E and B are the radial components of electric and magnetic fields respectively; r is the sphere radius; k and σ are some positive constants.

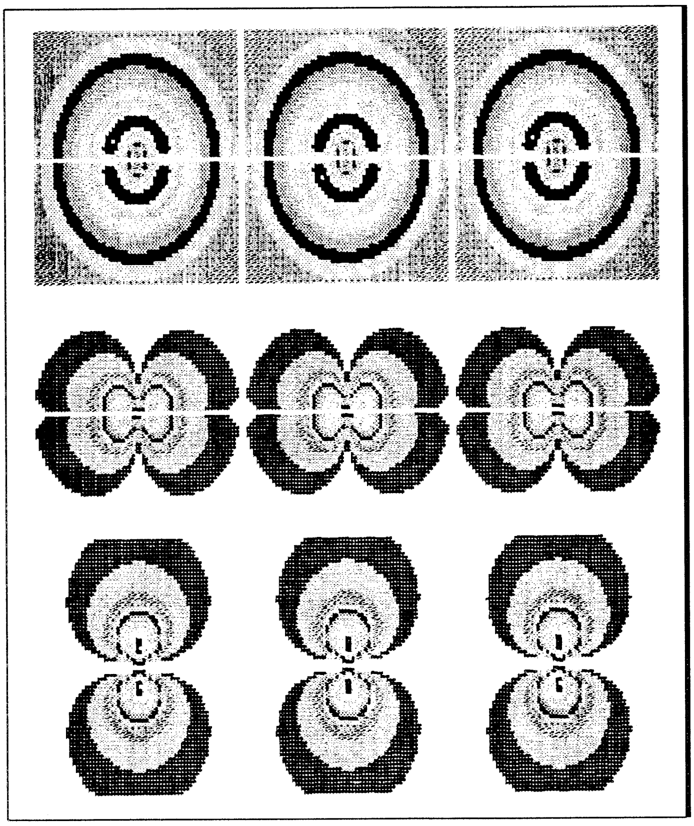

Figure 18.1: Stability of spherical solution

Numerical experiments with Equations 18.3 show that Solution 18.4 is stable (see Figure 18.1).

18.4 Conclusions

> Empirical consciousness (vijnanena) apprehends [separate] objects ... Transcendental knowledge (jnanena) should be a knowledge of universal Relativity (sunyata-alambana) ... Indeed the essence (rupa=svarupa) of absolute knowledge is such that it escapes every formulation (sarva-prapanca-atita) ... No one can realize it, consequently it is [logically] impossible (na yujgate).
>
> *Chandrakirti*

Analyzing philosophical aspects of human knowledge Werner Heisenberg formulated the main *gnostic* problem as follows:

> *If we try to explain reality we need to reduce it to a single principle, but it is not clear whether one can deduce empirical manifold from this principle*
>
> .

Proposed theoretical construction could be considered as an illustration of possible ways to perform deduction of variable phenomenal forms from a set of basic principles. These fundamental principles appear to be a formal representation of true formless empty reality — sunyata.

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Chapter 19

Farkhad A. Niyazov: Modern Aspects of Kondratiev's Cycle Theory

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Abstract

This paper deals not only with the analysis of Kondratiev's long waves, but also with research into the nature of economic and social processes, objective laws and regularities of the development of human association, as a complex dynamic system. In spite of the fact that there are too many models, a large blank exists in understanding the essence of economic processes.

A historical review of Kondratiev's ideas on long waves in the economy and "the latest" components of Kondratiev's cycles are analyzed, based on USA data. 2.5 Kondratiev's cycles are added together to make up four complete ones.

The paper also deals with the superimposition of events and political cycles on long waves using USA data, and an hypothesis on critical points in system development processes is offered. The psychological aspect of the big cycles theory is also considered. A structure of employment of the USA is given from the point of view of the long cycles thery.

The development of Kondratiev's ideas on market equilibrium in terms of models of classical mechanics and thermodynamics is presented. Principles of classical physics based on the notion of reversibility and closed systems are insufficient. We introduce the new term *ecoentropy*. Market equlibrium models are presented as a set of logistic equations.

The methodological and ideological base for building our big computing simulation model of cyclic mechanisms in socio-economic systems and for its "rough tune" is the main result of this paper.

19.1 Introduction

A lot of mathematical models of economy cyclic development have appeared recently. But usually they deal with the consequences of the fundamentals being the essence of the economical dynamics, not touching on the main concepts of the motion. In this paper we'll try to consider this essence making a modern analysis of the old problems. One of the pioneers in the field of economic dynamics and its relation with the natural science was N.D. Kondratiev, who discovered long waves. In 1926 N.D. Kondratiev read his well-known report [Kond 26] in Moscow. Analyzing numerous indices (per capita as a rule) of England, France, Germany and the USA for the period of 1780–1920 he made a supposition of a hypothetical equilibrium trend motion of these indices having called the motion a secular one. The method of cycles revealing was to find and study real values of deviations (fluctuation) from the secular motion. 9-points smoothing of the rows obtained is made for eliminating short cycles. Thus N.D. Kondratiev revealed 2.5 large cycles with 40–60 years period (see table 19.1). It should be noted that the terms "raising wave" and "falling wave" are taken from Kondratiev.

Cycles	Raising wave	Falling wave
I.	1786–93 to 1810–17	1810–17 to 1844–51
II.	1844–55 to 1870–75	1870–75 to 1890–96
III.	1891–96 to 1914–20	1914–20 to ?

Table 19.1: Kondratiev's large waves

His opponents denied the secular motion. Afterwards the affirmation of both equilibrium secular motion and fluctuating processes of capitalist indices availability had been incriminated to Kondratiev as the capitalism apology. In 1987 N.D. Kondratiev was rehabilitated posthumously. But his ideas are still considered to be bourgeois. For example it is written that the interest to N.D. Kondratiev is related with the incapability to explain the crisis of the seventies by the traditional trends in bourgeois political economy [Izum 88]. It seems to us that the interest in long waves is also caused by the modern physics tendencies that reached their peak in the same seventies (H. Haken, I. Prigogine).

In the present work several models using the USA data are suggested, each of them shows definite point of view at the cycles. Revealing special critical points where internal (structural) changes take place with great speed is common for them. Let's divide these models into two conventional groups:

1. structural models of economy,

2. self-organization models.

The former includes the description of all possible structural sections of economy as well as simulation models with a lot of parameters and variables where the particular exact solutions are determined. The latter includes the models where internal mechanisms of systems regulation and self-regulation are revealed. As a rule they are simple models with a small number of variables and parameters. Interaction of these variables and change of parameters are studied qualitatively there. Such models are required to define general properties of the system and to predict its dynamic behaviour when the conditions of its functioning are changed.

Our main task at this stage is to make an adequate description of the economy corresponding to our ideas of cyclicity and critical points existence, to reveal main parameters and variables of the socio-economic systems of the state with market relations and to obtain a methodology of the models construction with a reasonable interpretation.

19.2 Kondratiev's Method and Modern Analysis

Secular motion existence and its physical essence will be discussed in the third paragraph. When revealing large cycles statistical analysis is of subsidiary nature because of insufficient length of indices rows, their break up and incomplete reliability. Therefore simple procedures (see [Menshi 89, p. 65]) as for instance Kondratiev's method are more suitable. So the existence of large cycles won't be proved on the base of row processing but like Kondratiev we'll consider the probability of large cycles existence not to be 100 % but high enough, and illustrate the long waves existence in the XXth century and calculate their approximate parameters. Before dealing with the figures let's note the following on the secular motion finding:

- as a rule a visual research of a 9-points smoothing of a row and a "rough" smoothing by the method of least squares with the polynomials of the 1-st, 2-nd or 3-rd degree have been prelimenarily done (see also fig. 19.1–19.16). If long waves were found more accurate smoothing was done by the same method with the Chebyshev polynomials of the degrees from 2 to 4 and the remainder of it was found.When doing it we tried not to introduce the unnecessary wave visually correcting the remainders with the "rough" ones. The example of the initial row and smoothing with Chebyshev polynomial is shown in fig. 19.5 below and the chart of the deviation from the secular motion is given above;
- charts 19.13–19.16 are taken from [Menshi 89]. Moving averages of the logarithmic indices are shown on the charts 19.13, 19.14, and 19.16; the waves are sought as the deviation from the straight line of these logarithms;
- when 9-points smoothing, the ends of the rows obtained are lost. These ends are given on the charts smoothed by the special methods of ends smoothing. It doesn't influence on the long waves finding;
- as a rule the charts are presented in different scales but in a manner to mark the curves' similarity.

The calculations are made on the USA data as the most readily available in the open press [Stat], though in this case earlier indices are cut off (they are presented mainly on England). All the rows are divided into the following groups:

1. purely value indices;
2. mixed value-natural indices;

3. purely natural indices;
4. labour indices and others.

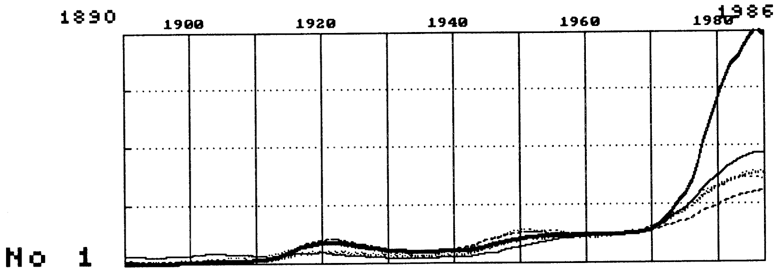

Figure 19.1: Wholesale price indices (1967=100) of farm, food, textile, metal products, and fuel in one scale

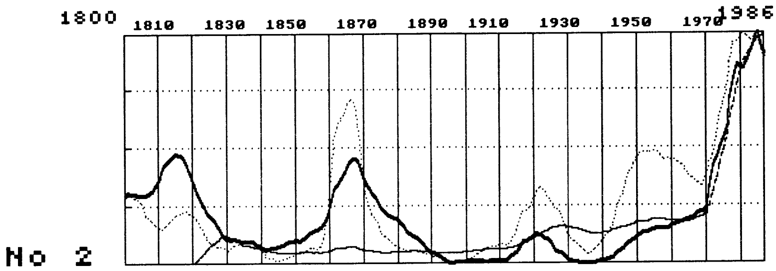

Figure 19.2: Wholesale prices of sugar, cotton, coal (anthracite) in different scales

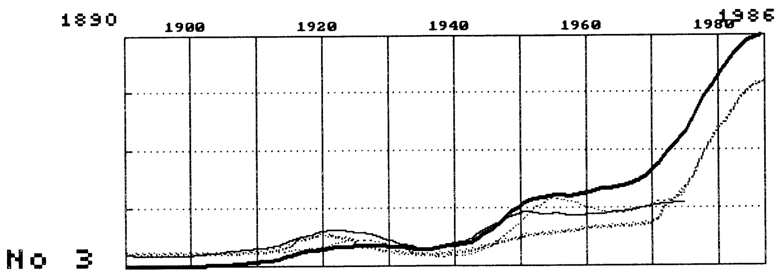

Figure 19.3: Retail proces for meat, butter, coffee, and sugar in one scale

Group I. Kondratiev noted that prices and their indices have the tendency to fluctuate near the constant value and so they can be observed visually without finding the secular motion. In general, such a pattern is observed in the 20-th century as well, though the prices have risen. Fig. 19.1–19.4 show 9-points smoothed indices: fig. 19.1 presents wholesale price indixes (1967= 100 %) of farm, food, textile, metal products and fuels for 1890–1986 in one scale; fig. 19.2 — wholesale prices of sugar, cotton, coal(anthracite) for 1800–1986 in different scales; fig. 19.3 — retail prices for meat, butter, coffee and sugar for 1890–1986 in one scale; fig. 19.4 — average values

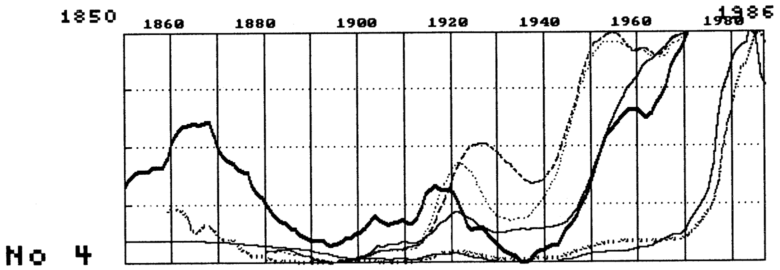

Figure 19.4: Average values for bituminous coal, Pennsylvania anthracite, iron ore (shipments), crude oil (at well), copper and silver (both in New York) in different scales

for bituminous coal, Pensylvania anthracite, iron ore (shipments), crude oil (at well), copper and silver (both in New York) for 1850–1986 in different scales. These charts agree well with the table 19.1. After 1920 the peak of maximum is 1980–83 and that of minimum is 1934–40. Since 1980–83 the falling wave has started. Small peaks are observed in $\approx$ 1955. One can see that the feature of 1860, 1920, 1980 peaks is independency. But 1955 is on the slope of a raising wave. The only exception is the cotton price (fig. 19.2), but it is difficult to interpret it. Below we'll call the waves of the charts 19.1–19.4 as P-waves (Price-waves).

