Mathematical Modelling of GIS Tailored GUI Design with the Application of Spatial Fuzzy Logic

by

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Frontmatter

Vorwort

Diese Dissertation entstand an der *Universität Koblenz-Landau* im Rahmen des Projekts *ReGLaN-Health and Logistics*, welches die Optimierung der Gesundheitsversorgung in ländlichen Gebieten Südafrikas zum Ziel hat. In dieser Dissertation wird mathematische Modellierung für adaptive graphische Benutzerschnittstellen, die ein angepasstes Verhalten in Abhängigkeit von Geographischen Informationssystemen besitzen und durch räumliche Fuzzy-Logik gesteuert werden, thematisiert.
Zusätzliche Materialien und Informationen zu dieser Dissertation sind unter folgenden Links zu finden:

- Animationen und Prototypen, welche im Kontext des Systems zur Adaption einer Benutzerschnittstelle (GUI) an einen Benutzer, Kapitel 9 und Kapitel 10, entstanden:
  - `http://mathematik.uni-landau.de/download/Platz/Animations.zip` (Stand 16.02.2014)
  - `http://mathematik.uni-landau.de/download/Platz/Prototype/Riskmap.html` (Stand 16.02.2014)
  - `http://mathematik.uni-landau.de/download/Platz/Prototype/FrameGUI.html` (Stand 16.02.2014)
- Material zur empirischen Pilotstudie, welche in Kapitel 11 thematisiert wird:
  `http://mathematik.uni-landau.de/download/Platz/Geo-GUI.zip` (Stand 16.02.2014)
  Die in diesem Kontext entwickelten GUIs sind unter folgenden Links zugänglich:
  - *expert GUI*:
    `http://mathematik.uni-landau.de/download/Platz/GUI/GUI1.html` (Stand 16.02.2014)
  - *more advanced GUI*:
    `http://mathematik.uni-landau.de/download/Platz/GUI/GUI2.html` (Stand 16.02.2014)
  - *advanced GUI*:
    `http://mathematik.uni-landau.de/download/Platz/GUI/GUI3.html` (Stand 16.02.2014)
  - *novice GUI*:
    `http://mathematik.uni-landau.de/download/Platz/GUI/GUI4.html` (Stand 16.02.2014)
- Das gesamte Dokument ist unter folgendem Link zu finden:
  `http://mathematik.uni-landau.de/download/Platz/Dissertation_Platz.pdf` (Stand 16.02.2014)

- Das Video zur Präsentation der Inhalte der Dissertation im Rahmen der Disputation ist unter folgendem Link zu finden:
  `http://mathematik.uni-landau.de/download/Platz/Disputation_Platz.mp4` (Stand 16.02.2014)

Dass diese Disseration gelingen konnte, habe ich besonders meinen Freunden und meiner Familie zu verdanken, die mich stets moralisch unterstützten und inspirierten.
Besonderer Dank gilt meinen Betreuern *Engelbert Niehaus* und *Marlien Herselman* für ihre Geduld und ihre Unterstützung, für die investierte Zeit für Besprechungen und für die Gastfreundschaft in Südafrika und die Hilfe bei der Beschaffung von aktuellen lokalen Informationen über Südafrika.
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Landau, Februar 2014 Melanie Platz

Preface

This PhD-thesis arose at the *University Koblenz-Landau* in the framework of the *ReGLaN-Health and Logistics* project. The goal of this project is the optimisation of health service delivery in rural areas of South Africa. This thesis deals with mathematical modelling of Geographic Information System-tailored Graphical User Interface (GUI) design with the application of spatial Fuzzy Logic for adaptive GUIs tailored to different user groups.
Additional material and information concerning this dissertation are available at the links below:

- Animations and prototypes, that arose in the context of the system for adapting a GUI to a user, Chapter 9 and Chapter 10:
  - `http://mathematik.uni-landau.de/download/Platz/Animations.zip` (retrieved 16/02/2014)
  - `http://mathematik.uni-landau.de/download/Platz/Prototype/Riskmap.html` (retrieved 16/02/2014)
  - `http://mathematik.uni-landau.de/download/Platz/Prototype/FrameGUI.html` (retrieved 16/02/2014)
- Material concerning the empirical pilot study which is addressed in Chapter 11:
  `http://mathematik.uni-landau.de/download/Platz/Geo-GUI.zip` (retrieved 16/02/2014)
  The developed GUIs are available at the links below:
  - *expert GUI*:
    `http://mathematik.uni-landau.de/download/Platz/GUI/GUI1.html` (retrieved 16/02/2014)
  - *more advanced GUI*:
    `http://mathematik.uni-landau.de/download/Platz/GUI/GUI2.html` (retrieved 16/02/2014)
  - *advanced GUI*:
    `http://mathematik.uni-landau.de/download/Platz/GUI/GUI3.html` (retrieved 16/02/2014)
  - *novice GUI*:
    `http://mathematik.uni-landau.de/download/Platz/GUI/GUI4.html` (retrieved 16/02/2014)
- The whole document can be found at the link below:
  `http://mathematik.uni-landau.de/download/Platz/Dissertation_Platz.pdf` (retrieved 16/02/2014)

- The video about the presentation of the contents of the dissertation thesis in the context of the defence can be found at the link below:
  `http://mathematik.uni-landau.de/download/Platz/Disputation_Platz.mp4` (retrieved 16/02/2014)

That this dissertation could succeed, I especially owe to my friends and my family who always supported me morally and inspired me.
Special thanks to my supervisors *Engelbert Niehaus* and *Marlien Herselman* for their patience and support, for the time invested for discussions and for the local hospitality in South Africa and the help in obtaining current information about South Africa.
I would especially like to thank *Prof. Dr. Gabriele E. Schaumann*, who has chaired the doctoral committee and *Prof. Dr. Hugo Lotriet* who has agreed to act as an additional supervisor.
I thank the project members of the *Reglan-Health and Logistics* project and the *AT6-FUI* for the constructive discussions at the project meetings and the helpfulness of each individual member.
I also thank the *Institute of Mathematics* of the *University of Koblenz-Landau, Campus Landau* for constructive criticism.
I would like to thank the *Integrierte Gesamtschule Landau* for the good support in the implementation of the empirical pilot study, and especially the pupils and teachers of the class 5 d (in the school year 2012/2013).
I also thank *Alexa Barnby*, who enriched this dissertation by corrections on the linguistic level.
I thank *Dr. Stefan Jergentz* for the provision of the altitude map of the university campus.
Finally, I thank at this point the program *NaWi* and the *Förderung des Wissenschaftlichen Nachwuchses* of the *University of Koblenz-Landau*, which made it possible for me to participate in thematically relevant conferences by financial supporting me and gave me therefore an opportunity to discuss my dissertation topic with African expert audience.

Landau, February 2014 Melanie Platz

Kurzfassung

Mathematische Modellierung für adaptive graphische Benutzerschnittstellen, die ein angepasstes Verhalten in Abhängigkeit von Geographischen Informationssystemen besitzen und durch räumliche Fuzzy-Logik gesteuert werden

Diese Dissertation entsteht im Rahmen des Projekts *Research-Group Learning and Neurosciences* (ReGLaN)-*Health and Logistics*, welches die Optimierung der Gesundheitsversorgung in ländlichen Gebieten Südafrikas zum Ziel hat. Es besteht dabei eine Kooperation mit dem *Council for Scientific and Industrial Research* (CSIR) *Meraka Institute* mit *Prof. Dr. Dr. Marlien Herselman*, Pretoria, Südafrika, als zentrale Ansprechpartnerin. Die Dissertation befasst sich mit der mathematischen Modellierung für adaptive graphische Benutzerschnittstellen (GUI), die ein angepasstes Verhalten in Abhängigkeit von Geographischen Informationssystemen (GIS) besitzen und durch räumliche Fuzzy-Logik gesteuert werden. Innerhalb der Arbeit geht es um die mathematische Visualisierung von maßgeschneiderten Risiko- und Ressourcenkarten für epidemiologische Fragestellungen mit GIS und adaptives GUI-Design für eine Open Source (OS)-Anwendung für digitale Endgeräte zur räumlichen Entscheidungsunterstützung zugeschnitten auf unterschiedliche Benutzergruppen. Zur Evaluation und Initialisierung der GUI-Elemente wurde empirische Lehr-Lern-Forschung zum Umgang mit Geomedien und GUI-Elementen eingesetzt.

Abstract

Mathematical Modelling of GIS-Tailored GUI Design with the Application of Spatial Fuzzy Logic

This PhD thesis is situated within the framework of the *Research-Group Learning and Neurosciences* (ReGLaN)-*Health and Logistics* project. The goal of this project is the optimisation of health service delivery in the rural areas of South Africa. Cooperation takes place between *ReGLaN-Health and Logistics* and the South African *Council for Scientific and Industrial Research* (CSIR) *Meraka Institute*, with *Prof Dr Dr Marlien Herselman* of Pretoria, South Africa, as the central contact person. This thesis deals with the mathematical modelling of Geographic Information System (GIS)-tailored Graphical User Interface (GUI) design with the application of spatial fuzzy logic. This thesis considers the mathematical visualisation of risk and resource maps for epidemiological issues using GIS and adaptive GUI design for an Open Source (OS) application for digital devices. The intention of this thesis is to provide spatial decision support tailored to different user groups. In order for the GUI elements to be evaluated and initialised, empirical teaching-learning-research on dealing with geomedia and GUI elements was conducted.

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Abbreviations

ACO Ant Colony Optimisation

ACT Artemisinin-Based Combination Therapy

AED Automated External Defibrillator

AIDS Acquired Immunodeficiency Syndrome

ANN Artificial Neural Network

ANOVA Analysis of Variance

AR Augmented Reality

ART Adaptive Resonance Theory

ASR Automatic Speech Recognition

AT6 FUI *Action Team 6 Follow up Initiative*

ATM Automated Teller Machine

BP Backpropagation

CASE Computer-Aided Software Engineering

CS Closed Source

CSIR *Council for Scientific and Industrial Research*

DALY *Disability Adjusted Life Year*

DDT Dichloro-Diphenyl-Trichloroethane

DSS Decision Support System

DST *Department: Science and Technology*

ED Emergency Department

EHR Electronic Health Record

EU *European Union*

EWARS Early Warning and Response System

FLC Fuzzy Logic Controller

FSD Functional Specification Document

GIS Geographic Information System

GIST GIS Tailored

GUI Graphical User Interface

GPL General Public License

GPRS General Packet Radio Service

GPS Global Positioning System

HCM Health Care Management

HIV Human Immunodeficiency Virus

HMD Head-Mounted Display

HOT *Humanitarian OpenStreetMap (OSM) Team*

ICT Information and Communication Technology

ICU Intensive Care Unit

ILS Integrated Logistic Support

IP Internet Protocol

IT Information Technology

ITN Insecticide-Treated Mosquito Net

ITS Intelligent Tutoring System

IRS Indoors Residual Spraying

KIT *Karlsruhe Institute of Technology*

LCD Liquid Crystal Display

OC Open Content

OO Object Orientation

OOA Object-Oriented Analysis

OOD Object-Oriented Design

OOP Object-Oriented Programming

OMG *Object Management Group*

OR Operation Room

OS Open Source

OSM *OpenStreetMap*

PARC *Palo Alto Research Center*

PC Personal Computer

PDA Personal Digital Assistant

PHA *Public Health Agency*

ReGLaN *Research-Group Learning and Neurosciences*

RMS Risk Management System

SAGU System for Adapting a GUI to a User

SDSS Spatial Decision Support System

S-I-R Susceptible-Infected-Recovered

SQI Spatial Quality of Information

TCIU Tool for Collecting Information about the User

UML Unified Modelling Language

UPS Uninterruptible Power Supply

UX User Experience

VCR Video Cassette Recording

WHA *World Health Assembly*

WHO *World Health Organisation*

I | Introduction and Objectives

Sub-Saharan Africa is experiencing a significant waste of resources. This region, while representing 9% of the world's population, currently accounts for 26% of the burden of disease as measured by the *Disability Adjusted Life Year* (DALY). The solution of such problems, which plague the health systems of developing countries, is therefore important in the context of decision making and management (cf [BKA+97]).

Resources in this context can refer to, inter alia, medication, a medical staff member or information concerning, for example, actions to be taken and preventive measures. Such information can be delivered via Information and Communication Technology (ICT), for example smartphones in areas where its distribution within the target audience is high. This is discussed in more detail in SECTION 2.2. In this thesis, the term "health" is defined according to the following definition of [Wor48]:

> "Health is a state of complete physical, mental and social well-being and not merely the absence of disease or infirmity."

The optimisation of health service delivery is an important issue. This thesis arises within the context of the *Research-Group Learning and Neurosciences (ReGLaN)-Health and Logistics* project, which will be further described in CHAPTER 1.

The project founder, *Gerhard Ackerman*, identified the fact that the sustainability of an optimised health care system would only be achieved if that system could be adapted to a changing environment and the requirements and restrictions of that environment, and if the process could be repeated quickly. Accordingly, an easy-to-use and fault-tolerant Decision Support System (DSS) tool is required, where the user can see a short-term benefit and which leads to a high level of acceptance. This is dependent on sustainability and ease of maintenance on a daily basis (cf [Ack07]).

An important milestone in the project was the development of an application for a digital device delivering early warning and decision support tailored to different user groups using a Graphical User Interface (GUI) that is adapted to the user. A mathematical modelling of Geographic Information System (GIS)-tailored GUI design, with the application of spatial fuzzy logic, results from these requirements and constraints. This thesis concerns the mathematical visualisation of risk and resource maps for epidemiological issues using GIS and adaptive GUI design for an Open Source (OS)-application for digital devices. The intention here is to provide spatial decision support tailored to different user groups. The aim of this thesis is to develop the mathematical structure behind a system for an adaptive GUI tailored to different user groups for an application that serves as an early warning and decision support system in hazardous situations. For the evaluation and initialisation of the GUI elements, empirical teaching-learning research on dealing with geomedia and GUI elements was conducted (see CHAPTER 11). In the following chapters we consider decision support for the user; for example in or in preparation for a hazardous situation, such as the epidemiological outbreak of a disease, ecotoxicological or radioactive exposure or an accident situation. For this purpose we have to generate risk and resource maps, as well as adaptive GUIs tailored to different user groups, that support the comprehension of the spatial sense for optimal decision support. A risk map assigns a risk to a geographic location (see FIGURE 1), while a resource map assigns the availability of resources to a geographic location. These risk and resource maps can be combined with fuzzy logic to illustrate

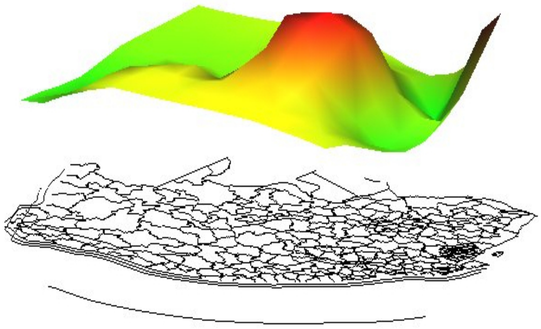


Figure 1: Exemplary risk map. (Figure generated with *GRASS GIS*.)

the degree of validity of a property at a certain geographic location. The maps are processed and visualised using a GIS (see p.14) and decision support is delivered to the user via a digital device using an adaptive GUI. The procedure for generating maps for decision support is described in SECTION 10.1, while GUIs are expounded on in CHAPTER 5. In order to generate risk maps, we have to predict the spatial risks of, for example, infectious diseases. Infectious diseases are considered in SECTION 2.1, with the focus being on the infectious disease, malaria (see CHAPTER 10). Malaria is transmitted by mosquitoes and occurs in our pilot region of South Africa.

The risk and resource maps and, finally, the maps for decision support are developed using mathematical modelling. The term "mathematical model" is used in this thesis in line with the definition of [GAJ04], p.4:

> "A mathematical model is a representation in mathematical terms of the behaviour of real devices and objects."

In order to design a good model we have to find out which environmental parameters are important. Such parameters could include temperature, rainfall, wind speed, wind direction, human population, animal population, pathogens (e.g. protozoa), and the mosquito life-cycle. Accordingly, the Susceptible-Infected-Recovered (S-I-R)-model may be important for the generation of a malaria risk map. In this regard we have to model the distribution of resources and combine the risk maps with maps of available resources for optimal decision support and early warning and response. Important resources for preventing malaria include mosquito nets, insect repellents, insecticides and information about the disease.

As "all models are wrong, but some are useful" ([Box76]), it is helpful to determine the quality of the maps by, among other things, observing the Spatial Quality of Information (SQI) (see p.185) in order to evaluate the information we obtain from a map. Even if the SQI at a geo-location is not very good, but not bad either, it is still better than having no information at all. The information gained is at least a clue. The functionality of a Spatial Decision Support System (SDSS) and an Early

Warning and Response System (EWARS) is described on pages 16 and 13. The demand for different GUIs arises from the differing demands of different user groups. User groups can include, for example, decision makers such as governmental facilities, *Public Health Agency* (PHA)s, vector control units, medical doctors and nurses or users who find themselves in hazardous situations. If we focus on the decision maker user groups (e.g. a governmental facility) and users who are in a hazardous situation, we have the following requirements: The decision maker wants to know where to locate resources, and users in hazardous situations want to know how they can protect themselves. Exemplary GUIs for both these users are illustrated in FIGURE 2. One user can belong to different user groups, for

(a) Exemplary GUI for a user who is situated in a hazardous situation.

(b) Exemplary GUI for a decision maker.

Figure 2: Exemplary GUIs for different user groups on different digital devices. (Figure generated with *GIMP*, *Libre Office Draw* and *GRASS GIS*.)

example a person who works for a vector control unit and has to travel to a hazardous area to apply pesticides. While travelling, this user could use the GUI of a user who is in a hazardous situation. Chapter 10 considers this in more detail. A GUI for a mobile device, for example a smartphone, is useful for a user in a hazardous situation. In Section 2.2, several digital devices are investigated with respect to their distribution in South Africa. The application of OS software is a requirement for the development of this application. The benefits of OS are mentioned in Chapter 3.

As the application for a digital device should be operational offline, because internet access cannot be guaranteed everywhere, these systems should be relatively fault-tolerant, that is, even if remote services are temporarily unavailable (e.g. lack of internet connection). In such an event, the system should be able to provide at least basic functions such as diagnosis, treatment and the handling of basic medical processes (cf [Ack07]).

In terms of a System for Adapting a GUI to a User (SAGU), the processes can be automated with the use of fuzzy logic, that is, membership functions are used to provide automated decision support. To adapt the GUI to the user, we use mathematical tools such as fuzzy logic, Artificial Neural Network (ANN)s and measure theory. Fuzzy logic and ANNs are discussed in Chapter 7 and measure theory in Chapter 8. The system is modelled using Object-Oriented Analysis (OOA), which is discussed in Chapter 6.

The determination of design faults in software development before realisation is made possible by the modelling technique OOA. As a result, design implementation costs and risks can be controlled (see [Nie07]).

The coarse functioning of the SAGU is illustrated in Figure 3, while the mathematical structure behind such a system is described in Chapter 9. According to [AHD09], the term "information" can

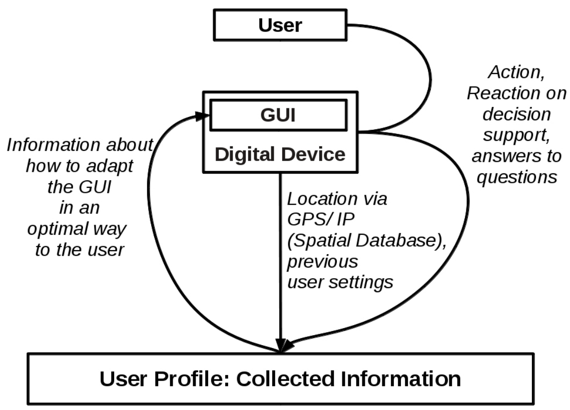


Figure 3: Coarse functioning of the application. (Figure generated with *Libre Office Draw.*)

be placed somewhere between "data" and "wisdom", if we consider the amount of knowledge:

> "Data are facts that can be analysed or used in an effort to gain knowledge or make decisions. [...] Information is knowledge or facts learned, especially about a certain subject or event. [...] Wisdom is the ability to discern or judge what is true, right, or lasting; insight. Wisdom is the sum of learning through the ages; knowledge."

Information about the user is only collected in a database with the user's permission (data protection).
In the next part, PART II, the requirements and constraints are analysed for the development of an application for a digital device delivering early warning and decision support tailored to different user groups using a GUI adapted to the user. Therefore, the layout of this part is as follows: CHAPTER 1 describes the *ReGLaN-Health and Logistics* project. The region of South Africa is used as pilot region (CHAPTER 2). The focus here falls on the South African health system as well as the current socioeconomic situation and the diseases occurring in South Africa. It also discusses the current situation relating to Information and Communication Technology (ICT) in South Africa. In CHAPTER 3 OS and Open Content (OC) are addressed, while CHAPTER 4 presents an interim conclusion and focuses on risk perception.
In PART III, the concepts that are merged in order to develop an SAGU are described. This includes topics like GUI for an application delivering early warning and decision support in CHAPTER 5; Object Orientation (OO) for preparing the creation of a prototype of the GUI for an application delivering early warning and decision support in CHAPTER 6; fuzzy logic and ANNs for implementing adaptivity in CHAPTER 7 and measure theory on topological spaces for evaluating fuzzy membership mappings on non-Euclidean spaces in CHAPTER 8.
In PART IV, the concepts addressed in PART III are applied to the development of a mathematical structure of an SAGU. Initially, the mathematical structure of an SAGU is presented in CHAPTER 9. CHAPTER 10, meanwhile, gives an object-oriented overview of the system and addresses maps for visualisation and exemplary GUIs tailored to different user groups. Finally, the GUIs are evaluated in CHAPTER 11.
A conclusion, PART V, completes the thesis with a summary, a discussion of the results, prospects and closing remarks.
FIGURE 4 gives a graphic representation of this thesis.

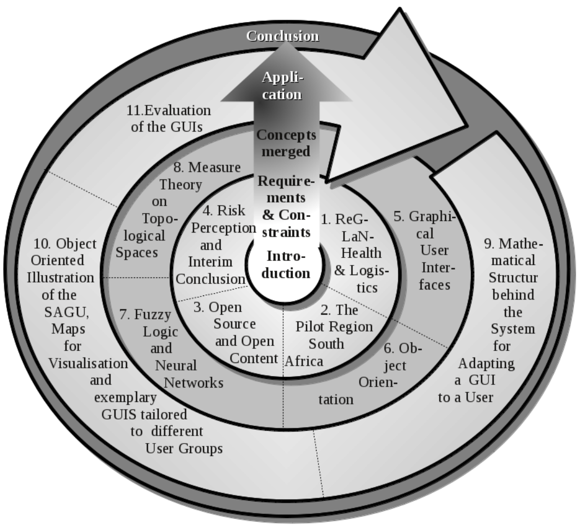


Figure 4: Structure of this thesis. (Figure generated with *Libre Office Draw.*)

II | Requirements and Constraints Analysis

1 ReGLaN-Health and Logistics

The context in which this thesis is embedded is the *ReGLaN-Health and Logistics* project and the *Action Team 6 Follow up Initiative* (AT6 FUI). The goal of the *ReGLaN-Health and Logistics* project is to optimise health service delivery in rural areas. This project is based on an idea by South African-born *Gerhard Ackermann* and is aimed at the optimisation of health care in the rural areas of South Africa. Accordingly, medical doctors, psychologists, logistics experts, software engineers, researchers in communication and information technologies and mathematicians work together in order to find a way in which to optimise health service delivery in rural areas. Within this group, there is cooperation between *ReGLaN-Health and Logistics* and the South African *CSIR's Meraka Institute*, with *Prof. Dr. Dr. Marlien Herselman* as the central contact person. The project aims for the development of an SDSS with tutorial components by exclusively using OS software.

The best support for people in rural areas can presumably be provided by a multidisciplinary, holistic approach. *ReGLaN-Health and Logistics* develops such a multidisciplinary approach, using Integrated Logistic Support (ILS) as a core principle. In this regard, ILS is a science which ensures the optimal availability of a system at a minimum cost (cf [Ack07]).

In Figure 1.1 the collaborating disciplines within the concept and the concept of the project itself are visualised. The project homepage is accessible via the link below:

`http://reglan-health-logistics.weebly.com` (retrieved 16/02/2014).

Within the *ReGLaN-Health and Logistics* project, the optimisation of health service delivery is an important issue. One important milestone for this project is the development of an application for a digital device (e.g. a mobile device such as a smartphone) delivering early warning and decision support tailored to different user groups, using a GUI that is adapting to the user. The aim of this thesis is to develop the mathematical structure behind a system for an adaptive GUI tailored to different user groups. The research addresses the mathematical visualisation of risk and resource maps for epidemiological issues using GIS and adaptive GUI design for an OS application for digital devices, including mobile devices, in order to provide spatial decision support tailored to different user groups. This is done by means of the mathematical modelling of GIS-tailored GUI design with the application of spatial fuzzy logic. The GUIs are developed for a software application delivering an SDSS within the framework of an EWARS. In this context, the terms "risk" and "resource" are defined as following:

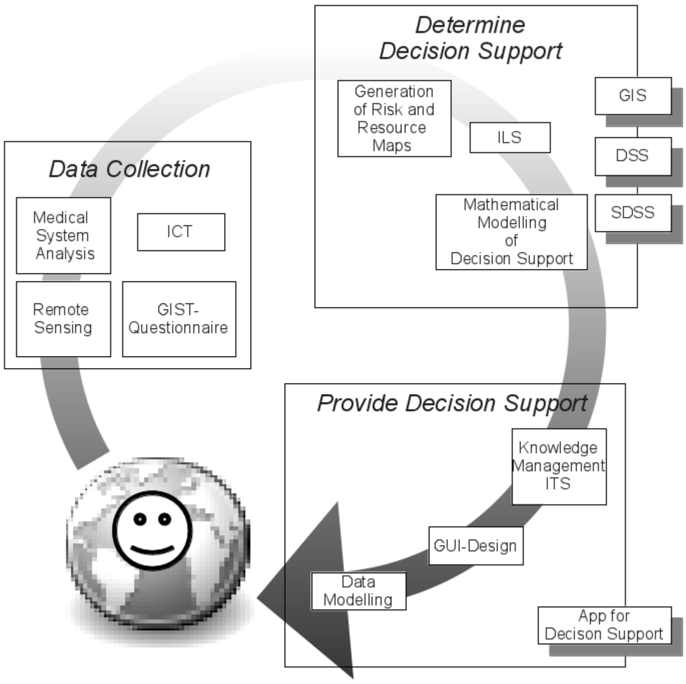


Figure 1.1: Collaborating disciplines in and concept of the *ReGLaN-Health and Logistics* project. (Figure generated with *Libre Office Draw* with supplements from `http://openclipart.org` (retrieved 16/02/2014).)

Definition 1.0.1 *Risk([NJ04])*
Risk is a quality that reflects both the range of possible outcomes and the distribution of respective probabilities for each of the outcomes. This can be expressed:

$$Risk \;=\; Probability\ (of\ an\ event)\ \cdot\ Impact\ (or\ severity)\ of\ an\ event.$$

In this thesis, the focus is on epidemiological risk, for example malaria risk.