When analysing other indices of prices we observe long waves, though many prices have sharp peaks in 1950–55. It makes these waves look like 30-years ones.

Group 2. Natural indices in terms of money, value indices in natural or percentage terms are involved in this group.

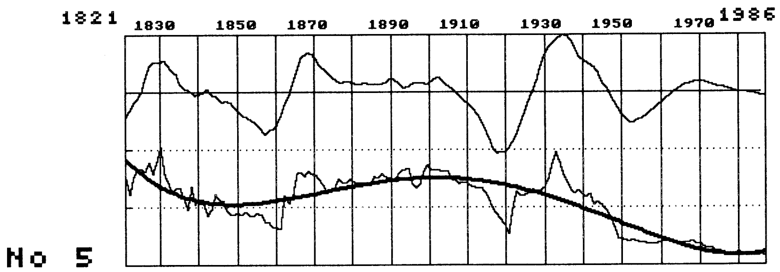

Figure 19.5: The ratio of duties

Chart 19.5 gives the ratio of duties from 1821 to 1986. Let us consider it visually relatively to the horizontal line. 1861, 1920, 1980 are minima and 1830, 1900, 1934 (and 1970 if smoothed with polynomials) are maxima. The import policy is sure to be related with the prices and the wave basically agrees with the tables 19.1–19.2 and P-waves (with the opposite sign).

Chart 19.6 represents the average liability per failure within 1857–1986. Visual examining shows the peaks availability (not on the slope) in 1873, 1920, 1982, that agrees with P-waves. Smoothing with the polynomials of the 4-th degree was the only one with such a high degree. But it resulted in an interesting picture: extremum points of the P-waves coincided with intersection points of the wave and the trend (plus a short peak down 1940). But if up to 1940 the wave advanced the P-wave by a quarter

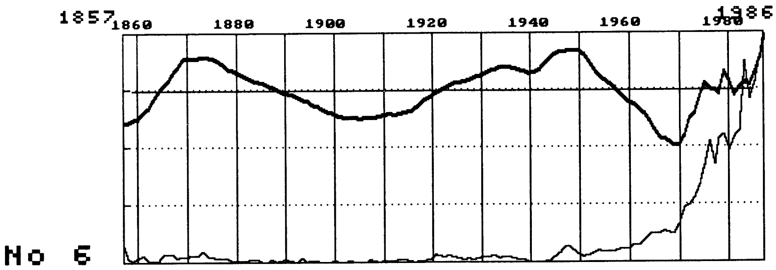

Figure 19.6: The average liability per failure

of a period, then after 1940 it lagged by the same quarter. It gives the impression that the model switching took place in these years.

Sometimes indices show 90–120 years waves, e.g. the export-import difference (1) gold; (2) silver.

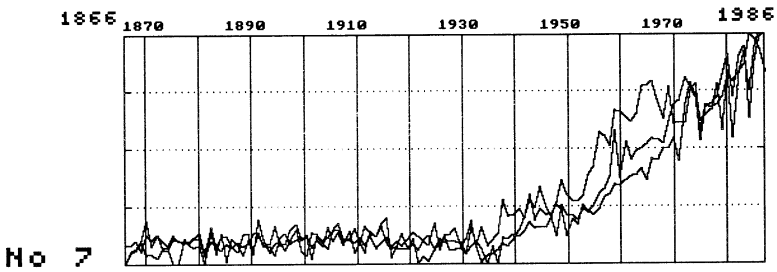

Figure 19.7: Corn, wheat, and cotton crop yields in different scales

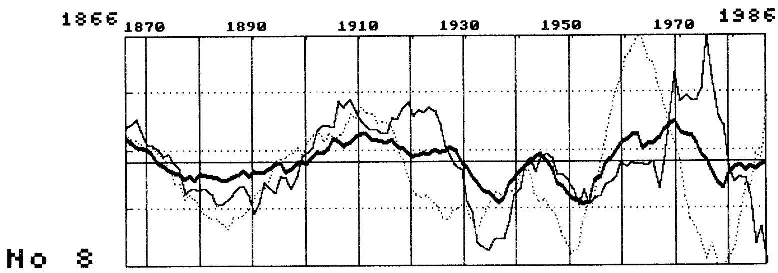

Figure 19.8: The remainders of indices in fig. 19.7

Group 3. Chart 19.7 represents in different scales corn, wheat and cotton crop yields in 1866–1986. Visual inspection of the 9-points smoothing (not shown here) reveals long waves. They are clearly seen on chart 19.8 of the remainders from Chebyshev polynomial. These waves as if precede P-waves.

Chart 19.9 represents in different scales the number of banks, national banks and insurance companies per capita in 1790–1986. Visual inspection shows waves availability with the peaks in ≈ 1794, 1870, 1920, 1960.

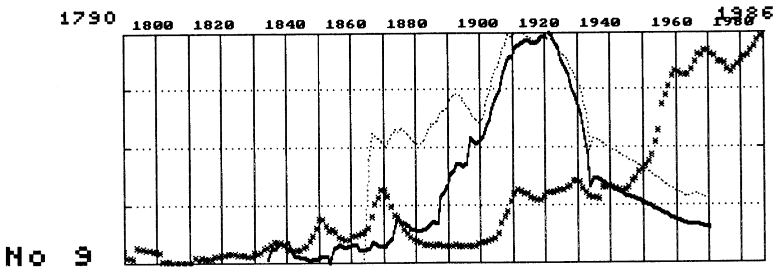

Figure 19.9: The number of banks, national banks, and insurance companies per capita in different scales

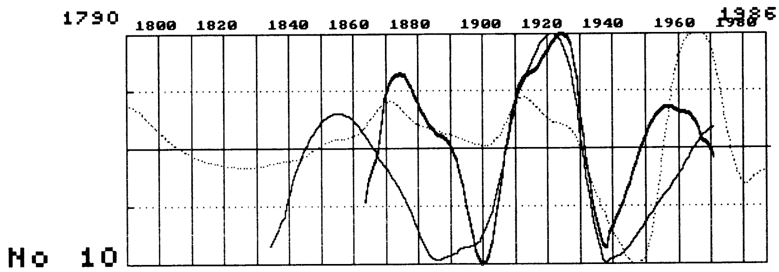

Figure 19.10: The remainder of indices in fig. 19.9

Chart 19.10 of the remainders shows maximum points 1790–93, 1857–70, 1912–20, 1960–70 and minimum peaks — 1810, 1885–1900, 1937–45. These indices symbolize business activity; an agreement with P-waves (or a small lagging) is seen.

Chart 19.11 represents in different scales the number of patent issues in 1790–1986 (dotted line: per capita). Long waves are seen visually and on the chart 19.12 of the remainders we see maximum points: 1812–20, 1887, 1930, 1970–74; minimum points: 1850–60, 1897–1900, 1946–48, 1980–83. These waves precede P-waves.

When analysing other indices 30-years waves are perceptible (as in some prices), e.g. coal, oil, iron ore production.

Group 4. This group includes the indices related with the labour and capital from 1889 to 1987 (1893–1983 on the charts). Gross private internal product (GPIP) is taken as the base of periodization [Menshi 89]; see chart 19.14, curve 3, and the

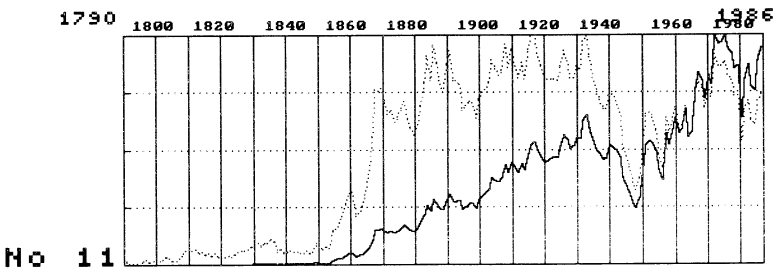

Figure 19.11: The number of patent issues in different scales

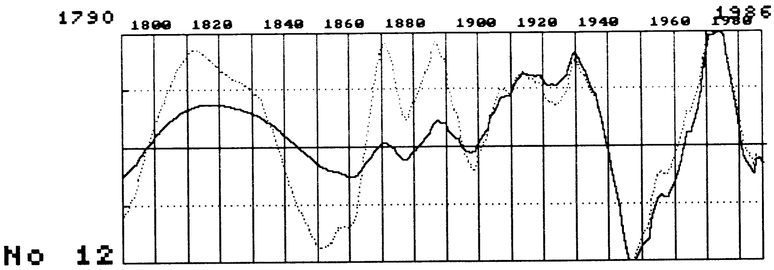

Figure 19.12: The remainder of indices in fig. 19.11

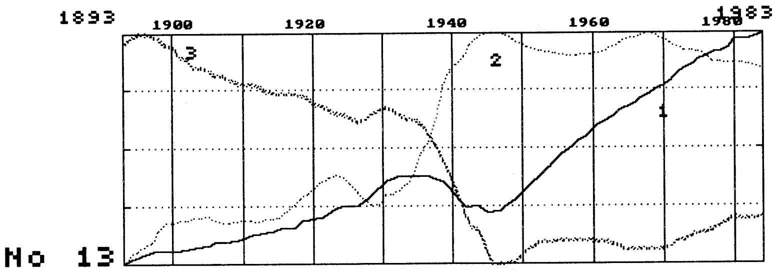

Figure 19.13: Capital availability per worker (1), capital productivity (2), organic structure of capital (3)

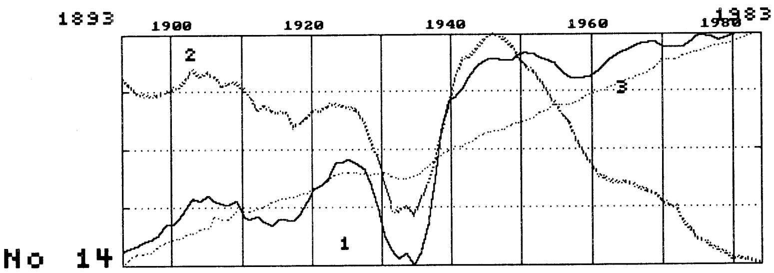

Figure 19.14: Rate of profit (1), profit per man-hour (2), GPIP (3)

deviation in chart 19.15, curve 6: 1893–98 — restoration; 1898–1924 — rise; 1924–38 — a great crisis; 1938–52 — restoration; 1952–74 — a long rise; 1974–8.. — a large crisis (uncompleted). As one can see agreement with P-waves is observed. In respect to that periodization the following indices are considered:

- chart 19.16 below (deviation is above) — labour productivity. Preceding the GPIP-wave in a favourable period it coincides and even lags in crisis;
- chart 19.13, curve 1 (deviation in chart 19.15, curve 1) — capital availability per worker. It precedes the GPIP-wave by 20–30 years;
- chart 19.13, curve 2 (deviation in chart 19.15, curve 2) — capital productivity.