Definition 1.0.2 *Resource ([AHD09])*
A Resource is something that can be used for support or help, an available supply that can be drawn on when needed.

In this thesis, mosquito nets, insecticides, medical devices and information (about action measures) are used as examples of resources.
Some of the terms used within the context of the development of an application for a digital device delivering early warning and decision support tailored to different user groups with a GUI that is adapted to the user are explained in more detail below. We also examine the applications of or current research concerning these terms and topics:

- spatial epidemiology
- optimisation of health systems
- mobile devices for optimisation processes
- EWARS
- GIS
- SDSS
- *AT6 FUI*

We will see, that, in contrast to our developing concept that builds on the exclusive usage of OS software and OC, most of the mentioned applications use proprietary software and proprietary content. The system developed in this thesis links ANNs, a Tool for Collecting Information about the User (TCIU), a Fuzzy Logic Controller (FLC) and error determination for adapting the GUI of a digital device to a user. The application is supposed to deliver early warning and spatial decision support to the user. Therefore an EWARS and an SDSS as combination of DSS and GIS are used. Measure theory on topological spaces, ANNs and fuzzy logic are used as mathematical tools for mathematical modelling. OO is used for the development of an OS application. The system developed in this thesis is supposed to deliver a contribution to health optimisation.

Spatial Epidemiology. The mathematical visualisation of risk and resource maps for addressing epidemiological issues is discussed in this thesis. In SUBSECTION 10.1.1, the generation of risk maps that visualise the risk for anopheles mosquito survival on the basis of rainfall and humidity is elucidated.
In order to understand the causes and consequences of spatial heterogeneity in infectious diseases, particularly in zoonoses, that is, diseases that are transmitted to humans from non-human vertebrate reservoirs, spatial epidemiology has arisen as one of the principal scientific disciplines ([OGK05]). Therefore, in this regard spatial epidemiology is of importance.

Definition 1.0.3 *Spatial Epidemiology ([OGK05])*
Spatial epidemiology is the study of spatial variation in disease risk or incidence.

[SRVB⁺13] deals with the mapping of environmental suitability for malaria transmission in Greece. In this research, only environmental suitability for transmission is considered and not the risk for transmission. This model is based on the results of a standardised questionnaire administered by a health officer for malaria patients in Greece, which was intended to determine the origin of infection. [LdPK⁺13], in which a study was conducted in an area of Brazil, found that by using a generalisation

of mathematical modelling, namely the *Ross-Macdonald model*, biodiversity can help prevent malaria outbreaks in tropical forests, which can then be described by a set of differential equations. The model used was identical to the *Ross-Macdonald model*, if it is considered that wild warm-blooded animals were either absent or did not interact with the vector, if non-vector mosquito species were absent or did not interact with the vector, if humans did not react to mosquito bites and if vector abundance was constant.
In this thesis we use mathematical modelling, including fuzzy membership mappings and measure theory on topological spaces, to develop risk, resource and decision support maps relating to epidemiological issues, inter alia, in non-Euclidean spaces. The focus of this thesis is the disease malaria as it occurs in the pilot region of this study, that is, South Africa.

Optimisation of Health Systems. The focus of the *ReGLaN-Health and Logistics* project is the optimisation of health service delivery. Some ideas for the optimisation of health systems are in development or have already been implemented.
[KKLC⁺07] uses systems engineering for this and describes systems engineering as "the process of identifying the system of interest, choosing appropriate performance measures, selecting the best modelling tool, studying model properties and behaviour under a variety of scenarios, and making design and operational decisions for implementation" in the United States health care delivery system. In this paper, the term "system" is defined as follows:

Definition 1.0.4 *System ([KKLC⁺07])*
A system is a set of possibly diverse entities[1]*, each performing some set of functions. The interaction of these entities as they perform their various functions gives rise to a global system behaviour.*

The following examples for systems engineering applications to health care delivery are treated by [KKLC⁺07]:

- Therapeutic optimisation. Models have been developed for
  - controlling the modulation mechanism so that the cancerous region receives the prescribed dose while collateral damage to surrounding health tissue is minimised
  - kidney allocation problems
  - Human Immunodeficiency Virus (HIV) treatment
  - seizure warnings
  - vaccine protocols ([KKLC⁺07]).

[1]e.g. patients, nurses, physicians, etc.

- Hospital operations modelling. Similar models of all hospital departments are developed. These models will be merged into one hospital model that will capture resource dependencies that propagate across the system to enable a statement about
  - how discharge policies affect daily operations in the Emergency Department (ED) and Operation Room (OR).
  - how ED congestion and OR scheduling policies affect medical surgical wards, Intensive Care Unit (ICU)s, and lab facilities over the short-term ([KKLC+07]).

[KKLC+07] mentions that systems engineering techniques have extensive data requirements. Accordingly, the management of this data requires expensive integrated information systems that are often not available in the health care sector and that managerial support for systems improvement can be difficult to obtain. "One possible approach is to use appreciative inquiry to help uncover enablers and motivators that will help systems engineering methods gain wider acceptance." ([KKLC+07]).
The idea put forward by *ReGLaN-Health and Logistics* goes even further. The developed OS-application should not only work within health facilities, but also everywhere else and it should be developed for more user groups. This use independence of spatial conditions can be realised by providing the application for several digital devices, inter alia, mobile devices, such as smartphones.
Another approach to optimise medical care, which focuses on the patient, is described by [BBL+12]. [BBL+12] provides decision support for medical professionals via data monitoring and feedback treatment. Additionally, [BBL+12] developed a computer-implemented method for modelling patient outcomes resulting from treatment in a specific medical area.
Such an approach would fit well into the application for early warning and decision support developed by *ReGLaN-Health and Logistics*, to relieve medical professionals and maybe enable even more patients to be helped.
Pursuing the goal of increasing cardiac arrest coverage and decreasing the distance to the closest Automated External Defibrillator (AED) by optimising AED deployment, [CLL+13] identified locations for public access defibrillators using mathematical optimisation. Therefore, geographical clusters of cardiac arrests were identified and prioritised by [CLL+13] using mathematical modelling.
ReGLaN-Health and Logistics uses logistics optimisation and mathematical modelling, including fuzzy logic, to optimise resource distribution.

Mobile Devices for Optimisation Processes. Mobile devices, for example smartphones, are widely used for optimisation processes, including crowd sourcing, GIS and decision support.
The article by [MZ08] is concerned with the reduction in the number of road accidents, especially in Malaysia, by using an intelligent road accident system. This system can be used for a comprehensive intelligent GIS-based solution for accident analysis and management. Accordingly it is able to send online data to a road accident database using a Personal Digital Assistant (PDA) or smartphone by police officers with real-time access.

[CH11] mentions smartphone and *iPhone* apps that make it easier for users to make green decisions with the use of GIS. Through the increase in smartphone users, apps can be used to assist with widespread reporting and awareness. The agricultural industry uses GIS for mapping and input, and machine control to minimise fertilisers and pesticide usage, cutting costs while also reducing the pollutants released into the environment.

[SMS+10] developed an information system as a web portal that allows users to collect, share and consult real-time data. Agronomists can consult formatted data to support their decisions on their regional pest and disease control. An *API Google map mashup* presents dynamic maps of the French regions where data was collected. This combines dynamic colour icons and charts, allowing for data mining and facilitating interpretation (disease or pest infection level). In addition, in order to accelerate data collection, mobile web pages using the latest mobile technologies (XHTML) for field data transmission by smartphone were added.

In contrast to our developing concept, which builds on the exclusive usage of OS software and OC (like the use of *OSM* instead of *API Google map mashup*), these above-mentioned applications use proprietary software and proprietary content. By applying *OSM*, offline use is made possible, which is important for the development of our application, because in hazardous situations, internet connection may not be easy to find. Additionally, and in contrast to the above-mentioned research, we are using mobile devices in the sense of mHealth.

EWARS. An EWARS is used to give advance notice of some impending event or development and to support the provision of a response. It is needed because outbreaks and micro-organisms recognise no borders and can spread rapidly if action is not taken in time. An EWARS can provide support for the government, the health system, vector control units, users who find themselves in hazardous situations, among others. FIGURE 1.2 shows an EWARS for a viral disease transmitted by mosquitoes,

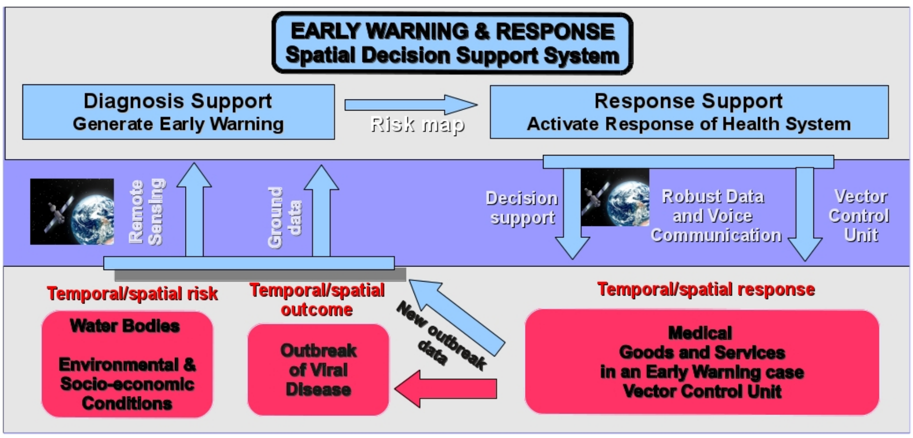


Figure 1.2: Early warning and response cycle. (Compare: `http://www.oosa.unvienna.org/pdf/pres/stsc2012/tech-10E.pdf` (retrieved 16/02/2014).)

for example chikungunya[2]. First, data is collected (lower left), then an early warning is generated (upper left), afterwards response support is delivered (upper right) and the distribution of resources is done (lower right). Then the starting point of the early warning and response cycle is reached with the collection of ground data (lower left) and the cycle starts again.
An EWARS is used by the *European Union* (EU):

Definition 1.0.5 *EWRS ([ECD13], p.5)*
EWRS is a confidential computer system allowing member States to send alerts about events with a potential impact on the EU, share information, and coordinate their response.

The EWRS has already been successfully used for previous outbreaks of SARS, Pandemic Influenza A (H1N1) and other communicable diseases. ([ECD13], p.5).
In Figure 1.3, a disaster management cycle known as the Risk Management System (RMS) is illustrated. This cycle is useful for acute events such as earthquakes or volcanic eruptions. As we are concerned with chronic events, such as epidemiological or ecotoxicological events, the early warning and response cycle illustrated in Figure 1.2 is more suitable.

Figure 1.3: Disaster management cycle. (Figure generated with *Libre Office Draw*. Compare: `http://www.degreescape.com/college-majors-and-career-fields/emergency-management` (retrieved 16/02/2014).)

GIS. Humans try to describe the earth's surface so that they can describe any point and any geometric constellation on it clearly and reproduce it at will numerically or graphically. However, on the one hand, they have a problem of developing a mathematical formula apparatus that makes it possible to describe an exact location and, on the other hand, that makes it possible to find a way of illustrating the obtained results in a way that they are clearly readable and understandable without the knowledge of such an apparatus. For a long time recording and reproducing the surface shape of

[2]The arbovirus chikungunya is transmitted to humans primarily by the mosquito species aedes aegypti and aedes albopictus, two species which can also transmit other mosquito-borne viruses, including dengue. The name, chikungunya, originates from the Makonde dialect of Tanzania, and refers to the patient's contorted posture as a result of severe joint pains. Records also show that the same disease has been referred to in India as aakdaya and maakdya, meaning "stiff-man" and "monkey-like" respectively. ([TSW09].

the Earth, that is, the topography and cartography, was the primary task of such systems. By the end of the 18th century, it was increasingly recognised that a large number of natural phenomena are a local issue and that their accurate recording and reproduction among different aspects of human society is not only valuable but also of vital importance. The data of geographic information are strictly site specific and describe characteristics of certain points, areas or spaces that are defined by coordinates (cf [Qua04]).

Within our application, the following data space is used:

$$X_{\Gamma_0} := S^2_{R_0}(z) \times T$$

with $S^2_{R_0}(z) = \{\omega \in [0, R_0]^3 \mid \|\omega - z\|_2 = R_0\}$ with

$$\omega = \begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix} = \begin{pmatrix} R_0 \cdot \sin(\theta) \cdot \cos(\varphi) \\ R_0 \cdot \sin(\theta) \cdot \sin(\varphi) \\ R_0 \cdot \cos(\theta) \end{pmatrix}$$

with $\theta \in [-\frac{\pi}{2}, \frac{\pi}{2}]$ which corresponds to the latitude $[-90°, 90°]$ and $\varphi \in (-\pi, \pi]$ which corresponds to the longitude $(-180°, 180°]$, $z = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ is the barycentre of the three-dimensional object describing the earth, $T = \mathbb{R}$ the time and R_0 the distance from the barycentre z to the earth's surface at the timepoint $t \in T$ (averaged $6371km$).

$$\|\omega - z\|_2 = \sqrt{(\omega_1 - 0)^2 + (\omega_2 - 0)^2 + (\omega_3 - 0)^2}.$$

Complexity increases, when a certain topology considers non-Euclidean distance measures, which is the case within the SAGU, Chapter 9. In this case, tools of measure theory on topological spaces (Chapter 8) are used.

Over time not only has the demand for such data become significantly larger, but also the demands on data quality (see p. 185) and the fast and flexible determination and exchange of such data has increased. The once static map products, which are temporal horizontal sections for the illustration of an immediate state, have become dynamic products which the user can equip with content according to the problems and which can be configured and constantly updated as desired (cf [Qua04]).