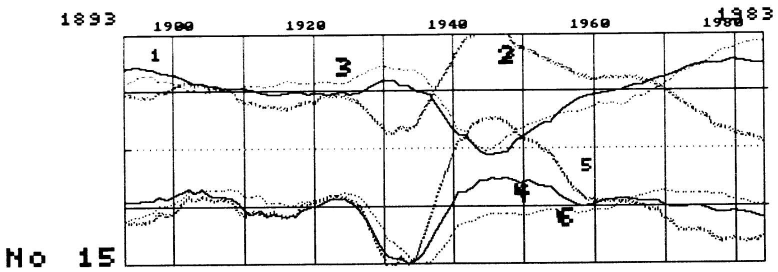

Figure 19.15: Deviations of capital availability per worker (1), capital productivity (2), organic structure of capital (3), rate of profit (4), profit of man-hour (5), GPIP (6)

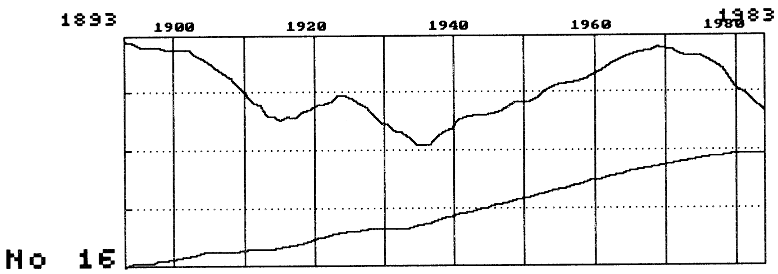

Figure 19.16: Labour productivity

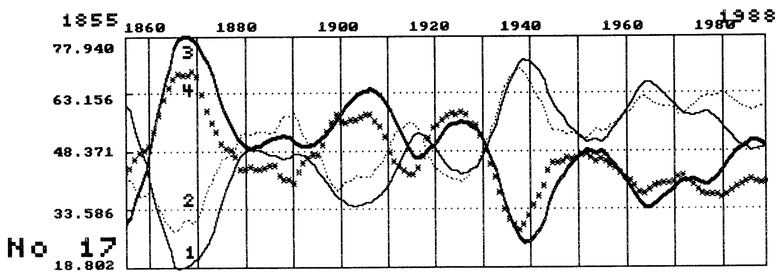

Figure 19.17: Percentage of Democrats and Republicans in the Senate and the House

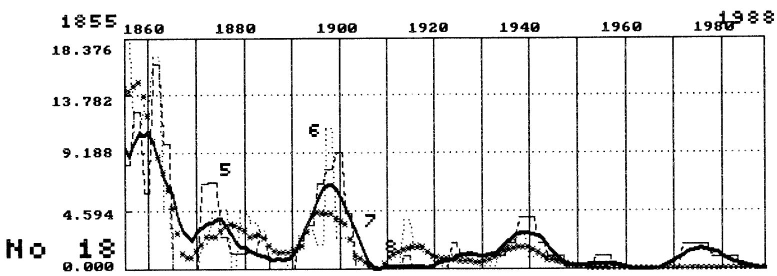

Figure 19.18: Percentage of other parties and independents in the Senate and the House

It precedes the GPIP-wave by 12–24 years;

- chart 19.13, curve 3 (deviation in chart 19.15, curve 3) — organic structure of the capital. It repeats the motion of the capital productivity;

- chart 19.14, curve 2 (deviation in chart 19.15, curve 5) — profit per man-hour. When rising it precedes the GPIP-wave by 15–20 years. In crisis the gap narrows;
- chart 19.14, curve 1 (deviation in chart 19.15, curve 4) — the rate of profit. It is similar to profit per man-hour.

Thus it can be summed up that by means of Kondratiev's method long waves in the USA are clearly observed and they are often seen even visually. Because of the lack of space a detailed analysis of the charts is not given. Taking P-waves as the base and using synchronism of all the waves we can present table 19.2 of large cycles periodization according to Kondratiev, which is a continuation of table 19.1. Table 19.2 agrees with the similar periodization of other authors and in particular ([Menshi 89],[Mensch 79]).

Cycles	Raising wave	Falling wave
III.	. . .	1914–20 to 1935–40
IV.	1940–48 to 1965–70	1970–75 to 1988–90
V.	1990–95 to ?	. . .

Table 19.2: An approach to continue Kondratiev's large waves

In economical analysis published there appears the information on coming crisis in the USA in 1991–93. According to table 19.2 it may be the continuation of the last falling wave. If we consider P-waves only, the rising wave IV lasts from 1940–44 to 1980–82 and then there is a recession.

In general in 1929–70 the following things are observed in comparison with the previous years: (a) some asyncronism in long-waves of different indices; (b) transition of long waves of some indices into 30-years waves (peaks in 1950-s). This is either (1) 1929 great crisis results or (2) World War II results or (3) results of the governmental anticrisis policy or (4) all taken together. Perhaps all these observations have caused the oblivion of the long waves theory but in recent years these anyncronisms have been vanished and the theory revived. If the USA statistical data are considered as "the results of a model" then the "switching" of this "model" is seen just after 30-s.

Kondratiev came close to the employment of the production function (PF) in the analysis of the economic dynamics, the trend problem, and the scientific and technological progress (STP). Using PF apparatus in long waves theory is the subject of our independent work. We would like to note the influence of Yu.P. Ivanilov's works on PF.

19.3 Critical Points, Historical Waves and STP

If the connection of economy internal processes with the wavy motion is considered, it is quite real to suppose the existence of the turning points where the processes preparing the change of the motion take place. These points are naturally supposed to be in the vicinity of the points of the wave extremum and in the vicinity of the points of inflection (intersection of the series chart with the secular motion). These points are called critical ones. On the base of tables 19.1 and 19.2 chart 19.19 is plotted. For the present the points of inflection can be calculated approximately as the middle of the rise and fall periods ± several (1–3) years — they are 1805.5, 1830.5, 1862.5, 1882.5, 1905, 1927.5, 1956.5, 1981.5. These points can be defined more exactly from the analysis of the charts.

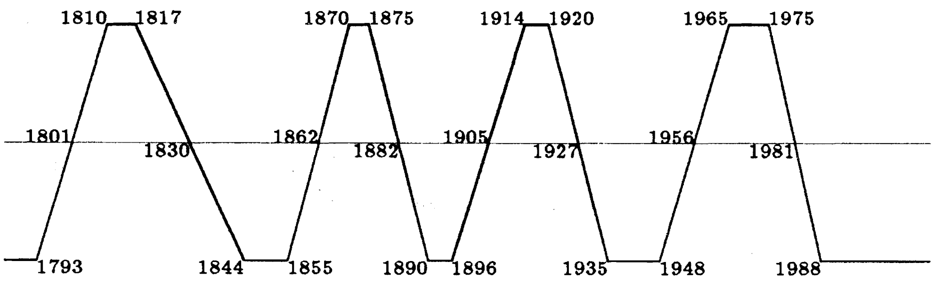

Figure 19.19: The long waves in the USA

We superposed USA historical events over the long waves [Niyaz 92]. The historical dates are written out from [Modern 88], having divided them into the following groups: (1) wars and interventions; (2) revolts and strikes; (3) important political events; (4) important economic events. Sometimes division into items 3 and 4 is conditional. Worldwide events are added to this list: 1990 conflict and 1991 war with Iraq. The classical statement of the cyclic recurrence is the fact that a majority of important social shocks comes in periods of rising waves. We made up table 19.3 in which all events are divided into the periods of the above-revealed cycles; transition periods being separated between the periods (doubled dates from table 2 and 3 are taken as their boundaries). Dates close to transition periods (from 1 to 3 years) are marked with "*", worldwide events are marked with "#", the dates from the inflection points — with "^" (see details and chronology in [Niyaz 92].

The following picture is opened up:

1. really, more events come to the periods of rising than those of falling;
2. main events (e.g. both world wars) come to the transition periods;
3. from the events coming to rising and falling general amount takes place in the years close to transition ones or inflection points. Thus in addition to Kondratiev's idea the following supposition suggests itself:

As was expressed in [Kond 26], more wars, social shocks come to the periods of large cycles rise than that of fall. But main events come to the critical transition years between the periods of rising and falling, falling and rising. Besides most of such events inside the periods of fall and rise are quite close to critical transition years (1–3 years) or to the inflection points.

Political cycles make up about 25–30 years [Schles 86]. They will be considered from 1855 to 1988. The figures in chart 19.17 designate: 1,2 — smoothed by 9-points percentage of democrats in the Senate and the House of Representatives respectively; 3,4 — the same with republicans. The fluctuations around 50 % mark are clearly distinguished. Years of intersection in the given mark are the change of power in one of

cycles	wars & interven-tions	revolts & strikes	import. politic. events	import. economic events
transition	# 1775–83	1786–87	# 1787 # 1789	1791
I-raise		* 1794 ˆ 1799	* 1807	
transition	# 1812–14			
I-fall		ˆ 1831 ˆ 1832 * 1842	* 1818–21 1820 # 1823 * 1841	ˆ 1828 * 1841
transition	# 1846–48		1845 1846	1848
II-raise	# ˆ 1861–65	ˆ 1859	1853 ˆ 1862 1865	ˆ 1862 *1868
transition		1874–75		
II-fall		1886		
transition		1892 1894	# 1895	1890
III-raise	# * 1898 # * 1899–01	1902 * 1912 * 1913		* 1913 * 1913
transition	# 1914–18 # 1915 # 1916–17 # 1916–24 # 1918–20	1914 1919 1919		1914
III-fall		1922 * 1932 * 1934	ˆ 1927 # * 1933–38	# ˆ 1929–33 # * 1933–38
transition	# 1939–45	1946	# 1945 # 1947 1947	1935 1935 1947
IV-raise	# * 1950–53	* 1949	# * 1949	ˆ 1957
	* 1962	1952 * 1963	* 1950 1953–54 ˆ 1955 ˆ 1957	

Table 19.3: Events during long waves (see text on p. 281)

transition	# 1965–73 # 1965–66	1964 1967 1968 1968–69 1969–70 1970 1971 1974	# 1969 1974	
IV-fall	# 1983	* 1976 * 1977	# ˆ 1981	ˆ 1981
transition	# * 1990 # * 1991			

Table 19.3: Events during long waves (see text on p. 281) (Continued)

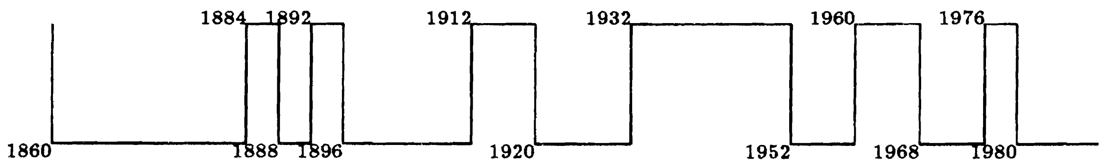

Figure 19.20: Dynamics of party change in presidential power

the congress chambers. They are 1857–60, 1875–82, 1889–96, 1911–18, 1931–32, 1946–54, 1981–88. A clearly marked wave in 1968–74 that unreached 50 % is added here. Look at tables 19.1–19.3. It is clearly seen that marked dates either enter into critical years or close to them. The activity of other parties may be supposed to increase in critical years that is visually confirmed by chart 19.18 (where 5,6 — percentage of other parties and independents representatives in the Senate and the House of Representatives; 7,8 — their meanings smoothed by 9-points). Peaks of maxima come approximately to the above-mentioned dates. And at last, having considered the dynamics of parties change in presidential power for democrats in chart 19.20 (the republicans have the opposite picture) we see that years of supreme power change 1860, 1884–96, 1912–20, 1932, 1952, 1960, 1968, 1976–80 are close either to critical or inflection years.

Even if we assume the conditionality (approximateness) of the numbers in tables 19.1–19.2, it will only set up the "fuzziness" of exact arrangement of critical points, but not refute their availability and concentration of historic events around them. Cyclic recurrence of the power change does not mean yet the cyclic recurrence of the development and that of economy, and doesn't directly lead to them (there are some other factors e.g. political "games" with electors, big business etc.).

STP and long waves are considered. Fig. 19.11–19.12 give a number of patent issues and their wave. Extremum region is given from chart 19.19 (tables 19.1–19.2), and the region of the extremum of the same name is given in parentheses from chart 19.12: 1810–17 (1812–20), 1844–55 (1850–60), 1870–75 (1870–87), 1890–96 (1896–99), 1914–20 (1930), 1935–48 (1945–47), 1965–75(1970–75). STP-waves are seen to coincide with those of Kondratiev, sometimes preceding them slightly. Kondratiev also noted that a larger part of important inventions and discoveries come to the period before the rising wave. It is a subjective statement and it won't be considered here. We keep to and study the models based on diffusion of technology and on cumulation of innovations.