Definition 1.0.6 *Geographic Information System (GIS) ([BV99], p.1)*
A Geographic Information System is a tool for planning development and environmental control, as well as an instrument for decision support. On the one hand, it consists of a georeferenced database, while on the other it comprises techniques for data acquisition, actualisation, processing and visualisation of the results. The semantic data are geometrically related to a homogeneous geo-referenced coordinate system, allowing for the controlled interrelation of the information.[3]

[3] FIG (International Federation of Surveyors) definition of Land Information System (LIS)

GISs describe two-dimensional and three-dimensional relationships of characteristics and situations using models in which descriptive parameters are linked with spatial parameters and basic geodata are linked with technical geodata (cf [Qua04]).
In a GIS database georeferenced data can be organised using different criteria, for example as thematic layers or spatial objects (cf [NM08], p.7) whereby objects can be understood as described in OO, Chapter 6.
Single thematic layers can be evaluated using fuzzy theory (Chapter 7), which is illustrated in Chapter 9.
With the use of geo-mathematical and geostatistical analysis data, GISs become powerful tools that provide the user with quick, reliable and scale-independent information on regularities and local circumstances even for more comprehensive and thematically complex databases (cf [Qua04]).
Over the past ten years GIS have developed into a technology that influences almost every aspect of our lives: from the search for driving directions to the management of natural disasters. While a few years ago only a small group of researchers, planners and government employees were able to use GIS, it is now possible for almost anyone to create custom maps or overlay GIS data. Many complex problems related to urban and regional planning, environmental protection, or business management, require sophisticated tools and specialised know-how - in our case mathematical tools such as fuzzy logic and measure theory. That is why the latest GIS technology includes a wide range of applications from viewing maps and images on the internet to spatial analysis, modelling and simulations. Besides the widely used proprietary systems, an OS GIS plays an important role in the adaption of GIS technology by simulating new experimental approaches and by providing access to GIS for the users who cannot or do not want to use proprietary products (cf [NM08], p.1).
Within this thesis, the OS-GIS GRASS GIS is used.

SDSS. Within the project an SDSS will be developed. In the following section, we examine the concept of an SDSS.

Definition 1.0.7 *Decision Support System (DSS) ([SWC+ 02] and [MCI09], p.1 and p. xviii f)*
A DSS can be defined as a computer-based tool used to support complex decision making and problem solving. It provides access to a wealth of information pertaining to a specific problem. The following types of information are available: information content, maps and data. This information may be contained in databases and GIS. DSSs may help to answer questions about the level of risk[4], the remedial technology options, the costs or whether the regulatory targets can be achieved. DSSs can provide powerful functionalities for analysis, visualisation, simulation and information storage that are essential to complex decision processes. Moreover, information and options can be presented within an ordered structure, visualised from a space-time perspective, elaborated in simulated scenarios

[4] Definition 1.0.1

and therefore more easily discussed by the interested parties to reach a common rehabilitation objective.

Definition 1.0.8 *Spatial Decision Support System (SDSS) ([YPXC07])*
A SDSS is a combination of GIS and DSS.

Example 1.0.9: *A SDSS for, for example, epidemic disease prevention, includes epidemic spread models in a GIS. Spatial information systems containing spatial and temporal data on epidemic diseases and their application models are intended to help health care facilities to implement and optimise disease surveillance and disease monitoring. ([YPXC07]).*

FIGURE 1.4 illustrates the system structure of an SDSS. The application developed by the *ReGLaN-*

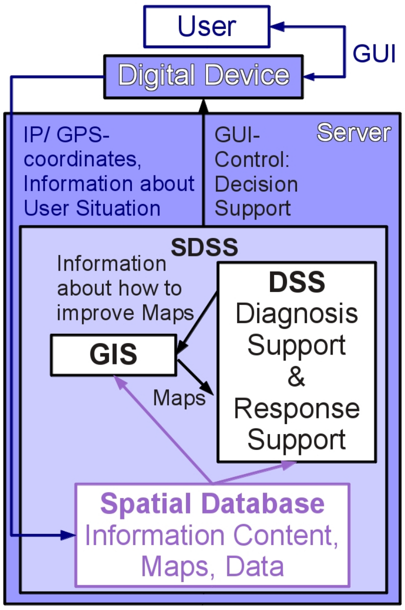


Figure 1.4: General structure of an SDSS. (Figure generated with *Libre Office Draw*.)

Health and Logistics project goes back to information delivered by the SDSS. Using measure theory, the quality of a decision delivered by the SDSS can be measured.
OO is often used for software development and the following research combines OO with GIS and fuzzy methods for decision support:
([FJ96]) describes a *WaterWaresystem* which is an object-oriented information system and DSS for river basis management. Classes of objects, including river basin elements, models and model scenarios, and tasks or decision problems are combined with a hybrid GIS by the basic data framework.
In ([BHW+04]) principal strategies of OOA are explained, the way in which the combination with fuzzy methods allows for the implementation of expert knowledge is discussed and a representative example of the proposed workflow from remote sensing imagery to GIS is described. ([BHW+04]) uses

the Closed Source (CS) software *eCognition* (`http://www.ecognition.com` (retrieved 16/02/2014)) an object-oriented image analysis software, which provides an appropriate link between remote sensing imagery and GIS; while ([BHW+04]) considers that there is a large gap between theoretically available information in remote sensing imagery and that extracted and used in order to support decision-making processes. Therefore, the approach of ([BHW+04]) focuses on:

- the extension of a signal processing approach for image analysis by exploring a hierarchical image object network to represent the strongly linked real-world objects
- the usage of polygons for suitable interface to GIS
- fuzzy systems for improved and robust modelling of real-world dependencies and a detailed quality check of the resulting product
- sensor and information fusion to use all available synergies.

In contrast to the *ReGLaN-Health and Logistics* project, these applications make use of proprietary software and proprietary content, such as *eCognition*. Furthermore, *ReGLaN-Health and Logistics* delivers DSS by combining, inter alia, the fields OO, GIS and fuzzy methods for users in hazardous situations and to improve the health system. Additionally, a first method to bridge the fields measure theory on topological spaces and OO is developed in this thesis.

AT6 FUI. The *ReGLaN-Health and Logistics* project and the *AT6 FUI* pursue similar goals. The *AT6 FUI* follows *UNISPACE III recommendation 6* to improve public health through the application of space technology. The proposed and discussed follow-up of *AT6 FUI* extends the main objectives in *recommendation 6*:

- Support the multinational concept of sharing in an OS, OC, Open Community Environment.
- Identify national objectives that overlap with objectives for the mitigation of structurally equivalent public health problems in other member states (sharing of developmental workloads).
- Enhance cross-national collaboration for public health problems and their mitigation through the application of space technologies.
- Through capacity building, support member states so that they can successfully mitigate the public health problem on their own. This initiative could operate as a networking hub and as a back office established through the concept of a living lab.

More information about *AT6 FUI* is accessible at the following link:

`http://at6fui.weebly.com` (retrieved 16/02/2014).

2 The Pilot Region: South Africa

South Africa is a Country in Africa consisting of nine provinces (see Figure 2.1) namely, Eastern Cape, Free State, Gauteng, KwaZulu-Natal, Limpopo, Mpumalanga, Northern Cape, North West, and the Western Cape ([LL99], p.35), with approximately 50 million inhabitants, ([Leh10], p.4).

Since the end of apartheid in 1994, South Africa has recognised eleven official languages: Zulu, Xhosa, Afrikaans, Pedi, English, Tswana, Sotho, Tsonga, Swazi, Venda and Ndebele ([LL99], p.12).

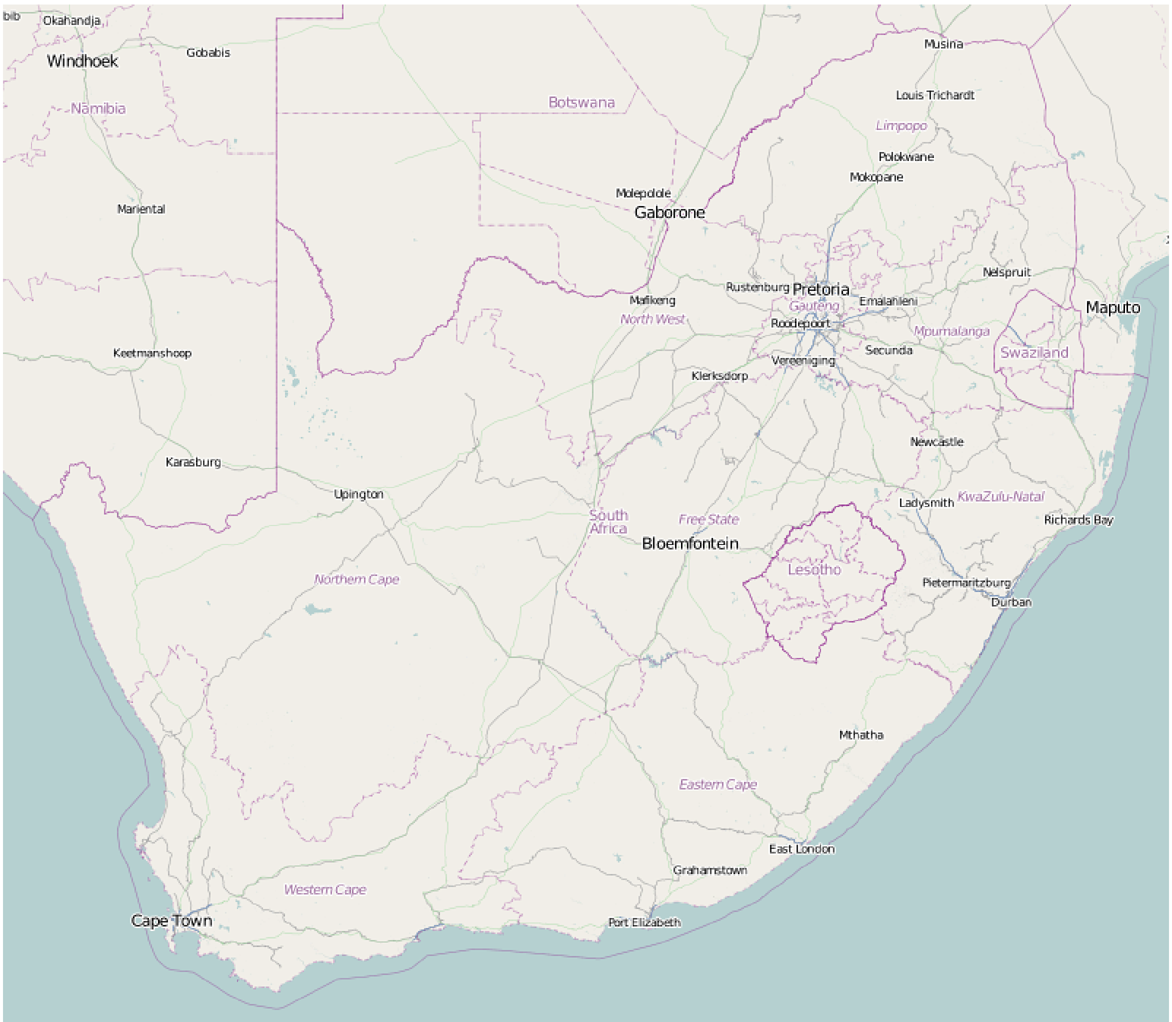


Figure 2.1: Map of South Africa. (Source: `http://www.openstreetmap.de/karte.html` (retrieved 16/02/2014).)

2.1 Health and Socio-Economic Situation in South Africa

In [CJB+09], the structure of the South African health sector is described as follows: The national Department of Health is responsible for the national health policy. Nine provincial departments of health are responsible for developing provincial policy within the framework of national policy and public health service delivery. There are three tiers of hospital: tertiary, regional, and district. The primary health-care system - a mainly nurse-driven service in clinics - includes the district hospital and community health centres. The local government is responsible for preventive and promotive services. The private health system, on the other hand, consists of general practitioners and private hospitals, with care in the private hospitals mostly funded through medical schemes. In 2008, 70% of private hospitals were situated within three of the country's nine provinces, with 38% being located in Gauteng province (Johannesburg and Pretoria) alone.
The political history of South Africa (colonial subjugation, apartheid, dispossession, post-apartheid period), which included racial and gender discrimination, the migrant labour system, the destruction of family life, vast income inequalities and extreme violence, provided the roots of a dysfunctional health system, as well as the collision of the epidemics of communicable and non-communicable diseases can be found. After the end of apartheid, macroeconomic policies were instituted that support growth rather than redistribution and thus favoured, despite a large expansion of social grants, the persistence of economic disparities between the races. The health sector has become an integrated, comprehensive national service. However, errors in leadership and responsibility, and poor management have led to inadequate implementation. Crucial aspects of primary health care are not available and there is a significant human resources crisis in the health sector. Furthermore, the HIV epidemic both contributes to and accelerates these challenges (cf [CJB+09]).

- Overall HIV prevalence rate in South Africa: 10, 5%, that is, 5, 24 million people living in South Africa are infected with HIV.
- The total number of new HIV infections in South Africa for 2010 is estimated at 410000. Of these, an estimated 40000 is assumed to be among children.

Life expectancy in South Africa is estimated at 53, 3 years for males and 55, 2 years for females. The infant mortality rate per 1000 live births is estimated at 46, 9 ([Leh10], p.3).
The South African Constitution obliges the state to work on the progressive realisation of the right to health. However, South Africa still has to struggle with serious health disparities. There are significant differences in the rates of disease and mortality among races, reflecting the racial disparities in access to basic household living conditions and other determinants of health. Accordingly, there are significant health inequalities between provinces and within provinces. Differences in the health status between men and women are also visible. Despite the fact that women have a higher rate of HIV infection, the mortality rate among men is 1, 38 times higher than among women (cf [CJB+09]).
[CJB+09] frames the following challenges for the new government:

- reduce the inter-provincial health inequities
- reduce urban-rural differences in access to health and related services
- reduce rapid urban drift and squalid urbanisation.