Let's note about PF. A new method of revealing the technological changes by means of PF analysis is proposed in [Voron 88]. The following years of technological changes in the USA within 1901–1960 have been revealed: 1907, 1920, 1932, 1937, 1945, 1949, 1957. All these points are critical by our definition.

It should be noted that Kondratiev's "empiric regularities" are more or less confirmed (the definitions are modified by us):

1. the qualitative wave on invention coincides with that of Kondratiev's;

2. the greatest amount of important social shocks comes to the periods of risings and besides to the critical points defined above;

3. falling waves are accompanied by the long depression in agriculture. It is seen from charts 19.7–19.8 of grain and cotton yields. These waves coincide with the Kondratiev's ones. In addition we studied an effect of (1) yielding on industry; (2) weather (4 stations data: Blue-Hill, Dickinson, Calhoun, Logan) and solar activity on the yield. Direct relation is not observed but complicated dependence is traced, particularly up to 1930.

4. average cycles (20–30 years) coming to the falling period of a large cycle must be characterized by the peculiar length and depth of the depression, shortness and weakness of the rise. And those coming to the rising periods are characterized by the opposite features. Sharply outlined peaks of 1950 (marked above) are exactly the rising average.

In conclusion Kondratiev's words are cited: (a) regularities permit exceptions; (b) large cycles are not explained but characterized here.

19.4 Equilibrium and Nonequilibrium

The development of physics has always defined the notion on the nature of phenomena in various fields of science. The first serious analogies of physics in economy started with classical mechanics. Economics considers statics and dynamics. If statics is a "snapshot photography" of the current state then dynamics is defined as the motion, as transition from one state of equilibrium into another. Kondratiev's collaborator T.I. Rainov compared the nature of mechanical equilibrium in classical mechanics and that of economic equilibrium [Rain 27]. He introduced the notion of analogy: if the phenomena

$$A = \{a_1, a_2, \ldots, a_n, \ldots\} \textit{ and } B = \{b_1, b_2, \ldots, b_n, \ldots\}$$

are given then A and B are similar if between a_i and a_j there are the same relations as between b_i and b_j . The analogy is considered not simply as the similarity-difference of mathematical representation methods of economic or mechanical equilibrium but as the identity-difference of the real structure, real form of both phenomena. T.I. Rainov concluded that there was no analogy. He studied market equilibrium as the variational problem [Rain 28]. T.I. Rainov considered variational principles of Euler-Lagrange, Helmholtz, Bertran, Thompson and in economy — the principle of maximum satisfaction (effect = expenses/output → min). There is no such notion as expenses and effect in varionational principles so there is no analogy as well. However,if market is considered not from the "value" point of view but that of "natural" one then the motion of goods has the analogue of mechanical equilibrium. In the works of N.D. Kondratiev, V.A. Bazarov, S.A. Pervushin in addition to mechanical models the analogues of thermodynamical models were considered. But Kondratiev noted their being insufficient and first of all their isolation and reversibility.From the contemporary works [Abram 91] is the latest. It compares modern optimization market models and production planning with classical mechanics and thermodynamics. Quite subtle analogies in mathematical expression between economical equilibrium and mechanical, thermodynamical ones are obtained. But N.D. Kondratiev's principal objections can't be eliminated within this class of models. Modern physics comes to assist, i. e. thermodynamics ideas of irreversible processes and self-organization. However Kondratiev's objections remain valid.

Modern science gave not only new ideas and methods but the understanding of classical postulates as well. In this respect [Leon 68] in the light of system analysis gave the idea on complete identity of representation form of all the equilibrium interaction irrespective of their class. This led to the "classification" of basic thermodynamic parameters of the reversible processes. Peculiarities appear only in the connection with irreversibility. According to this classification ($i = 1, \ldots, I$):

1. Q_i — amount of action in power units. Interaction may be mechanical deformation, electrical,chemical and phase tranformation, thermal etc. (number I is the freedom level of the system);

2. system state coordinates x_i (they change when having interaction i and are constant in its absence) are volume, charge, mass, entropy respectively;

3. P_i potentials (when they are equal in the system: P_i^i and in the environment: P_i^e, interaction of the given type is not carried out) are minus pressure, electrical potential, chemical potential, temperature respectively. Any of these potentials is the single-valued coordinate function

$$P_i = P_i(x_1, \ldots, x_I), \quad i = 1, \ldots, I.$$

The condition of equilibrium state stability is written as

$$(\frac{\partial P_i}{\partial x_i})_{Xinv} > 0 \ \ or \ \ (\frac{\partial P_i}{\partial x_i})_{Pinv} > 0$$

where inv is invariance. The condition

$$dQ_i = P_i dx_i$$

is satisfied for all i, it being established only on the base of experiment. Change of internal energy dU (where $U = U(x_1, \ldots, x_I)$) is written as

$$dU = \sum_i P_i dx_i$$

and the connection among energy, work A and interactions as

$$dU = \sum_i dQ_i = -dA = -\sum_i dA_i$$

Generalized force X_i is written as $P_i = -X_i$.

If the potential difference $P_i^e - P_i^i = \Delta P_i$ is small ($\frac{\Delta P_i}{P_i} \ll 1$), then such interactions are called equilibrium, otherwise — non-equilibrium.

Equilibrium interactions are characterized as follows:

1. only the coordinate x_i corresponding to the given interaction changes;
2. as $\mid \Delta P_i \mid \ll 1$ it is possible to speak on the existence of the potential uniform field in the system.

Nonequilibrium interactions are characterized by the following:

1. new degrees of freedom appear: $dQ_i = 0, dx_i \neq 0$; but new effects are not observed at non-equilibrium heat exchange;
2. there is a non-uniform field of potential in the system;
3. non-equilibrium heat exchange in the isolated system is always accompanied with the entropy increase;
4. conversion of other forms of energy into heat is observed.

Classical thermodynamics considers quasi-static processes i.e. proceeding under the action of $\mid \Delta P \mid \ll 1$ with the infinitesimal speed and representing the continuous change of equilibrium states. They possess the reversibility property. When being isolated the process instantly stops and the system turns out in the equilibrium state.

Modern physics studies non-static processes i.e. proceeding in the system as the result of non-equilibrium interactions. They are characterized by the following:

1. there is no single-valued correspondence between the interaction number and that of coordinates. Additional effects emerge. Heat release and entropy increase are obligatory even in the heat-insulating system;

2. thermodynamic parameters of the system are not the same in its various parts and the state of the system at each moment t is non-equilibrium;

3. non-static processes are irreversible.

The potential difference being maintained, non-equilibrium-state regime acquires the ability to self-organization: positive feedback mechanism can assimilate and even strengthen small (random) local deviations.

Before considering complex models it is necessary to introduce the variables conforming to the thermodynamic model. According to the Rainov's analogy these variables must satisfy the given classification. Let $x_i,\ i = 1, \ldots, n$ — be the amount of i-product and $p_i,\ i = 1, \ldots, n$ — the price of i-product. It is seen that x_i is a system state coordinate and P_i — potential is — p_i (p minus is like pressure minus in the mechanical deformation system). In market economy at x_{inv} or p_{inv} the condition

$$\frac{\partial p_i}{\partial x_i} > 0$$

is satisfied. The amount of action i changes as

$$dQ_i = -p_i dx_i$$

As x_{n+1} system state coordinate we have chosen the welfare value W. Let's express it as the function of $x_i,\ i = 1, \ldots, n$:

$$W = W(x_1, \ldots, x_n)$$

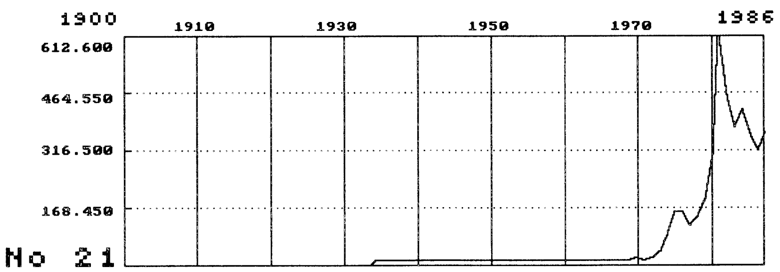

Figure 19.21: The price of an ounce of gold in $

If labour resource is noted separately then W is the classical production function. Physical measure of W is expressed in conventional energy units. Kilocalorie is taken as the unit of product measurement [Odum 76]. But it is not convenient. We propose to take the gold equivalent as the measure. As the P_{n+1} — potential we propose E — the price of 1 unit of conventional energy of W. In [Odum 76] one dollar in 1975 was evaluated as 25000 kcal, i.e. the price of 100000 kcal was E=4 $. Because of inflation E

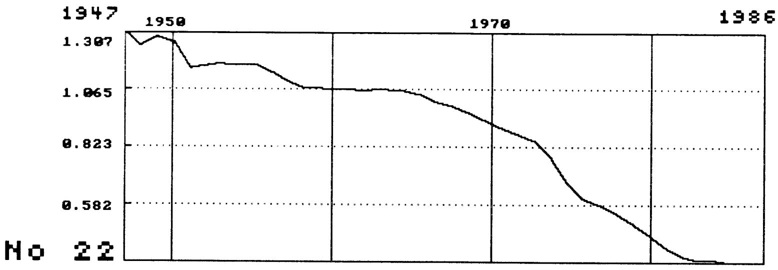

Figure 19.22: Purchasing power of the dollar in the USA (producer price, 1967=100

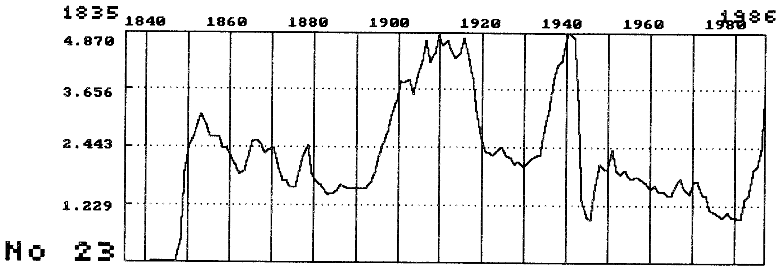

Figure 19.23: Gold mining in the USA in 1835–1986 in m. fine troy ounces

increases. If E is taken as the price of gold the price of product in gold equivalent (e.g. weight) is less subjected to changes and fluctuations, particularly for the complicated product with longer production period. And dollar constantly changes — see fig. 19.21: the price of an ounce of gold in dollars and fig. 19.22: purchasing power of the dollar in the USA in 1947–86 (producer price, 1967=1$).

The condition

$$(\frac{\partial E}{\partial W})_{Xinv} > 0$$

is satisfied here: when the external price of the welfare unit increases, welfare W in the system increases by ΔW and then the internal system price E increases by ΔE.

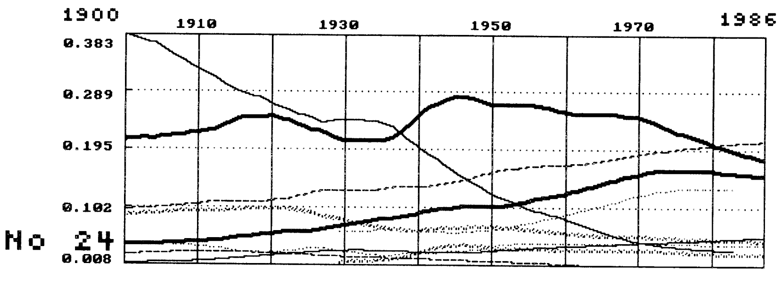

Figure 19.24: Employment structure in the US economy; percentage of persons employed in (1) in agriculture, (2) in manufacture, (3) in trade, (4) in transportation and public utilities, (5) in governmental institutions, (6) in contract construction, (7) in mining, (8) in various financial activities (from left top to bottom

The amount of actionis

$$dQ_{n+1} = EdW.$$

Are W and entropy S, E and temperature analogical in the isolated system?

1. S and W are not directly measured and observed but calculated;
2. when non-equilibrium changing of E, new effects (state coordinates) do not occur;
3. the proof of the entropy increase in the non-equilibrium isolated system supports on

   $$S = S_1 + S_2$$

   for two bodies. In our model when two systems are united,

   $$W = W_1 + W_2$$

   and the proof of the welfare growth is analogical to that of the entropy growth e.g. in [Leon 68]. And that is the reason why such variables as welfare per system unit of measure (per capita) and average life expectancy of man are not suitable here as W;
4. at non-equilibrium interaction other forms of energy transfer into heat i.e. $dS > 0$. The productive function has the property

   $$\frac{dW}{dx_i} > 0, \quad i = 1, \ldots, n.$$

So when introducing the notion of products energy or their equivalent the welfare W with high probability can be compared with the entropy. Let's call W the *ecoentropy*. The analogy of heat transfer is the processes of money inflation-deflation. Though gold is also a commodity (Kondratiev noted the periodicity of gold production-mining, see fig. 19.23. However more reliable energy equivalent has not been invented yet.

19.5 The Employment Structure in the USA Economy from the Large Cycle Theory Standpoint

We are building the simulation model of the structural changes in the USA from the long waves theory standpoint. In the present work the model will not be touched on, but it would be interesting to see the employment dynamics which plays an important role in the main model.

	Raise 1896–1914 1948–65	Transition 1914–20 1965–75	Fall 1920–35 1975–88	Transit. 1935–48
Agriculture	— = — — = —	= =	— + = — = =	— !
Manufacturing	= + +! = — =	= —	— — != — = —	+!
Trade	= + = + = =	= +	+ = + = + +	=
Transport. & publ.util.	= + = + — =	+ =	— — ! — — = —	=
Government	= + + = + +!	+! +	+ = +! = — —	+
Contract construction	= — — + = =	= +	+ = — — = =	+
Mining	+ = = = — —	— =	= — = = — =	—
Finance etc.	+ = + + = +	+! +	= + = = + +	—

Table 19.4: Employment structure and large waves

Fig. 19.24 represents the charts of employment in the main spheres of the USA economy (in shares to total employment) since 1900, in from left top to bottom order: (1) in agriculture, (2) in manufacture, (3) in trade, (4) in transportation and public utilities, (5) in governmental institutions, (6) in contract construction, (7) in mining, (8) in various financial activities.

Charts in Fig. 19.24 reveal indices with two characteristic features:

- steady growth (fall) as a norm and curve smoothing to the horizontal level in critical years e.g. agriculture,
- on the contrary,horizontal curve as a norm and growth (fall) in critical years e.g. trade, transportation.

Let's "superimpose" the waves from tables 19.1 and 19.2 over the charts of Fig. 19.24 and study the curves in each period of long cycles. The results are recorded in Table 19.4. Herein the sign "+" designates growth and "−" — fall of the index curve; "!" with the sign designates sharp increase of the growth or the fall; "=" — zero growth (horizontal curve) of the index. Extreme critical points are separated in the table 19.4, and there are three stages inside each period of rise and fall: initial, inflection (approximately middle but not obligatory), final. Analysing table 19.4 shows marking out of extremum and inflection critical points that confirms their special significance.

19.6 Mathematical Research of the Cycles

The usual approach in theoretical mathematical study of cycles is to find the periodical fluctuations in linear differential equations systems. The following model ([Menshi 89], p.117) is considered as a typical example:

$$\begin{aligned} \frac{dy}{dt} &= -\alpha(y - bk) \\ \frac{dk}{dt} &= -\beta(k - gp) \\ p &= y - k \end{aligned} \tag{19.1}$$

where y — rate of labour productivity growth, k — rate of growth of capital availability per worker, p — rate of profit norm growth, α , β , d, g — structural coefficients. The characteristic equation of the model is written as

$$\lambda^2 + [\alpha + \beta(1 + g)]\lambda + \alpha\beta(1 + g - bg) = 0.$$

Depending on the value of this equation's roots different types of the equilibrium state are revealed in the system 19.2. Without touching the essence of the model we note that $b = 1$ is supposed here. Regular cycles appear when $g = -2$ and $\alpha = \beta$. When $\alpha = \beta = 0.1(0.12)$ their periodicity is 50–60 years, when $\alpha = \beta = 0.34$ — 20 years, $\alpha = \beta = 1$ — 7 years, $\alpha = \beta = 2$ — 3.5 years. Similar methods are kept up both at the modifications of the model and its disaggregation.

Research of the economic models by means of canonical transformations of variables to the "angle-action" type of variables, and study of resonance and non-resonance tori, and Poincare mappings are made in detail in [Niyaz ap]. It is also correct for non-linear models. For the model 19.2 under the given assumptions $b = 1$, $\alpha = \beta$ and $g = -2$ the Hamiltonian function can be written as:

$$H(k, y) = \frac{1}{2}(\alpha k^2 + 2\alpha y^2) - \alpha ky. \tag{19.2}$$

The system's divergence equals to zero, $div = 0$, the volume of the arbitrary region of the phase space (Liouville's theorem) is constant, therefore the system 19.2 under the given assumptions is conservative. As $H(k, y)$ doesn't obviously depend on time, the Hamiltonian function is the motion integral, i.e. for the given initial conditions $E = H(k, y) = \text{const}$ is the total energy of the system. The model's economic interpretation problem arises and in the first place — the division of the variables into generalized coordinates and generalized momenta. In the model 19.2 at the assumptions $b = 1$, $\alpha = \beta$ and $g = -2$, the rate of labour productivity growth y is taken as a generalized coordinate and the rate of growth of the capital availability per worker k — as a generalized momentum. The total energy of the system is written in the form 19.2 and though it might be paradoxical for economists this expression is a constant.

If the number of degrees of freedom is the number of independent generalized variables (q, p), the phase space for the system under study is $2n$ dimensional. In our case $n = 1$, $(q, p) = (y, k)$. By canonical transformation

$$\begin{aligned} y &= \sqrt{R}\, cos\ \Theta \\ x &= \sqrt{R}\, (cos\ \Theta + \sqrt{1 + sin^2\Theta}) \end{aligned}$$

(where Θ has the period 2π) we obtain

$$\begin{aligned} H' &= \alpha R \\ \frac{dR}{dt} &= 0, \quad R = R_0 = const \\ \frac{d\Theta}{dt} &= \alpha, \quad \Theta = \Theta_0 + \alpha t. \end{aligned}$$

From the latter we have

$$\alpha = \frac{2\pi}{t}.$$

Substituting t by economic cycles periods 3.5; 7; 20; 60 we obtain the α-values coinciding with the above-mentioned.

The applicability of analytical mechanics methods to the problems of optimal control is considered in [Ivanil 83].

The models below are described in more detail in the work to appear [Niyaz ap]. Methodologically they are a gradual complication of a simple initial model. Let x_i , $i = 1, \ldots, n$ be competitive products (goods) in the market (identical but produced by different firms), N — marginal saturation of the market. The initial model of goods dynamics at the market is written as

$$\begin{aligned} \frac{dx_i}{dt} &= \alpha_i x_i\ (Nk_i - x_i) \\ & N > 0, \quad k_i \geq 0, \quad \sum_j k_j = 1, \quad i = 1, \ldots, n, \end{aligned} \tag{19.3}$$

where the α_i are logistic parameters. Let the market of each commodity buyers have been formed and $\sum_i x_i \leq N$ (if $x_{0i} < Nk_i$, then it follows from the model). Then all the equations in the system would be independent and every commodity would be in "its niche". The k_i coefficients are bonds of the system.

Now let's suppose that

$$k_i = \frac{x_i}{x_1 + \cdots + x_n}.$$

Substituting it into 19.3

$$\begin{aligned} \frac{dx_i}{dt} = \frac{\alpha_i x^2}{x_1 + \cdots + x_n}\ (N - x_1 - \cdots - x_n) \\ N > 0, \quad i = 1, \ldots, n \end{aligned}$$

is obtained. For the simplicity let's consider the case $n = 2$. x_1 will below be denoted by x; x_2 — y; α_1 — $h\alpha$; α_2 — α. Equilibrium points are $(N, 0)$; $(0, N)$; $(w, N-w)$, $0 < w < N$ (i.e. a straight line $x + y = N$). In the all points the system is not rough (one of the roots of the characteristic equation of the linearized system equals to zero). There are no transitions to the cycles. As was mentioned above each commodity is in "its niche".

Now let's suppose

$$k_i = \frac{x_i + \epsilon_i}{x_1 + \cdots + x_n},$$

i.e. in eq. 19.3 the condition $\sum_j k_j = 1$ is violated and depending on the signs at competition can strengthen or weaken. Substituting it into eq. 19.3

$$\frac{dx}{dt} = \frac{\alpha_i x_i^2}{x_1 + \cdot + x_n} (N - x_1 - \cdots - x_n) + \frac{\alpha_i N \epsilon_i x_i}{x_1 + \cdots + x_n}$$
$$N > 0, \quad i = 1, \ldots, n$$

is obtained. Let's consider the example for $n = 2$. ϵ_1 is denoted by ϵ and ϵ_2 — by ϵ_m. Four states of equilibrium $(0, \frac{N \pm \sqrt{(N^2+4\epsilon m}}{2})$ and $(\frac{N \pm \sqrt{N^2+4\epsilon}}{2}, 0)$ make a saddle and a node. The property of the other two points

$$(\frac{N \pm \sqrt{N^2 + 4\epsilon(1+m)}}{2(m+1)}, \; m\frac{N \pm \sqrt{N^2 + 4\epsilon(1+m)}}{2(m+1)})$$

is determined by the values of coefficients u (at λ) and v (free term) of the characteristic equation of the linearizied system. From the condition $u = 0$ we determine ϵ

$$\epsilon = -\frac{N(m^2 + h)(m + h)}{(2m^2 + mh + m + 2h)^2}$$

and if $m > -h$ then $\epsilon < 0$, if $m < -h$ then $\epsilon > 0$. For a cycle to appear it is necessary that $u > 0$. In [Niyaz ap] it is shown that at $h > 0 u$ is always < 0. The cycle appears at some values of m and at $h < 0$, i.e. if one of the firms produces goods and others consume it.

That is why for $n = 3$ (and more) one of the equations must describe the consumption dynamics and hence its variable means demand. For this equation coefficient $h < 0$ holds. Preliminary calculations [Niyaz ap] show that for $n = 3$, α_1, $\alpha_2 > 0$, $\alpha_3 < 0$ cycles appear and bifurcations are possible. It is necessary to note that the third equation has some changes.

We also study more complicated models of the type

$$\frac{dx}{dt} = \alpha_i x_i \; (N k_i - \sum_{j=1}^{n} (\beta_{ij} x_j))$$
$$N > 0, \quad \beta_{ij} = \{0, 1\}, \quad i = 1, \ldots, n,$$

models in terms of money where N is monetary mass of the system and x_i are substituted by $p_i x_i$, i.e. price parameter p appears.

Similar equations have independently been explored by Troitzsch [Tr92].

19.7 Some Psychology

Economic "agents" behaviour in market models would rather belong to micro economics. Marxist political economy that disliked the cycles theory pathologically criticized the models of fluctuating mechanism based on the firms interaction, considering them to be a mere piling of "behavioral functions" and it is either impossible to deduce cycles from such models or the matter comes to the hypothetical case with a firm and a customer (as for us it reminds thermodynamics in micro and macro sections). But it should be noted that by any hypothesis the connecting link among different scales of the models is the psychology of behaviour — the reaction of the economy "living" elements on the changes in situation, psychological acceleration and retardation of the system's internal processes. Usually, this psychology of the behaviour is not accentuated and underlined but it is invisibly present in putting the particular problems (e.g. choice of a functional, choice of consumption dependence either from the income or the stock of an agent etc.).

The behaviour of some individuals depends certainly on their income and social status but in a general case it is their patience, obedience and disobedience, ability to unite and protest (in the simplest case as voting) that create a "litmus paper" for the behaviour of all the structures above.Psychology of the individuals economic behaviour as was mentioned above is similar to that of "agents".