Achieving these goals requires

- More government spending on
  - health
  - education
  - social services and
  - a better redistribution of resources.
- Political leadership
- Social interventions (e.g. in schools) aimed at promoting a more responsible, caring and non-violent masculinity to reduce violence, crime, and Acquired Immunodeficiency Syndrome (AIDS).
- Renewed efforts to protect children and imaginative social programmes to support families are needed to redress the harm caused by years of apartheid and migrant labour.
- A strategy for land and rural development as well as employment creation and urban development.

[WPSH09], p.3, assumes that clinical care and public health can be improved by ICT. Accordingly, the appropriate use of ICT can, among other things,

- improve access to health care, by a redistribution of resources and communication about the location of the nearest resource
- improve the effectiveness of public health and primary care interventions, by providing tailored information provided on digital devices.

The feasibility and benefits of mHealth technology in South Africa has been described in several studies (cf [LS12]).

Definition 2.1.1 *mHealth ([LS12])*
Mobile health information technology (or mHealth) is a subsection of eHealth[1] *in that it refers specifically to the use of mobile information technology to improve health service delivery. MHealth technology*

[1]"EHealth is a broad term for the full spectrum of technological applications of ICT in health. The main objective of eHealth is to use ICT tools to improve health-care service delivery. EHealth captures all the components that may be part of an ICT system, including static or mobile devices, mechanisms for transporting signals and for management

involves portable hardware devices[2], as well as software applications and satellite, internet and wireless networks that allow for the rapid transmission, storage and retrieval of electronic data.

MHealth can also be used in conjunction with other non-mobile eHealth interventions, for instance, where a clinician can use a portable device to access electronic patient records, for prescribing, ordering, diagnostics or managing patient referrals. ([LS12]).
In light of the gaps in the evidence of the effectiveness of mHealth and the system challenges for the implementation of mHealth, it was found that for South Africa, a development approach to the implementation of mHealth was suitable, as this supports the implementation of smaller, phased and heavily evaluated projects within the routine service environment (cf [LS12]).
By developing an OS application for digital devices delivering early warning and decision support for health issues, for example in the form of telemedicine or mHealth, rural development is supported and hopefully rapid urban drift and squalid urbanisation can be reduced. Within the application, the focus is on the optimisation of resource distribution in order to bring about an improved redistribution of resources. The use of mobile devices (e.g. smartphones) and digital devices (e.g. digital doorways) is promising in this regard (compare Chapter 2.2). A first step for enabling the development of an executable prototype of this application for digital devices in the future is done in this thesis. For the development of reliable risk and resource maps, reliable data is fundamental. In this thesis, we focus on infectious diseases transmitted by mosquitoes, particularly malaria, to demonstrate, how early warning and decision support can be carried out with respect to preventing infection with an infectious disease.
Infectious diseases are diseases that are caused by an infection. Their severity depends on the infectivity of the pathogen and the preparedness of the organism. Between the infection and the outbreak of the disease is a varying duration of incubation. Prevention measures in this regard include the isolation of patients, disinfection, hygiene and vaccinations ([Pau00], p. 408).
Infectious diseases can be clustered in the following way (cf [ER88]):

- Infections caused by protozoa, for example malaria.
- Virus infections, for example chikungunya, dengue fever, influenza, hepatitis, AIDS.
- Infections caused by bacteria and fungi, for example leprosy.

In Chapter 10 we will concern ourselves with malaria as an example of infection caused by protozoa. This disease is transmitted by mosquitoes and is also found in South Africa. In this thesis, infectious diseases are considered because they influence the spatial mathematical modelling of public health

of information such as satellite receivers, the internet and computers. There is a wide range of eHealth interventions where the endâuser is the health professional, such as Electronic Health Record (EHR), e-Prescribing, ordering and communicating results of diagnostic tests and radiology, decisionâmaking job aids at the point of care and for delivering care at a distance, such as in telemedicine." ([LS12]).

[2]such as cellphones, digital pens, PDA or other handheld devices

issues. Epidemiological risk maps can be generated and early warning and decision support can be delivered to the population in hazardous areas and, for example, to health facilities in order to optimise health service delivery. To develop a risk map for a disease, accurate information about the disease is essential, such as the transmission of this disease. Furthermore, treatment, prevention and possible mitigation strategies for a disease are also considered, because, if this information were to be communicated to the population via an appropriate GUI on a mobile device, it could help to reduce the spread of the disease.

2.2 ICT-Situation in South Africa

The application will be developed for a selection of digital devices: for mobile devices, Personal Computer (PC)s and digital doorways. The reasons for this selection are addressed below.
In addition to the development of the application for PCs, the application will also be developed for mobile devices for the following reasons:
[GDM09] found that there are at least 3,3 billion mobile phone subscriptions worldwide, while 1,4 billion people access internet via PCs.
"Handsets with General Packet Radio Service (GPRS) and data capabilities are becoming less expensive and more prevalent every year. [...] Data, too, is becoming accessible, particularly to prepaid costumers, who form the vast majority of mobile users in the developing world. Data costs as little as 13 US¢ per megabyte in South Africa. Estimates from India and South Africa suggest, that there may already be more mobile internet connections than traditional PC internet connections operational in each country." ([GDM09]).
The most common Information Technology (IT)-related household goods in households in South Africa include the radio (approximately 77% of households own a radio), followed by cellphones (approximately 73% of households own a cellphone). This means that most people can be reached via radio and cellphone. For our approach, the response possibility of a user is of great importance for delivering decision support adjusted to the user's needs. Consequently, a user of a radio cannot communicate his/her needs to the system. Therefore, the application will be developed, inter alia, for mobile devices, namely, cellphones, such as smartphones, to reach as many potential users as possible. However, as there are still some differences between mobile devices, the design of a GUI is faced with a challenge: in contrast to smartphones, not every mobile phone has a miniature QWERTY keyboard nor do all mobile phones support the function of reading business documents in a variety formats ([HHP+09]). Additionally, it is possible that a cellphone may only have the potential to visualise textual information on its display. However, since the application should be developed for a maximum number of mobile devices, the characteristics of mobile phones have to be considered even if "it is expected that the unique features of each mobile device will disappear and the boundary of mobile devices will become ambiguous in the near future" ([HHP+09]).

Definition 2.2.1 *Mobile Phone ([HHP+09])*
Mobile phones are portable, self-contained information and communication systems. They are characterised by three features affecting the design of interfaces:

- *They are used primarily in a user's hands.*
- *They are operated without cables.*
- *They support the addition of new application and internet connection.*

There are three features of user interfaces or constraints that can influence the usability of mobile phones:

- The screen of mobile phones are too small to display a lot information at the same time; therefore information organisation and navigation can be critical usability issues. (Recently, the quality and size of the displays of mobile devices have increased.)
- A physical button or key has generally more than one control function. Thus the functions of a single key are dependent on the types of modes. (Newer mobile devices use touch-screens to prevent buttons from having more than one control function.)
- Processing power and available memory is limited.

Other features include multimodality, different display resolutions, and restrictive data entry-methods (cf [HHP+09]).
Another device that the application should be developed for is the digital doorway, which is widely distributed in South Africa.

Definition 2.2.2 *Digital Doorway (cf `http://www.digitaldoorway.org.za` (retrieved 16/02/2014).)*
The digital doorway was invented by the Meraka Institute[3] together with Department: Science and Technology (DST)/CSIR, to introduce computer literacy into the ambit and experience of all South Africans through the implementation of the concept of Minimally Invasive Education. The aim is to provide people in rural and disadvantaged areas with freely accessible computer equipment and OS software, enabling them to experiment and learn without formal training and with minimal external input.

In the following example, EXAMPLE 2.2.3, a number of different digital doorway designs are presented.

Example 2.2.3:

[3] African Advanced Institute for Information and Communication Technology

- *A digital doorway container is a self-contained unit designed to be located in areas without power or suitable shelter. The roof of the housing is covered with solar panel strips, which charge batteries secured within the unit itself. Three terminals are accessible inside the container.*
- *The one-terminal desktop configuration is based around a PC running the Xubuntu Linux distribution. The hardware is housed within a rugged steel enclosure with a vandal-proof metal keyboard, a Liquid Crystal Display (LCD) screen (protected by reinforced glass), a webcam and speakers.*
- *The latest three-terminal configuration is based around a server running the Xubuntu Linux distribution, and two "fat clients" without hard drives, which rely on the server for file access. The server also acts as one of the terminals, making a total of three terminals. The hardware is housed within a rugged steel enclosure with vandal-proof metal keyboards, LCD screens (protected by reinforced glass), webcams, speakers and a Uninterruptible Power Supply (UPS).*
- *The hardware of the four-terminal configuration consists of a rugged steel terminal comprising four sides; each side contains an LCD screen, vandal-proof keyboard, touchpad, webcam and speakers. A server is also located inside the steel housing. The server is used for video capture. Internet connectivity is provided by various means (e.g. GPRS). Cables run from the unit through a pipe into the ceiling.*
- *The accessible digital doorway has two terminals specially adapted for wheelchair access. In addition, grab handles are installed to assist users who are unstable when standing in front of the unit. Two of the four terminals are equipped with joysticks instead of mouse pads. Specially adapted software works in conjunction with the hardware modifications.*
- *The hardware of the single terminal system consists of an outside terminal embedded in a steel housing. The terminal is served by an LCD screen, vandal-proof keyboard and touchpad. A server is also located inside the steel housing and is used for video capture. Internet connectivity is provided by GPRS. (cf `http://www.digitaldoorway.org.za` (retrieved 16/02/2014)).*

In conclusion, the application is generated for a selection of digital devices, that is, all types of cellphone, digital doorways and PCs, to reach many users. Besides the requirement to reach many users, another important requirement for the digital devices for which the application is generated is the following: To deliver spatial decision support matched to the user's needs; for example the needs of users who find themselves in hazardous situations and where the location of the user is important. The geo-coordinates of the user's locations can be delivered to the system via a Global Positioning System (GPS) (mobile devices) or Internet Protocol (IP) address (digital doorway, PC). Older cellphones without GPS and internet can provide their approximate location by determining the distance from the user to three transmitter masts by measuring the duration that is needed to send a signal from the mobile device of the user to the transmitter mast or vice versa. With triangulation, the geo-location of the user can be approximated.[4] In FIGURE 2.2 the early warning and response

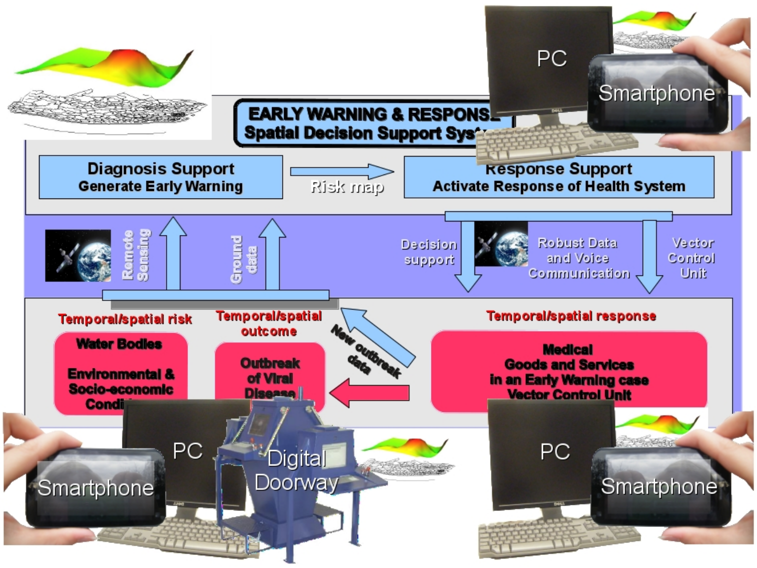


Figure 2.2: Early warning and response cycle with supplements. (Figure generated with *Libre Office Draw*.)

cycle, which was visualised in FIGURE 1.2, is illustrated with supplements. Within this figure, digital devices are assigned to the different stages of the early warning and response cycle based on the user groups involved and their needs. Risk, resource and decision support mapping is here represented by an exemplary risk map.

[4]The game "Cellhunter" with the goal to find as much unknown mobile cells as somebody can and submit them, uses crowd-sourcing to "get a whole list with all cells and when there is enough data per cell we will get the position of it. With the position we can provide "location based services" or something like that." (`http://ch.omoco.de/cellhunter/?lang=en&orderby=&beginat=` (retrieved 16/02/2014)) Thus, the method for determining the geo-location of a user using an older cellphone can be supported by the information about the geo-location of mobile cells and thus of transmitter masts from "Cellhunter".

3 Open Source and Open Content

In the *ReGLaN-Health and Logistics* project, the use of OS software and OC is essential. The developed application will be OS software and thus it will be delivered free of charge. This is important in order to make the application available to everyone. Accordingly, the financial circumstances of the user do not matter, which in South Africa is of great importance, because many potential users would be unable to afford it. By delivering an OS application, we can use already developed components of other OS software and OC, and our application can, in turn, be used in other OS software and OC. The advantage of this is faster development, improvement and, eventually, distribution of the application. First we will define OS.

Definition 3.0.4 *Open Source (OS) ([DOS99], p.254)*
Open source does not just mean access to the source code. The distribution terms of an open-source program must comply with the following criteria:

- *Free Redistribution*
  The licence may not restrict any part from selling or giving away the software as a component of an aggregate software distribution containing programs from several different sources. The licence may not require a royalty or other fee for such sale.