Thus, the psychological environment of the whole system is determined by the following four structures: state structure of management, political structure, economic agents and individuals. The influence on the average and long cycles is considered to be determined by the state management and economy agents i.e. their interaction. "Laissez faire" (free trade) periods are changed by the stiff state regulations which can be seen for instance on the cyclicity of the duties to import ratio change (see Fig. 19.5). Political cycles which depend on the term of election must form 4 years in the USA. Ruling parties change with 25–30 years periodicity. Some authors suppose this period to be conditioned by the change of ruling forces generations [Schles 86]. At that time struggle for power increases and one may speak about critical points. Chart 19.17 of votes dynamics for the parties allows to speak on the periods of voters full confidence when it is possible to take even unpopular decisions,and the periods of the confidence loss that affect the psychology of administration.

The investigations on individuals influence on the long fluctuations of economy date back to the works by J.Mills [Mills1867] and A.Pigou [Pigou 20]. But they consider the "emotions" of the society that cause the economic changes while in our approach one can speak only of their effect on the cycles and mainly on the interaction of all structures of the system. Let's touch on this problem. As it was mentioned above the behaviour of individuals for the whole system is a reflexion on that of the other structures, that is why psychological climate of the society, its feeling and readiness to accept economic and political (sometimes daring or unpopular) decisions are of importance. We make assumption that this psychological climate changes periodically from conservative to radical one and vice versa. Society is bored with "monotony" and it raises radical reformers but it also gets tired with transformations. Probably, these periods are connected with the figures 25–35 years — change of a generation. The chain obtained can be closed by a feedback of administration influence on society but it is of constant character and studied in special literature.

19.8 Are there Large Cycles?

The "cautious" name of large cycles is long waves. Long wave existence is generally accepted, but the approach to large cycles was cautious. The question of the cycles introduction at smoothing with the function transfers into that of trend existence in general. J. Forrester [Forr 71] model being considered, his prediction curves are nothing else than the trend. Time interval is the other matter (we'll supplement it with the 20-th century). The functions change with time increasing but the wave character remains invariable (that is why we don't give figures, the most important thing is the qualitative results).The existence of longer waves (200–250 years and more) are not excluded.

The other mechanism which was guessed by Kondratiev supplements the first one. Each of the main branches of the economy has its "pulsation". Their interconnection forms the structure of the whole system. There are reversible processes in the system at the normal course of the circumstances. Conflict of the branches (and their "pulsation") takes place in the critical points mainly because of the resources (energy). The potential difference increases and non-reversible process begins.Slightest disturbance leads to the jump,breaking of all the old structure and forming a new one. Then the process stabilizes and everything commences all over again.

The third mechanism — is models of self-organization based on the nonlinear differential equation analysis. It resembles the second one by the idea.The critical points here are the bifurcation ones. Here jumps and structures appearance take place as well.

Large cycles question has been and would remain open for a long time. But periodicity in the mechanisms mentioned and mainly in the real observation points to the greater possibility of large cycles existence.

On cycles in the USSR. Having no reliable data it is impossible to speak on the serious analysis. Besides, it is necessary to analyse different-pole crisis symptoms in the USSR and the USA e.g.simultaneous slump of production and rise in prices in the USSR. But we shall note their following periodicity:

1. political cycles also have about 30-years wave (change of leadership styles in 1917–24, 1953, 1985) that coincides with ([Schles 86],[Niyaz 92]);

2. the greatest economical changes in 1861 (land reform), 1920-s (new economic policy),1985–91 (perestroika) have the periodicity of 60 years. Vitte and Stolipin reforms (the end of the XIX-th and the beginning of the XX-th centuries) and those of Khrushchev (1950-s) are exactly in the middle.

It should be noted that some works on the cycles in the USSR have appeared lately, but unfortunately we can't give an exact reference to them.

In conclusion we note that our program system U-CYCLES [Niyaz 91] with database of the USA indices was of great help in this work. It made possible to calculate a great number of computations without noticing it and to concentrate on the analysis and the design of results.

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Chapter 20

Georg Erdmann, Zürich: Modeling Economic Aspects of Institutional Change in Eastern Europe

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Abstract

According to a result of a recently expanding branch of economics, the so called "New Growth Theory", there is no unique evolution of the economy, but rather an multitude of equilibrium trajectories (equilibrium growth paths) for a given set of exogenous variables. As analyzed by evolutionary economics, the macroeconomic selection takes place through self-organizing economic actors on the micro level. Self-organization stands also at the origin of occasional transitions between different equilibrium trajectories with their typical market instabilities.

The self-organized selection of equilibrium trajectories still becomes more ambitious if an institutional transition (cf. the shift from centrally planned to competitive market order) is under way. Economic actors thereby face a basically changed economic environment in which they are lead to redirect their search for sources of income and wealth without precisely knowing what would be an improved or superior search and adjustment strategy.

Experience of capitalist countries (especially in the developing world) shows that the establishment of a market order — including the creation of the required public institutions — is not sufficient to guarantee an evolution that leads to a well working free market economy. By describing the search oriented self-organization through a formal model, a deeper understanding of this crucial problem of institutional change might be achieved.

Thus my paper focuses on the complex linkages between institutional and economic change and addresses the following questions: What economic evolution will probably happen in former socialist countries when their institutions are changed from the centrally planned economy to the market economy? Can the former socialist economies take a pareto-efficient trajectory or will they be locked into a shadow market structure or some other inferior evolution path? Are there some political strategies to stipulate the evolution of the pareto-efficient evolution?

20.1 Requirements and Major Problems of Institutional Change in Former Socialist Countries

One of the aspects of institutional change which actually takes place in Eastern Europe is the transition from a centrally planned economy to a market economy. The key ingredients of the market economy that must be established by the legislations and politicians in charge are well known:

- the freedom of trade comprising the individual right to become owner of land, real estate and business companies (constitutional law);
- the establishment of a public guarantee to pursue and protect the individual property rights (civil law);
- the market order including the antitrust legislation (corporate law, and other elements of business legislation);
- the creation of tax systems and social contribution systems in order to finance the supply of the so called public goods and the social security network (fiscal law).

In order to stipulate the market economy it is not sufficient to create a new political constitution and the body of laws which belong to the required basic elements of the market economy. A workable market economy needs also institutions and organizations such as civil courts, criminal courts, administrative tribunals, a politically independent central bank (protecting the value of liquid assets and nominal claims), an efficient financial administration, organizations protecting the workability of markets[1] and augmenting the market share of the private sector through privatization, free syndicates employer's associations, political parties, and other formal interest groups, to mention only the most important public organizations.

All these organizations should be headed by people which agree with the idea of the market economy and accept the principle of competition as order principle. The staff should be guided in the spirit of of serving people.[2] It would help, if managers and staff would also have some experience about how the market economy practically works, what the threats to it are and what kind of instruments could prevent the market order of being undermined. Finally, the public organizations need some minimum degree of goodwill and confidence which they merit though good work.

If these public organizations cannot be established in reasonable time and in replacing the existent bureaucratic structures, the market economy exists on paper only and their rules and principles cannot influence the behavior of the economic actors.

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[1] These are for example anti-cartel-organizations, industrial inspection boards, public health offices, and chambers of industry and commerce.

[2] This statement is not normative but empirical: If an important number of institutions is not headed by people that agree with the idea of the free market economy it will hardly be introduced. The statement is not scientifically proven here, but it stands in line with the broad experience of all those that are engaged in building market economic structures in East-Germany and Eastern Europe.

Of course, the conditions just mentioned are hardly going to be met in most East-European countries. But this indicates where actually are the most critical problems of the fast transition from the socialist economy to the market economy. It is not so much the ability of the legislation to skip the old socialist law and to establish an appropriate set of market laws and rules. Several of these countries did made much progress in this direction in only a short period of time. Unfortunately, progress in this direction seems to be rather slow in most of the states emerging from the former USSR. But all East-European countries seem to have much problems to establish workable market oriented public organizations.

The basic problem lies in the fact that there are no appropriate elites in the notion of Josef Schumpeter [Schumpeter 1912]. While it seems not advisable to pass the responsibility of the market oriented institutions — including the still state owned companies — to the former socialist elites, there are rather few other domestic people in view which could become managers and administrators, because the former opposition had been decimated under political and economic pressure and could not develop the required management skills (perhaps with the important exception of the churches).[3]

As long as the market oriented institutions do not work properly, the economic recovery will hardly take place. But due to the lack of available skills it will take several years — in an optimist view — until public organizations can work according to the principles developed above. The evolution of these institutions is further challenged by the withdrawing former elite, which gives rise to the pessimist view that the market institutions may not be established at all in all East-European countries within the range of one generation.

The lack of native market oriented managers in Eastern Europe creates also a heavy burden on the accelerated privatization of industrial plants. Privatization requires not only risk taking capital but also entrepreneurs which are able to open up local, regional, national and international markets under a competitive environment. On the other hand such management skills are probably less demanding in agriculture, commerce, and some other services. Therefore the transfer of these sectors to private owners is much easier and creates faster benefits to the society.[4] Political reforms should set in here first.

This is a rather shortened description of the still required institutional change and its major problems. But I don't discuss this important topic more in detail here. I rather assume in the following that well working free market institutions have already been created (as is the approximately case for East-Germany). Assuming this I turn to the question, how the economy will adjust to the new opportunities created by that kind of institutional change. Thus my proposition is to ask what economic evolution will result if market institutions exist. The major message is that the answer is not unique and that a positive evolution depends largely on "trial and error".

[3]This "elite problem" seems to be an important reason for the delayed economic recovery even in East-Germany where already a considerable import of human capital had been taken place.

[4]Experience in former Soviet Union and in the Peoples Republic of China showed that a privatization of agricultural land guarantees an immediately augmented food production. In Ex-USSR private-owned land amounted around 5 % of the total agricultural land and produced 35 % of total food. The land reform of 1979 in the Peoples Republic of China increased the agricultural output during several years by two digit growth rates.

This answer may also give some hints concerning the establishment of the market oriented economic order and the the institution forming process under way in the other former socialist countries. According to my analysis these institutions must be formed through "learning by doing". This requires certain capacities to correct errors and this aspect should be reflected in the initial design of these institutions. In addition one has to take into account the aspect of co-evolution between the institution forming and the market forming process. But this topic is excluded from the following analysis.

20.2 Institutional Change and Economic Behavior

Any economic order contains a set of stimuli and opportunities for the economic actors — that are entrepreneurs, artisans, farmers, consumers, politicians and the other members of different societal groups. All these individuals and groups will adjust their market behavior according to the incentives and opportunities offered by the existing economic order.

This principle of economic self-organization is well known for the capitalist economies. Here profit opportunities stimulate entrepreneurs to undertake investments into business activities, and the outcome of these activities, a huge set of good valued products and services, offers large possibilities for consumers to satisfy their needs and wishes. But this principle of economic self-organization holds also true for socialist economies where "successful businessman" did also exist. Unfortunately, most of them improved their utility level not through production of goods thereby satisfying human needs but through hoarding, speculation, corruption, blackmail, and administrative power.[5]

Thus, while creativity, cleverness and venture exists in all types of economies, the macroeconomic outcome of this behavior depends, among others (such as religious and other cultural factors), on the specific business opportunities offered by the economic order. One of the major advantages of the market oriented economic order compared to the socialist order is the implicit creation of business opportunities where people can earn large sums of money without going through black and grey markets. People can become rich by making other people better off — an economic situation which above was called pareto-efficient.

This doesn't mean that legal type business is the only way in market economies to become rich. We know from experience that shadow economic activities exist in many market economies, too. In the OECD countries (perhaps with the exception of Italy) it is of minor, in some cases marginal importance (see [Schneider 1986]). But the existence of market economies with a dominant semi-legal or illegal business sector shows that the establishment of a free market order is a necessary but not a sufficient condition for achieving pareto-efficiency.

[5] In socialist countries people followed utility oriented strategies but due to the given restrictions (no property rights, for example) and incentives (party careers, for example) the economic outcome of individual behavior had been inferior (in quantitative and qualitative terms) to the outcome of market economies. This is the principle of self-organization applied to the former socialist countries.

When the creation of market oriented institutions opens up the possibility for several macroeconomic structures (trajectories)[6] the question becomes relevant what will happen in the course of institutional change to the market oriented economic structure actually going on in the former socialist countries. Can the former socialist economies reach a pareto-efficient trajectory or will they be locked into a shadow market structure or some other configuration? Are there some political strategies to stipulate the evolution of a pareto-efficient economy? In the following I will discuss these questions by using a semi-formal model which strongly focuses upon the self-organizing nature of the socio-economic system (for more details see [Erdmann 1990]).

20.3 Approaching a semi-formal model

The model assumes a limited and known number of economic scenarios as possible trajectories of economic evolution. Usually the number of trajectories that are possible evolutions of the economy is unknown. Thus the assumption stands on weak grounds. But a deeper mathematical analysis of the model shows that it is no crucial assumption. It is introduced here for didactical reasons.

Each of the economic scenarios stands for an equilibrium trajectory. That means: If the economy has evolved a bit in the direction of one of the scenarios it becomes rather difficult for the economy — in economic terms: costly — to shift to another trajectory. This idea can easily be translated into the mathematical language by using the concept of potential curves. Figure 20.1 shows the most simple, non-trivial situation. The potential curve has two local minima in x_1 and x_2 representing two different stable equilibria at time t. The depth of each minimum is assumed to correspond on the overall (social) benefits associated to that economic equilibrium. In figure 20.1 the situation x_2 is the pareto-efficient optimum, while x_1 represents a suboptimal equilibrium state of the economy.

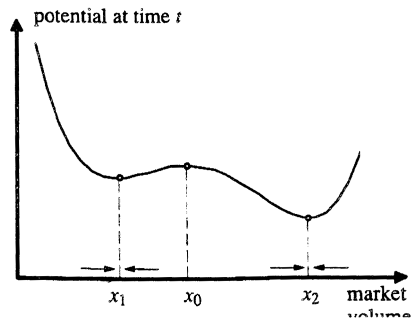

Figure 20.1: Potential function showing two equilibria

It may be useful to explain this idea here more in detail. Traditional (neoclassical) economic theory claims that overall welfare is best in terms of the aggregate producer's

[6]There is some analogy of this statement to the insights from the so called New Growth Theory. See [Romer 1986, Romer 1989, Lucas 1988].

and consumer's rents if markets are in equilibrium. New contributions to neoclassical theory prove that under certain realistic conditions multiple situations exist where markets are in equilibrium. Outside these configurations losses occur in terms of the consumer's and producer's rents. Most economists assume that the system of (interrelated) markets tends (through the force of competition) towards one of the equilibria. Nobody knows the exact position of these equilibria ex ante, but ex post it is possible to observe whether the markets have reached a coordinated state or not.

In a neoclassical world the adjustment is immediate or at least straight forward: In a series of small steps the economy approaches one of the fix points x_1 or x_2 depending on the initial state of the economy. In an evolutionary world search occurs that allows the economy to cross over to the attraction basin of another equilibrium. If this happens, a basic structural shift of the economy occurs and in a certain sense it may be called "the discovery of a market niche".

An special state of the system shown in figure 20.1 is the unstable equilibrium in x_0. If the economy is in this situation, small deviations (fluctuations) would execute a force on the economy to develop into one of the two directions. Thus the point x_0 represents a contingency of the economy or — mathematically — a bifurcation. But once the society evolves a little bit in one of these two directions, an irreversible effect occurs: It is not easy (i.e. free of social costs) to return to a situation, from where the economic evolution automatically takes the direction of another trajectory.[7]

From mathematical systems theory it is known that a dynamic model must be nonlinear if it should be able to cover contingencies and bifurcations. In the economic literature of the past ten years or so there is a growing body of studies aiming to identify and to analyze the basic economic reasons for these nonlinearities. Among others, [Hotelling 1929, Arrow 1962, Helpman 1985, David 1987] and [Stiglitz 1987] may be cited. In reviewing these studies, the following theoretical arguments explain nonlinearities in economies undergoing an institutional change:

- Learning by doing and/or economies of scale: If an economic choice has been made, the (individual) costs and benefits of the follow-up choices depend on the prior decision.
- Sunk costs: If economic actors are going to redirect their efforts towards other problems, some parts of their accumulated knowledge and know-how concerning technologies, management and markets is no more used and has to be depreciated in the economic sense. The associated costs represent a threshold for changing to a strategy that would be economically optimal if there would be no sunk costs.
- Network externalities: They are determined by technologies which often include spill-overs between different actors and markets. If several actors are engaged in similar technological paradigms and activities, they may benefit from each other through these types of positive externalities. If an individual decides in this situation to redirect its activities it looses some of these external benefits.

[7] This model is used here in the following discussion because there is the idea that the transition from the socialist order to the market order changes the economic incentives in a way that the economy finds itself in the unstable equilibrium point x_0. One of the stable equilibria may be called "the survival of the free market economy", the other "the survival of a strong shadow market structure".

- Finally, the herd behavior or synergetic behavior (see [Weidlich 1983]) is a socio-psychological reason for non-linearities in the economy.

20.4 Stochastics

A nonlinear dynamic model reproduces deterministic trajectories in the sense that the future evolution of the system depends exclusively on the starting conditions. Concerning the question of institutional change, it follows that the future evolution of former socialist economies would depend only on the conditions of the moment where the old regime breaks down and the new regime is going to be set up. Thus a deterministic concept cannot be a rather interesting approach for analyzing the complex linkages between institutional and economic change.

This is a case for introducing a stochastic element into the nonlinear dynamic model. The solution of such a stochastic model is a time path of a probability distribution over the relevant state space as shown in figure 20.2. What could be the nature of the stochastics? I propose the following idea: Take the large number of actors interacting in the society. The behavior of each of these actors depends on a enormous number of individual factors, or, in mathematical terms, degrees of freedom. Therefore the economic evolution in the aggregate also depends on a large number of degrees of freedom. Usually this has to be formalized by using a stochastic approach.

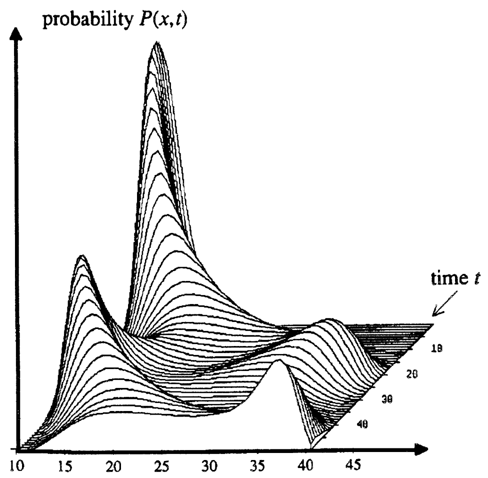

Figure 20.2: Example of a stochastic trajectory

A more detailed analysis may lead to an objection against this statement: In spite of the number of individual degrees of freedom, individual behavior follows some rules and structures due to which the aggregate result of individual actions follows a more or less structured order. If there would be no such self organized order at all, there would be no room for developing any social theory.[8]

[8]Remember that it is not the scope of social theory to describe or to anticipate how a singular actor will decide in certain situations. The focus is some ensemble of individuals.

A lot of social scientists assume that the target of utility maximization is dominating the individual behavior, even if there may be other relevant factors in individual behavior such as altruism, synergetic or herd behavior, solidarity etc. This axiom of the so called methodological individualism is, in my view, crucial for for any types of social theories even for those which don't rely explicitly on it. But the principle of utility oriented behavior alone cannot eliminate all the other degrees of freedom in individual behavior in every relevant situations.

Simon [Simon 1984], Kahneman, Slovic and Tversky [Kahnemann 1985] and other scientists developed a process oriented decision theory which can be used as an illustration for this point. As long as an individual finds itself in a well known situation where it has accumulated a lot of experience concerning the costs and benefits of different decisions it will probably turn to the best alternative in terms of its utility function. But in situations which are new and unknown to the individual (such as the installation of workable market institutions in former socialist countries) sound informations about costs and benefits are hardly available and the identification of the best decision in terms of an utility criterion becomes cumbersome. Thus, it is likely to assume that in an ensemble of independently deciding actors each decides quite differently from the other even if he would be in an objectively identical situation characterized by the same assets, restrictions, and preferences. In new and weakly transparent situations individual decisions doesn't only depend on preferences and restrictions alone but also on the process of problem perception, strategy generation and evaluation. This, in turn, depends on psychological and social factors that may be called intuition, creativity, cleverness, strength, and so on.

In conclusion, new and unknown situations imply an heterogeneous pattern of reactions on given economic and/or institutional stimuli. The spectrum of individual behavior reflects a large number of variables (or mathematical degrees of freedom) whereby the utility orientation is only one of them. If, however, the situation becomes simple and its solution transparent, all these psychological and social factors become of minor importance compared to the effect described by the standard economic model of expected utility maximization. The large numbers of degrees of freedom in individual behavior implies that the reactions of a single individual on certain stimuli (such as the creation of a new institution) cannot be predicted exactly. But an appropriate indicator to describe the degree of heterogeneousness can be the variance of a behavioral variable.

20.5 Implications from the model concerning the economic change

Now let us introduce the random forces reflecting heterogeneous behavior into the model represented by figure 20.1. As long as the variance of individual behavior is small compared to the scale of the relevant state variables x and the system is near to one of the two stable equilibria x_1 or x_2, the random forces have no effect except of noisy fluctuations around one of these equilibria.

However if the system is near to the unstable situation x_0, the fluctuations resulting from the heterogeneous individual behavior play a decisive role for the future evolution

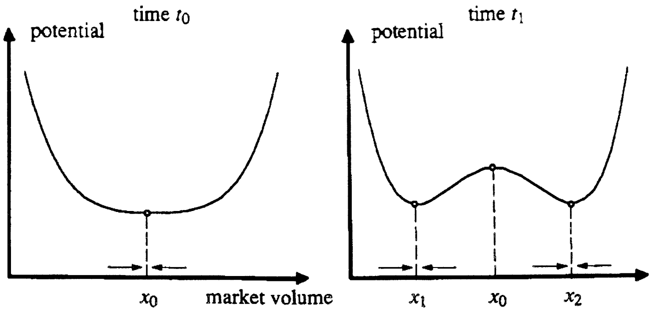

Figure 20.3: Phase-transition of the potential function leading to a bifurcation

of the system. In the absence of random forces, the system would stay in this point x_0 forever. But a random influence will necessarily move the system in the direction of one of the stable equilibria x_1 or x_2. Both equilibria can be regarded as market niches, which might be "discovered" by the effects of unpredictable individual behavior. The solution of this contingency happens through the effects of individual search, that create an impulse strong enough to force the economy irreversibly into the convergence basin of a stable equilibrium. Afterwards the new equilibrium is approached by the aggregate effects of utility oriented individual behavior.

Though the pareto-superior equilibrium x_2 may turn out more likely than the inferior equilibrium x_1 there is some chance for the inverse evolution to happen. (lock-in of an inferior equilibrium, see [Arthur 1988]). Random forces representing individually heterogeneous behavior — or the unpredictable search efforts of innovative individuals — are then important for correcting the direction of the economic evolution. If the economy follows an inferior trajectory and — equivalently — the state variables are in the attraction basin of the inferior equilibrium x_1, superior equilibrium x_2 can only be reached if a force is present supplying the energy to cross the the potential barrier at x_0. A possible origin of such a force is the search horizon (see [Nelson 1982]) of some members of the society. If their search activities are extended so far that system configurations beyond the potential barrier come into consideration, a basic change of the economic evolution can become possible.

But remember that such search efforts are costly and their effects cannot be known in advance. In other words they are of trial and error type and require an innovative spirit. But the experience shows the likelihood that always some economic actors are ready to take the risk associated with innovative activities and extend their search horizon. The macroeconomic role of these innovators that has first been described and analyzed by Schumpeter [Schumpeter 1912]. In some particular situations these distinct members of the society are able to create large scale economic effects which are in our model reflected by transitions of a state variable across a potential barrier.

In proceeding one more step in our analysis we have to assume that the shape of the potential curve is not time-invariant. In particular, the number of the stable equilibria may change. As mentioned, one source of such a particular structural shift of the potential curve is assumed to be the institutional change as it is actually going on in Eastern Europe. In dynamic systems theory, a structurally shifting shape of the potential curve is called a phase transition and it occurs if the parameters describing the relevant influences and forces acting upon the system under concern pass across some critical borders. A special case is shown in figure 20.3, where the transition from the left to the right drawing is called a bifurcation. A more sophisticated picture of the same idea is shown in Figure 20.4, where no less than four equilibrium states emerge out of one single equilibrium through a bifurcation.