- *Source Code*
  The program must include source code, and must allow distribution in source code as well as a compiled form. Where some form of a product is not distributed with source code, there must be a well-publicised means of downloading the source code, without charge, via the internet. The source code must be the preferred form in which a programmer would modify the program. Deliberately obfuscated source code is not allowed. Intermediate forms such as the output of a pre-processor or translator are also not allowed.

- *Derived Works*
  The licence must allow for modifications and derived works, and must allow them to be distributed under the same terms as the licence of the original software (rationale).

- *Integrity of the Author's Source Code*
  The licence may restrict source code from being distributed in modified form only if the licence

allows the distribution of "patch files" with the source code for the purpose of modifying the program at build time. The licence must explicitly permit the distribution of software built from modified source code. The licence may require derived works to carry a different name or version number from the original software (rationale).

- *No Discrimination Against Persons or Groups*
  The licence must not discriminate against any person or group of persons (rationale).
- *No Discrimination Against Fields of Endeavour*
  The licence must not restrict anyone from making use of the program in a specific field of endeavour. For example, it may not restrict the program from being used in a business, or from being used for genetic research (rationale).
- *Distribution of License*
  The rights attached to the program must apply to all those to whom the program is redistributed without the need for those parties to obtain an additional licence (rationale).
- *License Must Not Be Specific to a Product*
  The rights attached to the program must not depend on the program's being part of a particular software distribution. If the program is extracted from that distribution and used or distributed within the terms of the program's licence, all parties to whom the program is redistributed should have the same rights as those that are granted in conjunction with the original software distribution (rationale).

Below some of the benefits are mentioned that accrue from the use of OS software compared to commercial products. Most authors of OS software develop their software for own use, as the robustness is preferred for adding properties. The aim is to reduce complexity and improve the maintainability. An important feature of OS software is reliability: If there is a defect which causes incorrect operation, data loss or sudden failures, it can be fixed within a short time. When errors occur with CS software, it generally happens that a defect report has to be filed; and then there is a delay while the provider determines when and whether an updated version should be created. Another important property of OS software is stability: Software vendors can use a number of tactics to persuade their customers to update their system. The problem for users of the software, however, is that they rarely have much control over this process and that they are left on their own if they decide to stick with older versions. In OS products, the standards change slowly and interchange formats are often particularly stable. As a result, incompatible file formats are less of a problem. Access to the source code also makes it possible to rely to an old version and it probably opens more options and choices for the user in general. An important role is played by traceability: CS software forces the user to trust the provider, if claims to properties such as security, freedom from backdoors, compliance with standards and flexibility in the face of future changes are made. By releasing the source code of the OS software, the authors

enable the software users to have confidence that there is a basis for these above-mentioned claims. Flexibility and freedom are significant properties of OS software: Software flexibility is characterised by providing solutions suitable for the needs of users. Moreover, OS software is less dependent on related products, as it offers its users more freedom to buy products from other manufacturers. The fundamental benefit of OS software when it comes to support, however, is that it is always possible to support a company since the source code is freely available. In addition, users of the product will often discover and correct defects themselves. Other important features of OS software are, of course, cost reduction, greater security against viruses and cracker attacks, and suchlike (cf [PS01], [Wor00] and `http://open-source.gbdirect.co.uk/migration/benefit.html` (retrieved 16/02/2014)).
PCs and digital doorways (SECTION 2.2) can be equipped with the OS operation system *LINUX*, for example *OpenSUSE Linux*: This is a Unix-like computer operating system assembled in line with the model of free and open source software development and distribution (useful within empirical studies). For the development of an application for a digital device delivering early warning and decision support tailored to different user groups with a GUI that adapts to the user, the following OS software, among others, can be used (in alphabetical order):

- *ArgoUML*: A free, platform-independent Unified Modelling Language (UML) tool (a Computer-Aided Software Engineering (CASE) tool) for describing and modelling (software) systems and for code generation. It is written in Java.
- *GeoGebra*: (an amalgam of the words "geometry" and "algebra") This is an application software for creating and calculating mathematical figures made of sub-regional geometry, algebra and calculus. *Geogebra* is also known as dynamic mathematics software. The majority of *Geogebra* is free software. As *Geogebra* is written in Java it is available for multiple platforms.
- *GIMP*: The GNU Image Manipulation Program.
- *GNU Octave*: A high-level interpreted language, primarily intended for numerical computations.
- *GNUPLOT*: A portable command-line driven graphing utility.
- *GRASS GIS*: A hybrid, modular GIS software with raster and vector functions. GRASS stands for Geographic Resources Analysis Support System. It is available under the GNU General Public License (GPL) and is therefore freely available.
- *K3DSurf*: A program for visualising and manipulating mathematical models in three, four, five and six dimensions. *K3DSurf* supports parametric equations and iso surfaces.
- *Libre Office*: A powerful Office-Suite.
- *Maxima*: A computer algebra system developed as an open-source project under the GNU GPL.
- *R*: A free software environment for statistical computing and graphics.

- *vokoscreen*: A software for recording screen casts with audio (useful within empirical studies).

The application is generated for mobile devices with the *Android* operating system, in this way OS applications can be provided free of charge. The following OS applications are available, among others:

- *Navit*: A navigation application for offline navigation by car, bike or foot.

Among others, the following OC is used:

- *OSM*: a free wiki world map.

OSM is used in the sense of the *Humanitarian OSM Team* (HOT), which is an initiative for the use of open data and OS for humanitarian purpose. In this thesis, we need *OSM* in order to determine "the route a user can take", for example by including the infrastructure. For GIS, we can use *GRASS GIS* as a spatial database and to visualise the risk, resource and decision support maps. This visualisation can be supported by *GNUPLOT*. GUI element objects can be displayed in *GeoGebra*, while the OOA can be carried out using *ArgoUML*. The implementation of the algorithm can be rudimentarily transposed in *GNU Octave*, while *Navit* can be used as a navigation tool and modified for our purposes.

4 Risk Perception and Interim Conclusion

From the previous chapters, the following can be summarised: One important milestone within the *ReGLaN-Health and Logistics* project is the development of an application for a digital device delivering early warning and decision support tailored to different user groups using a GUI adapted to the user (Chapter 1). Consequently, we have to consider the mathematical visualisation of risk and resource maps for epidemiological issues using GIS and adaptive GUI design to produce an application for digital devices to supply spatial decision support tailored to different user groups. For the purpose of this thesis, the pilot region was South Africa. Therefore a literature study of the country, South Africa, had to be carried out and a GUI prototype adjusted to South African users is supposed to be developed in the future. In this regard, the health situation in South Africa was considered, as well as conditions pertaining to the ICT infrastructure and the economy (Sections 2.1 and 2.2). Moreover, a literature study was conducted on the field of medicine with respect to the health situation in South Africa. Inter alia, user groups belonging to the medical staff will be considered for the application development, as well as action measures for medical emergencies. The application is intended to work within the context of mHealth, with the aim of improving the health system. When investigating conditions pertaining to the socio-economic and ICT situation in South Africa, suitable digital devices and software components were identified in order to reach as many potential users as possible. Cellphones (e.g. smartphones), PCs and digital doorways have all been proven to be suitable, as well as OS software and OC for providing the application free of charge. In other words, we identified which OS software and OC is applicable directly or in a modified way for our concerns and how the software can be implemented.

Another literature study on the topics of epidemiology, early warning and SDSS was done (Chapter 1). When considering epidemiology, early warning and SDSS, risk perception plays an important role. Therefore, we refer to the *Harding Centre*. The goal of the *Harding Centre* is to help people in their struggle to understand and assess the risks facing them. The primary focus is on health and medicine, as well as on educating people from childhood onwards to understand statistics. By conducting studies, experiments and surveys, the *Harding Centre* investigates the problems people have with understanding numbers and, subsequently, find solutions to these (`http://www.harding-center.com` (retrieved 16/02/2014)).

One idea is to use icon arrays within the GUI to communicate medical or epidemiological risks, see

FIGURE 4.1.

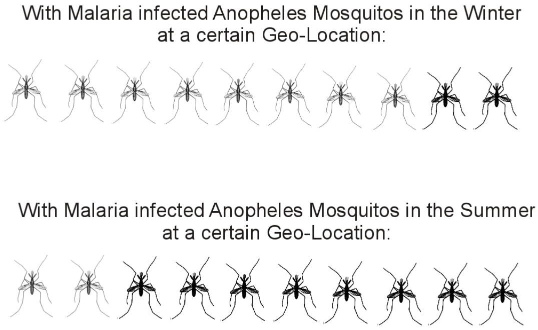


Figure 4.1: Exemplary icon array about the amount of with malaria infected anopheles mosquitoes. (Figure composed of icons from `http://openclipart.org/` (retrieved 16/02/2014) and generated with *Libre Office Draw*.)

Definition 4.0.5 *Icon Array*
Icon arrays are graphical representations consisting of a number of stick figures, faces, circles, or other icons symbolising individuals who are affected by some risk.

A study showed that icon arrays increased the accuracy of both low- and high-numeracy people, even when transparent numerical representations were used. With larger icon arrays (1000 instead of 100 icons) risks were perceived to be more serious, and risk reduction was larger. Risks presented by means of icon arrays were perceived as less serious than those presented numerically. Nevertheless, icon arrays are a promising way of communicating medical risks to a wide range of patient groups, including older adults with lower numeracy skills. ([GGRG09]).
Icon arrays can be used within the GUI for an application for a digital device delivering early warning and decision support to support the comprehension of risk. Accordingly, ideas for methods for making risk visible via risk maps and for appropriate GUIs will be presented (CHAPTER 9 and CHAPTER 10).
From the requirements and constraints identified, there results the need for a literature study about "good" GUI design, inter alia, by paying attention to special conditions relating to the culture in South Africa, in order to raise risk awareness and support the comprehension of risk with the application for delivering early warning and decision support. We will take a closer look at GUI design in the following chapter.

In the following part, PART III, the concepts merged for developing an SAGU are described. This includes topics like GUI for an application delivering early warning and decision support in CHAPTER 5; OO for preparing the creation of a prototype of the GUI for an application delivering early warning and decision support in CHAPTER 6; fuzzy logic and ANNs for implementing adaptivity in CHAPTER 7 and measure theory on topological spaces for evaluating fuzzy membership mappings on non-Euclidean spaces in CHAPTER 8.

III | Concepts merged for developing a System for adapting a GUI to a User

5 Graphical User Interface for an Application delivering Early Warning and Decision Support

Communication relating to the transmission of information between professionals and lay people can be problematic. This is especially true for health care, where the rising costs lead among other things to a reduction in the conversation time between doctor and patient. ICT solutions could provide a solution. (cf [Mar07]).
Therefore, the *ReGLaN-Health and Logistics* project includes an ICT solution for health care optimisation by developing an application for a digital device delivering early warning and decision support tailored to different user groups, using a GUI that adapts to the user. In this thesis, the GUI is a key issue.

Definition 5.0.6 *Graphical User Interface (GUI) (cf [AHD09])*
A GUI is an interface for

- *delivering information to a user and supporting the comprehensibility of the information using supportive visualisation.*
- *issuing commands to a digital device using a pointing device*[1] *that manipulates and activates graphical images on a monitor.*

An important milestone within the *ReGLaN-Health and Logistics* project is the development of an application for a digital device delivering early warning and decision support tailored to different user groups with a GUI that adapts to the user. One main task of the GUI for the application that is supposed to be developed for digital devices, is a risk representation tailored to the user for early warning and decision support by risk assessment. Resource allocation is optimised to optimise health service delivery. In this thesis we are concerned with the mathematical structure behind a system for an adaptive, comprehensible GUI tailored to different user groups that enables the user to comprehend

[1]such as a mouse

the (spatial) sense of a support-proposal, e.g. how to get out of a hazardous area possibly unscathed or how to act in an accident case.

Definition 5.0.7 *GUI design ([Wes98], p.17)*
GUI design is the design of the graphical interface of a software, the interface between a person and a machine. GUI design characterises what the user gets to see of the software: that what goes beyond simple operation.

A good GUI design transforms algorithms into applications and gives them a contemporary and ergonomic "face". GUI design has advantages: Good GUI design allows quick creation of applications, it allows to work more efficiently at higher satisfaction of the user and it allows reduction of operator errors. Poor GUI design reduces the acceptance, the attractiveness and usability of software. ([Wes98], p.17).
For a user who is situated in a hazardous situation, it is important to be able to work efficiently with the GUI, because a fast action of the user, which helps the user to protect himself in the best way, is essential.
Investigations of large German companies show that up to 70% of software development must be set for the development of a user-friendly human-machine interface. The technical system can obtain optimal design for a user, if we know the user with her/his abilities, limitations and requirements. A user-friendly design is not feasible without a user analysis. ([ZÖ5], p.10f and p.102).
Therefore a user analysis is done within the SAGU (Chapter 9) with the aid of the TCIU described in Section 9.4. Based on this user analysis, i.e. the information about the user, the adjustment process of the GUI is carried out, i.e. a GUI fitting to the user needs is assigned to the user. This process is dynamic and the GUI assigned to the user can change, if the user needs change.
The involvement of the user in the development process has great advantages. But there is not the one user, one has to deal with many different types of users. ([ZÖ5], p.27).
In this thesis, we consider different user groups, like the user group "decision maker" or the user group "user, who is situated in a hazardous situation". The developed system described in Chapter 9 dynamically adapts to each individual user, thus different types of users get different GUIs. For the developed GUI it is essential to involve the user to adapt the GUI to the user needs and assign the best GUI to a user.
A person and a machine are communication partners, which differ substantially in their characteristics and abilities. While the technical design of machines in terms of capabilities, operating concepts, automation, etc. can be varied in a wide range by the designer, a person as user is a communication partner of a machine, that is marked by mostly downer abilities and thus by its limits. On the other hand, the properties and behaviour of a machine over its lifetime is usually constant, while a person is able to constantly adapt to new requirements. A person is indeed limited in perception

and information processing, yet highly flexible in its ability to solve new unknown problems. ([Zö5], p.79ff).
To enable the SAGU (Chapter 9) to solve new unknown problems, we include ANNs. The developed system has to deal with the challenge to be flexible and change its properties and behaviour to deliver the best result for a user, i.e. a GUI that is comprehended by the user and adapted to the user's needs. This is done, inter alia, with the aid of an ANN used within the system, which is described in Section 9.3. In addition, the system should be constantly maintained and improved. Therefore User Experience (UX) can be included.