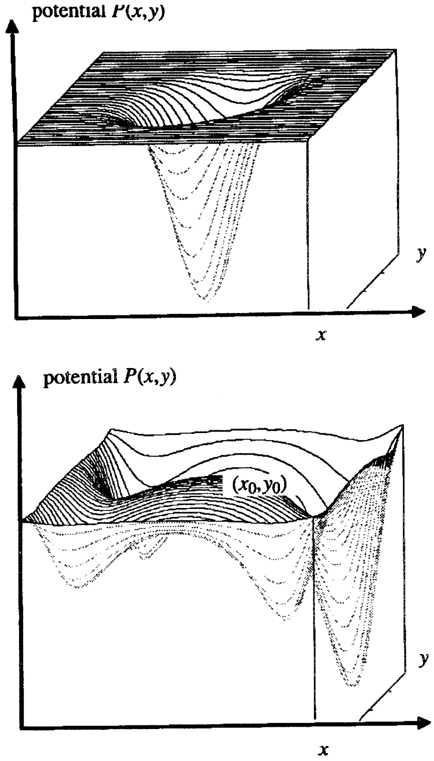

Figure 20.4: Potential curve before and after a basic institutional change

I propose here the following interpretation: The old economic order is represented by the potential function shown in the upper part of figure 20.4. The politico-economic forces hold the system close to the stable equilibrium (x_0, y_0). Through the institutional change this state becomes unstable, thus the situation shown in the lower part of figure 20.4 becomes relevant. The first effect of this change is that the system still located close to (x_0, y_0) is now located far from any stable equilibrium. This means that the economy becomes heavily instable. It also means that a lot of economic actors are becoming to be rather confused as multiple directions are open to the economic evolution and nobody really has an precise idea which direction will hold.

But, sooner or later, random forces resulting from the individual heterogeneous search efforts will drive the economy towards one of the four stable equilibria and thereby solve the contingency. But again: As nobody can have a detailed knowledge about these efforts the further direction of the evolution cannot be predicted ex ante. It is structurally impossible to make sound predictions on the basis of past observations.[9]

[9]Market participants cannot know more than only small segments of the rather complex potential

These observations do not tell anything of importance about the further evolution of the system, or, more technically, about the further evolution of the equilibrium path of the economy under concern.

To tell the same story in a biological language, we may identify a stable equilibrium with an ecological niche. If the environment of a dynamic system is continuously changing, new ecological niches are likely to emerge. Driven by fluctuations the system is "searching" for new equilibria. If the system discovers an equilibrium it is moving in that direction through the (biological or economical) forces acting upon it. In other words: The dynamics of the system is the result of necessity, represented by the force acting upon it, and chance, represented by random fluctuations.

In terms of a synergetic interpretation, a changing environment leading to phase transition exerts a force on the elements of the system to do something totally new. The system must change into a novel structure, or a novelty or innovation is bound to happen. Again, if several options can be reached by the system, a decision concerning the future direction has to follow. But outcome of this decision is not imposed from the outside. The outside rather determines the necessity of a new solution. Therefore the system can be regarded as self-organizing (see [Haken 1988]).

20.6 Conclusions and political recommendations

A certain number of conclusions turn out from the previous analysis:

- Actually the incertitude about the future evolution is greater than ever. This largely accepted statement can be modelled by the proposed approach.
- Trial and error is the appropriate strategy for any actor to discover the chances and opportunities as well as the risks and bottlenecks of the given situation. This is formally reflected through the idea of a search horizon.
- Government agencies are not better informed than other market participants concerning the management of the given situation of change and transition. Therefore governments have to follow a trial and error type policy.
- As there is more than a dozen of countries trying to change their economic order in a similar direction they probably will choose more or less different strategies and will probably will have different degrees of success. This opens a broader base for experience and learning. Thus, the search can be be biased through learning from the experience of other countries.

There are some more recommendations how to improve the chance for the economy to follow a pareto-efficient trajectory. These recommendations are based on the idea

function, and that not by theoretical considerations taken from the office desk, but by actively testing the market through explorative search. In addition, the shape of the potential function — or the changing market opportunities for innovations — is continuously changing challenging the individuals which must continuously (re-) discover it and search for new solutions, which is subject to trial and error.

that the capacity of the society to correct inferior strategies should be strengthened. The most important factor for this strategy is the overall search horizon of the market participants, which is represented mathematically by the variance of certain behavioral variables. A greater search horizon — or a greater variance of the relevant behavioral variables — can be a source for correcting an inferior evolution.[10] This leads to the following proposals:

- Individual freedom seems to be a necessary condition for the variance of individual behavior. A policy favouring egalitarian behavior eliminates the variance among individual actors and thus creates a strong burden for the economic evolution.
- Any decentralized decision process will more likely be successful in finding innovative solutions than centralized decision making. This is an argument for a strategy of accelerated privatization in agriculture, commerce and industry (after the industrial complexes are split up) — in addition to the motivational reason for shifting to private property.
- If the access towards informations is restricted, those who would draw different conclusions from a same bit of information will be prohibited to do so and to transform their different conclusions into individually different actions.
- It is equally necessary to promise to the lucky innovators some reward consisting of material and positional benefits if they succeed in their innovative activities and cause economically and socially advantageous breakthroughs. Otherwise, there would be also some innovative activity, but the creativity of individuals would be guided towards less favorable innovations.
- Equally, some insurance in favour of the unlucky innovators may be appropriate as this improves the readiness to take the risks of guiding individual efforts into principally uncertain and innovative activities.[11]

Of course, all of these statements describe necessary, not sufficient conditions for improving the chance to correct pareto-inferior market evolutions in former socialist countries. It should further be mentioned that some risk exists that the required freedom to individual search might again be challenged by the former nomenclatura which might have still the capacity to move the society through their search efforts in a wrong, pareto-inferior direction. This is an actual danger and a strong burden threatening the evolution of the emerging markets towards economic recovery and creating social wealth.

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[10] This idea can also be formulated as follows: Competition is not only a way for coordinating decentralized decisions but also serves as a procedure for discovery.

[11] Sinn [Sinn 1986] explains in his interesting paper how insurances (i.e. a redistribution of innovation risks) can improve the innovativeness in a society.

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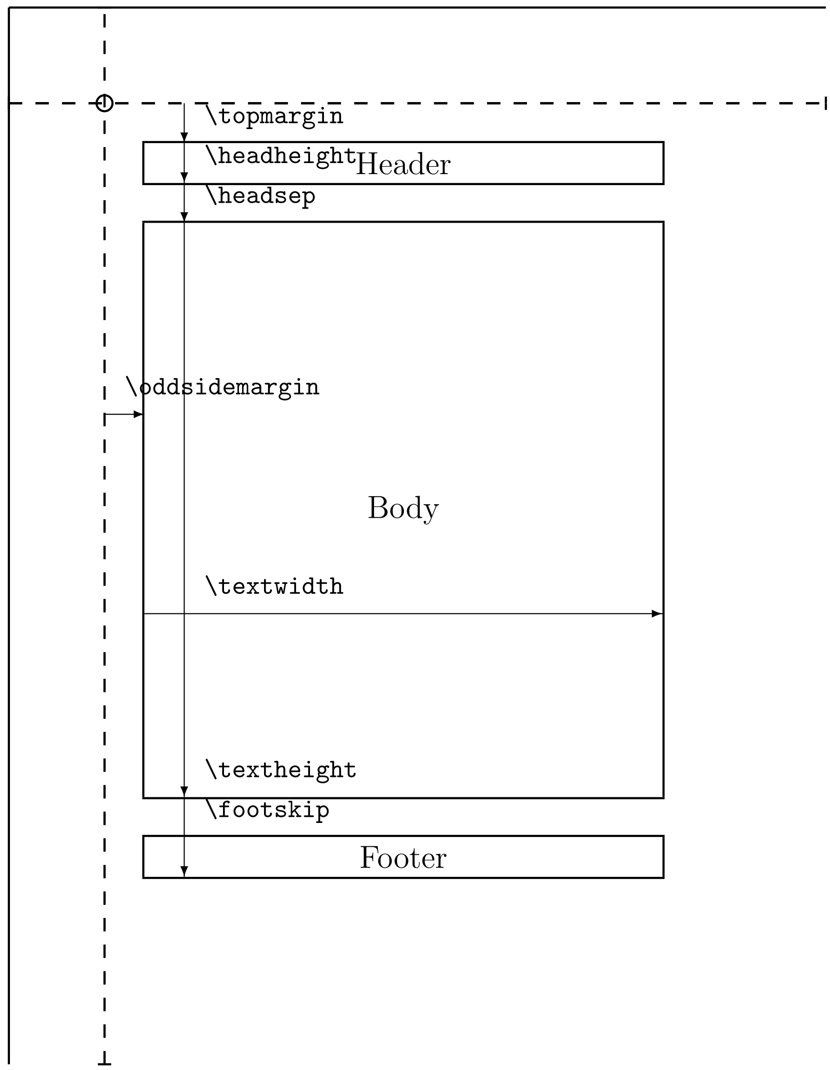

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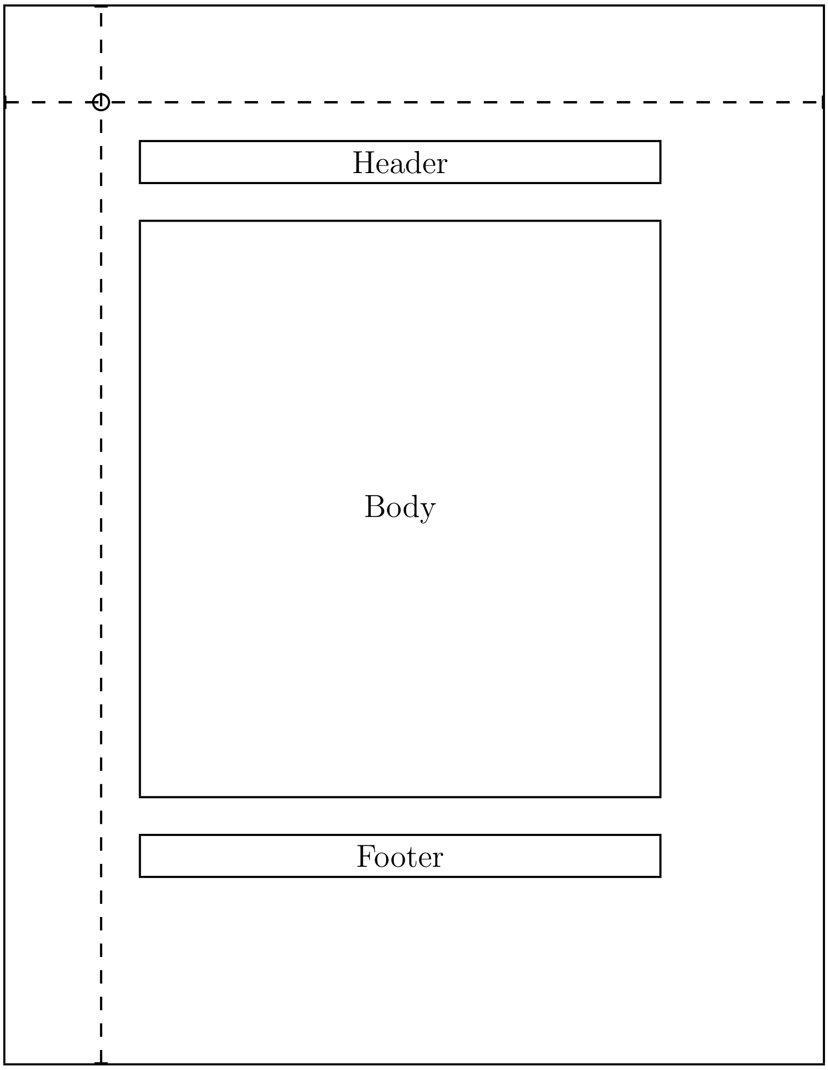

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