Definition 5.0.8 *User Experience (UX) ([AT03] and [Rot06])*
Basically, UX refers to the experience that a person gets when he/she interacts with a product in particular conditions. In practice, there are numerous different kinds of people, products, environments and infrastructures that influence the experience that interaction evokes. The user and the product interact in the particular context of use that environmental, social, temporal, cultural factors and (optionally) the task context for the experience are influencing. The user component refers to the mental and physical state of the person who interacts with the system[2]. Also, the product has influential factors[3]. All these factors influence the experience that user-product interaction evokes.

UX is considered in the TCIU (Section 9.4) and within empirical studies about the GUI-usability. An empirical pilot study about the assignment of an initial GUI is addressed in Chapter 11.

5.1 Design

A well-designed application makes it easier for the user to both understand the presented information as well as interacting with this information. For an optimal screen design guidelines concerning features of screen design and guidelines addressing the use of screens to help the user navigate through the system, or promote efficient memory should be considered. (cf [GC87], p.268).
The following are guidelines that promote the optimisation of the use of memory.
For adapting an operating system to the users as operators there is first of all a basic understanding of their information acquisition and information processing essential. This includes:

- the gathering of information, with that data is collected by the senses seeing (visual sense), hearing (auditory sense), touch (tactile sense), balance (vestibular sense), movement (kinesthetic sense), smell (olfactory sense) and taste (gustatory sense),
- the perception, i.e. the summation and interpretation of data collected by the senses to information (recognition of symbolic elements and structural patterns),

[2] which can be values, emotions, expectations and prior experiences, among others
[3] for example, mobility and adaptivity

- cognition, i.e. the purposeful arrangement of the compressed and interpreted information (actual intellectual performance with problem-solving strategies),
- action, i.e. the implementation of the results obtained during the cognition into action plans and
- motor functions, i.e. the execution of action plans by the “authors“ such as hand, foot, eye or voice.

The adoption that stimulus offers match optimally to the information processing mechanisms of the human brain, trigger pleasure, while too much or too little information leads to boredom or over-claim is contradicted by the theory of ”chunking“.

Definition 5.1.1 *Chunking (cf [SB78], p.48f)*
Information can be reduced by organising the provided stimulus objects, thereby the amount of information is reduced. This process is called chunking.

Human perception is therefore not a passive process. However, there does not need to flow an amount of information into the consciousness in each second: there flows virtually no information from the visual environment when you close your eyes. At the same time, e.g. the pleasure of an immediately pre-recorded piece of art can last. A person has to rely to find regularities in her/his environment because then she/he can receive the environment with less memory overhead, and thus more easily. The discovery of such chunks has a satisfactory effect. ([SB78], p.48f).
Some performance parameters of the human brain can be determined very well by appropriate experiments. Thus, from experiments can be calculated that a person gathers about 10 Mbit/s, or 80% of its total recorded information over the visual sense, i.e. taken up by the eyes. This huge flow of information, however, is already strongly reduced in a first phase in the brain. The external stimuli of the sensory system makes it to demarcated areas of the brain, where neural networks are performing pattern recognition. The incoming information is subjected to a kind of data analysis and tested for its relevance to the current action goal of the user. Information that could be relevant is directed for further processing to the so-called presence of memory, irrelevant information will immediately disappear, however. This process is controlled subconsciously by the brain. The flow velocity into the presence of memory of our brain is on average 15 bit/s, this value depends, however, strongly on age. Maximum values of $18 - 20$ bit/s are achieved only at the age of about 18 to 30 years and this only in a balanced state of mind and consciousness. Under stress, such as the influence of alcohol, the values descend rapidly. It was proved in many experiments, that one should not offer more than seven choices at the same time on a display. More than seven digits a person can hardly memorise, for example for entering a password, and sentences with more than seven meaningful terms are for this reason difficult to read. The following example delivers an explanation: Suppose we have a variety of objects and we

want to assort them anyway. Experiments with volunteers show that they usually create five to nine classes. In this case, each class is unconsciously associated with a particular property or combination of properties with that we can decide on a new item with yes/no questions, to which class it belongs. Five to nine yes/no decisions have to be made regarding to the allocation. Thus our present memory is charged with $2^5 = 32$ bit to a maximum of $2^9 = 512$ bit. $2^7 = 128$ bit and thus corresponds to the optimal area of the storage capacity of our present memory. If we present information on a screen, for example, in a way that chunking is facilitated on the one hand, and if we limit the number of chunks to about seven on the other hand, we took a first step towards human-oriented design ([ZÖ5], p.82ff). For our concerns, we have to evaluate the different GUI element objects with respect to the chunking quality. Each GUI element object corresponds to a number of chunks depending on the chunking quality of the GUI element object. A chunking-facilitating GUI element object corresponds to few chunks and a not-chunking-facilitating GUI element object corresponds to a lot of chunks. The chunk values of the GUI element objects used within a GUI are added up. The quality of the GUI depending on the number of chunks can be represented with a membership mapping, see FIGURE 5.1. Membership

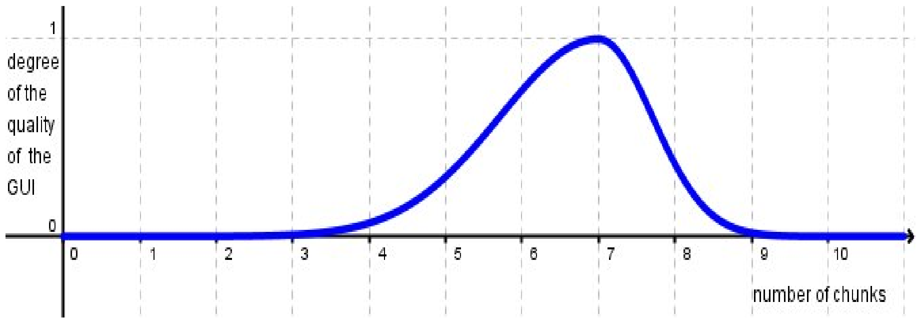


Figure 5.1: Quality of a GUI depending on the number of chunks displayed within the GUI. (Figure generated with *GeoGebra*.)

mappings are explained in more detail in CHAPTER 7 about fuzzy logic. The value "1" means "good quality" the value "0" means "bad quality".
For GUI design optimisation, the organisation of...

- ... commands into structures that make sense and are easy to remember,
- ... a task so that all necessary measures for performance are presented in an optimal sequence,
- ... user directories in systems that support information storage and retrieval,

should be improved. (cf [GC87], p.269).
In the following the keywords menus, toolbars and icons are defined, based on the *GNOME Human Interface Guidelines 2.2.2* by the *GNOME Usability Project* (`http://developer.gnome.org/hig-book/stable` (retrieved 16/02/2014)):

Definition 5.1.2 *Menu (`http://developer.gnome.org/hig-book/stable` (retrieved 16/02/2014)) Menus present the whole range of an application's commands to the user, and often a subset of its preferences.*

To make it easier for the user to learn, it is useful to place common menu items in the same locations as they appear in other applications, when designing a new application. (cf `http://developer.gnome.org/hig-book/stable/` (retrieved 16/02/2014)).
In contrast, e.g. within the OS-application *Navit*, a navigation application for mobile devices for offline navigation by car, bike or by foot, the menu-structure is not common. The items, which are used mostly, are at the top level.

Definition 5.1.3 *Toolbar (`http://developer.gnome.org/hig-book/stable` (retrieved 16/02/2014)) A toolbar is a strip of controls that allows convenient access to commonly-used functions. Most toolbars only contain graphical buttons, but in more complex applications, other types of controls such as dropdown lists, can also be useful.*

By giving direct access to functions that would otherwise be hidden on a menu it is possible to speed up the user's task. The number of toolbar controls should be limited, as too many toolbar controls reduces the amount of screen space available to the rest of the application and reduces their efficiency by making them harder to find (cf `http://developer.gnome.org/hig-book/stable` (retrieved 16/02/2014)), this problem was already addressed above.

Definition 5.1.4 *Icon (`http://developer.gnome.org/hig-book/stable` (retrieved 16/02/2014)) Icons are a graphical metaphor presenting a visual image that the user associates with a particular object, state or operation.*

Some things are easier to communicate with a picture, even a very small one. A good icon reminds immediately of the item it represents. An icon can represent e.g. an application in the panel menu. Icons can assist the user in rapidly scanning a large number of objects to select the desired item. A user can identify an icon more rapidly than a text label, particularly after a user is accustomed to an icon's appearance. Icons can augment text by providing visual suggestions to accompany the descriptive text. Icons can compactly represent a large number of objects when there is insufficient space to display textual descriptions (such as in a toolbar). (cf `http://developer.gnome.org/hig-book/stable` (retrieved 16/02/2014)).
[Fen94] concerns with logical pictures and found out, that the degree of enhancement of text-comprehension yielded by the logical picture depends on the success of the translation of the spatial metaphor into

the illustration, whereby the term logical picture can be defined as follows:

Definition 5.1.5 *Logical Picture ([Fen94])*
Logical pictures are transformations of spatial metaphors into visual analogues, mapping the spatial allusion of the verbal metaphor into a two-dimensional representation. This visual analogue increases the efficiency of the metaphor of a heuristic tool in providing visual control of the construction of mental models[4].

Logical pictures are similar to icons, with the difference that within logical pictures spatial metaphors are transformed which can be implemented with Augmented Reality (AR) (SUBSECTION 5.5.2), e.g. for visualising invisible risks or for making complex processes like medical help delivery in an emergency case more comprehensible for a lay person with appropriate visualisation, e.g. in the context of telemedicine (SUBSECTION 5.5.1). The contents mentioned above are taken into account for the illustrative design of GUI-elements tailored to different user groups described in CHAPTER 10. Each GUI-element can be found in the menu, sometimes used GUI-elements are included into the toolbar and often used GUI-elements are placed on the screen in form of an icon. This grade of use (never, sometimes, often, always) can be represented by a fuzzy membership mapping.

5.2 Task-User Match and Feedback

Another interesting example of the perception of a person is the perception of spatial information. For the GUI for an early warning and decision support application, the investigation of the perception of spatial information is important inter alia for supporting comprehensibility of a navigation. Therefore, a GUI for an early warning and decision support application should use AR (SUBSECTION 5.5.2).
It can be shown that the visual system of a human makes use of twelve different techniques to detect depth. In [Zö5], p.96, they are subdivided into

- The oculomotor depth criteria, which evaluates the position of the eyes and the tension of the eye muscles. These criteria work, especially at very close objects at a distance less than 1.5 to 3 meters.
- The monocular depth criteria that also work with one eye and evaluate, for example, information about the overlap or occlusion of objects or on the distribution of brightness and shadow. Here flows mainly the experience into the analysis. Thus, the human brain assumes a light source illuminating the scene from above (analogous to sunlight) and expects corresponding distributions of brightness and shadow. Bright objects are seen closer than darker objects and if there are objects with similar dimensions, the smaller pictured appears to be farther than the larger pictured.

[4]see DEFINITION 5.2.1

- The motion-induced depth criteria, that evaluates additional information about relative motion between the objects and the viewer. Objects that move quickly are perceived as closer by the viewer than slow moving, especially the typical speed of comparable objects known from the experience slip in the recognition.
- The lateral disparity, which evaluates the two sets of pictures of the two eyes is the most obvious technique of deep recognition, but it only works at close range to about 20 meters.

On the perception follows the processing of information in the phase of cognition and of recognition. It includes the processes of understanding, learning, problem solving, decision making and planning. They are conducted depending on the complexity and, accordingly, depending on their cognitive mobilisation at different levels, the so-called levels of regulation of human behaviour. A person orients very strongly towards learned action schemes. These schemes are called mental models. ([ZÖ5], p.100f).

Definition 5.2.1 *Mental Models ([ZÖ5], p.100f)*
Mental models are personal images in the form of an internal representation of objects, of their function of the dynamic behaviour of technical systems and of patterns of behaviour and actions for the environment that is interested in individual work situations.

Mental models are according to the American psychologist *Donald Norman*

- incomplete: there are only a limited number of input and output variables taken into account, because, like any model they are merely an abstract representation of reality,
- unstable: i.e. mental models can be forgotten or changed,
- not distinguishable from each other: i.e. similar models can mingle,
- uneconomic: i.e. unneeded models still remain and
- to higher levels of action limited: i.e. they are used only, if no sensorimotor processes are available for the current tasks or problems.

The models are formed through perception, reasoning, learning, decision making, development of creativity, evaluation and dissemination of ideas and are thus of individual kind. Because of different personal ideas and the particular perspective completely different mental models about the function of the same technical device can result. So basically the developer of a technical system has a completely different view on a device than the future users, and thus the developer also has a different mental model (development model) than the user (user model, see DEFINITION 7.4.1). One might therefore equate user-friendly design with the combination of these two mental models. ([ZÖ5], p.100f).

The system developed in this thesis adapts to the user by adjusting to the user's mental models in a dynamical and flexible way. If the user's mental models change, the GUI changes equally. Additionally, the SAGU (Chapter 9) and the GUI work similar to mental models: It is incomplete, because an abstract representation of reality is visualised on the GUI e.g. via GUI-elements illustrated as logical pictures. It is unstable, because the ANN can forget (i.e. delete connections), which means that in the recent past rarely used GUI elements are suggested to become removed from the screen, even if they were used often in the distant past. The behaviour of the adaption process can change (i.e. create new connections). The GUI is supposed to be used as tool if the user does not know how to solve the current task or problem in an easy and fast way, i.e. the GUI delivers decision support. But the SAGU should work economic and the GUI elements or GUI element objects should be distinguishable from each other in contrast to mental models.
A person learns mental models by experience, which she/he makes in dealing with technical systems. She/he has a certain vision and initiates an action, with which she/he believes to come closer to her/his goal. She/he then observes the result of her/his actions and checks if she/he achieved her/his goal. If so, she/he continues the way, if not, she/he modifies her/his actions or even her/his goal. Shortened, this procedure can be described as a trial-and-error principle. ([ZÖ5], p.103f).
The ANNs included into the SAGU (Chapter 9) work similar.
To make the described procedure work, two things must be fulfilled:

1. The person may make mistakes.

2. the person needs feedback on the success of her/his action.

This feedback is extremely important and should satisfy certain requirements. It must be obvious, i.e. in recognisable context to action and it must happen in reasonable time. Here, multimodality helps, too. The feedback of pressing a button is detected much better if the user feels the break point, hears the cracking and if the user can see the changes visually on the screen. A person expects feedback on different levels. The clicking of a button is the so called action feedback, it confirms that the immediate action was successful. But in the long term a person expects a feedback on whether she/he has achieved the intended goal with her/his actions. This is known as a target feedback. And if a longer time elapses between action feedback and objective feedback, the user needs in general also a state feedback that shows her/him how long she/he must wait until the target feedback. On the display appears in this case at least an hourglass or a progress bar. ([ZÖ5], p.103f).
Error-tolerance is included into the SAGU (Chapter 9) via ANN-settings. Feedback is essential especially for blind users[5] (cracking when clicking on a button) or for deaf users or illiterates (see changes on the screen). The feedback is important to avoid or decrease panic of a possibly impatient

[5]For blind users not a GUI is used, but a special user interface for blind users. For the development of such a user interface, e.g. the idea of ASR who develop an electronic guide device for the visually impaired aiming at developing a navigational platform to help visually impaired persons to commute easily in campuses and schools (`http://as-research.org/initiatives` (retrieved 16/02/2014)) could be included.

user in a hazardous situation.

5.3 Error Management

People-friendly design also means to tolerate errors within certain limits. Today's unimodal[6] technology robs us most of the possible information and thus contributes to the fact that the communication is error-prone. ([ZÖ5], p.94).
The use of fuzzy logic, further described in CHAPTER 7, allows error-tolerance by considering the degree of an error, i.e. the degree of the difference of the expected user reaction to the actually user action. This degree of an error can be determined, i.e. the error can be measured, with measure theory (CHAPTER 8). The GUI that will be developed within the *ReGLaN-Health and Logictics* project is supposed to include, inter alia, telemedicine (SUBSECTION 5.5.1), AR (SUBSECTION 5.5.2), speech recognition and observation of the spatial location and the movement of a user via GPS-coordinates to make, according to [GC87], p.271, the communication of the user with the system easy by creating appropriate electronic analogues of the traditional ways of communicating or manipulating similar information in the non-electronic domain.

Definition 5.3.1 *Error ([ZÖ5], p.114)*
An error is in general described as the deviation of an actual value from a predetermined set value. Within the human-machine systems errors are described in relation to work tasks.

[ZÖ5], p.114, mentions, that since a false action can have serious consequences, the prevention of malfunction has to be an important goal in all technical systems. Mistakes are made accidentally, or even without the person in charge is aware of having made an error at all. When dealing with technical systems, a person is confronted with three basic system characteristics that cause the person problems again and again:

- system complexity
- system dynamics
- non-transparency.

An error determiner, which takes account of user skill levels and tailor error feedback accordingly (cf [GC87],p.273f) is implemented into the SAGU (CHAPTER 9), further described in SECTION 9.7. The SAGU makes mistakes, as well, because of non-transparency, i.e. too little information about the user.
The error determiner is supposed to consider the consequences of possible errors. Subsequently,

[6] E.g. a personal interview is known as multimodal, since multiple sensory channels are addressed simultaneously.

appropriate actions should be accomplished to ensure that the system supports the user as much as possible. One way to avoid mistakes of the user could be training and practise. Other possibilities to avoid mistakes are the optimisation of navigation to support mind consistency. (cf [GC87], p.257ff).

5.4 Emotions and Cultures

Authoritarianism, rumours and hearsay have been effective guidance in human history before there were books and medical research.[7] ([Gig07], p.170).
As the following experiment shows, it plays an important role, who presents results and how results are presented: A large American company that manufactures Automated Teller Machine (ATM)s among other things, has attempted a few years ago to replace the well-known text interface of ATM with a completely novel one, in which an artificial person - a so-called Avatar - welcomes the customer and "discusses" the customer's bank transactions with the customer. The customer identified herself/himself with her/his bank card, then the avatar appeared on the screen - for men, a good looking lady, for women an equally attractive man - greeted the customer and asked the customer what she/he could do for her/him. Now the customer could express her/his wishes and could be guided by the avatar in a natural language dialogue through the procedure. It was hoped that this form of interaction is not just a marketing gag, but also an introduction to the virtual advising of bank customers. But unfortunately, the experiment went wrong. They tested the prototype with selected employees, they observed them when using it and then questioned them about their opinions. To the astonishment of the developers the opinions were very divided. Thus a coloured female volunteer stated, she would only be reluctantly served by a white man, a different proband did not like his female avatar neither the voice nor the appearance. And others said that they would use in a possible selection rather the traditional ATMs than the new one. Even the small group of probands who had initially expressed a positive opinion changed after several test runs also to the "No-voters". After that unexpectedly negative outcome of the experiment for the developers, it was decided to drill down on the rejection. It was immediately clear that it were the emotions that had brought the experiment to fail. ([ZÖ5], p.117f).
Therefore, emotions and cultures concerning GUI design are addressed in the following.
If emotions have such a central importance, there can be no human-oriented technology, without taking the role of emotions in the design of technology into account duly. ([ZÖ5], p.120).
People believed earlier, that emotions are highly culture-dependent. Today we know however, that this is not actually the case. Studies have shown that the six basic emotions, fear, surprise, disgust, sadness, anger and joy are found in all cultures. What is different are the causative factors as well as at least to a part the forms of expression. So the Europeans have learned to connect the colour black to commonly feelings of sadness or even power. But in many Asian cultures with sadness the colour white is associated.([ZÖ5], p.124ff).

[7] Learning by experience can be fatal. So it is a bad strategy to figure out yourself which plants are poisonous.

The meaning of colours has to be considered for being able to develop designs of GUI element objects for providing a generative concept of the GUI.
The showing of emotions is highly culture-dependent. While a Southern European is much more expressive in general, i.e. shows feelings expressive, corresponding signals from northern Europeans and even Asians are much more discreet. ([ZÖ5], p.124ff).
Culture has, however, a number of other influences on the technical design of operating systems. Two important keywords in this regard are analogy and metaphor. "One can do a lot towards improving the ease of use of an electronic system by allowing users to capitalise on their existing bodies of knowledge about procedures and tasks in the non-electronic world." ([GC87], p.251)
Inter alia the use of AR (Subsection 5.5.2) allows users to capitalise on their existing bodies of knowledge about procedures and tasks in the non-electronic world.
While a transformation of language between German and Swedish, both technically and from the cultural background of the target users is comparatively easily possible, it is completely different with the Asian languages. A part of the Asian language such as the Chinese language is based on images and derived symbols. So it is not surprising when in particular in the technique so popular icons as we know it from any Video Cassette Recording (VCR) remote control, regularly lead to misunderstandings in Asian cultures. But there are also differences in the mental structures of the machine users in many cultures discoverable. While the typical users in the Western industrialised nations form clearly recognisable structures in their mental models, such structures are created of Asian users much broader and more connected. Control systems for the goal market of Europe should be structured as a unique tree structure with a maximum of five to seven items per level, while for Far Eastern markets, the width of the menu structures may be greater albeit at lesser depth of the menu. So a localisation, i.e. adaptation of the key product features to the respective goal market, is essential in most cases. ([ZÖ5], p.124ff).
The pilot region for the application the GUI is developed for, is South Africa. The following idea turned out helpful for (South African) users: If the concept of an Inbox and Outbox is familiar from an email-GUI, this concept could be implemented for delivering information to a user within the Inbox and giving the user the possibility to respond within the Outbox. (cf [Che05]).
This idea has proven as successful in [Che05] and can therefore be used as GUI element design for delivering text-messages to the user (e.g. action measures tailored to the situation the user is situated in or questions from the TCIU, Section 9.4) and giving the user the possibility to respond (e.g. for answers on questions for the TCIU or crowd-sourcing).
It is important to deliver the GUI in different languages, as there are eleven different languages spoken in South Africa. It could be possible, as well, that there are e.g. illiterates or blinds among the users. Therefore an Automatic Speech Recognition (ASR) could be useful. [BT06] carries out field trials with two communities in South Africa: disadvantaged Deaf users and an isolated rural community with the result that there have to be designed user interfaces that allow users to understand and cope

with delay. Additionally, social concerns seem to be often more important than the technical issues. The following results of [BT06] concerning the ASR can be concluded:

- "Deaf users use a different grammar (related to South African Sign Language) and this cannot be automatically corrected for hearing users.
- ASR is inadequate to the task of recognising South African English and other South African Official Languages and in the meantime we will have to mimic ASR by employing a person to provide the service.
- Presence indicators are needed to show continuation of the conversation when there is no visible activity due to delays."

Thus ASR has to be improved for a use within the GUI, that is supposed to be used within the application delivering early warning and decision support described in this thesis.

5.5 Designs of GUI Element Objects for Risk and Resources

In this thesis we are concerned with the mathematical structure behind a system for an adaptive, comprehensible GUI tailored to different user groups that enables the user to comprehend the (spatial) sense of a support-proposal, e.g. how to get out of a hazardous area possibly unscathed or how to act in an accident case. For those claims, AR is useful for risk visualisation and navigation aid and telemedicine can be used as resource, which requires certain IT-constraints (e.g. availability of a screen) and internet-connection.

5.5.1 Telemedicine

The telemedicine support deals with the opportunity to link real people via video conference with GUI information. It is difficult to define the term telemedicine, there is no generally accepted definition of telemedicine. Nevertheless, in the following a definition describing the meaning of the term telemedicine how it is used in the following context, is done:

Definition 5.5.1 *Telemedicine ([WPSH09], p.3)*
The literal meaning of telemedicine is "health[care] at a distance".
Thus, telemedicine may represent health care practised in real time[8]*. The type of health care interaction is perfectly general, and may encompass diagnosis and management, education - of staff, patients and the general population - and administrative meetings. Telemedicine is one aspect of the use of ICT in health care.*

[8] using a video link for example, or asynchronously, perhaps by email

One interesting idea of the application of telemedicine is the following: the collaboration between doctors and patients in a virtual hospital. All digital medical data of a patient can be collected in an EHR. A health portal provides integrated access to all data of this EHR on the internet. *Sanagate* is such a web-based information system, which has a three-dimensional GUI, called "virtual reality environment", which allows the patient an intuitive access to its medical data as it is a virtual hospital. In addition to retrieving and viewing text and image data, it is possible to generate individual spatial anatomical models from existing layer image data of the patient and examine them. The visualisation takes place in an internet-based client-server environment. This allows multiple PC-users from different places to meet in the virtual hospital through avatars, i.e. their virtual represented, and communicate with each other via the built-conferencing tools. Thus, the system can be used as a platform for telemedicine collaboration between doctors and patients. ([MÖ7]).
Telemedicine can be used within the GUI for an application for a digital device delivering early warning and decision support e.g. in an accident case, when first aid is needed, but no physician can reach the accident location in appropriate time to save an injured person. Then a lay person can be guided by a physician to keep an injured person alive, until a medical doctor arrives at the accident location.

5.5.2 Augmented Reality

Definition 5.5.2 *Augmented Reality (AR) system (cf [ABB+01])*
An AR system supplements the real world with virtual (computer-generated) objects that appear to coexist in the same space as the real world.
An AR system has the following properties:

- *it combines real and virtual objects in a real environment;*
- *it runs interactively, and in real time; and*
- *it registers (aligns) real and virtual objects with each other.*

AR can potentially apply to all senses, including hearing, touch and smell. Certain AR applications also require removing real objects from the perceived environment, in addition to adding virtual objects.

In *Sutherland's* work in the 1960s AR was first used in a seethrough Head-Mounted Display (HMD) which was used to present 3D graphics. Then, few work was done in this research field, until in 1997 a survey was published by *Azuma* which initiated the progress and growth of AR. Within this survey, the field was defined, many problems were described, and the developments until 1997 were summarised. Nowadays, *ARToolkit*, a software toolkit for rapidly building AR applications is available for free (`http://www.hitl.washington.edu/artoolkit` (retrieved 16/02/2014)) (cf [ABB+01]).
As we use inter alia mobile devices for the application of AR, the handheld display plays an important role for us which acts as a window or a magnifying glass that shows the real objects with an AR

overlay. (cf [ABB+01]).

But there can still occur some problem areas in AR displays, especially when older digital devices are used, because they do not have sufficient brightness, resolution, field of view, and contrast to seamlessly blend a wide range of real and virtual imagery. Some problems regarding error estimate visualisation and data density occurred. (cf [ABB+01]).

But increasingly improved displays are developed for mobile devices, aiming on solving the mentioned problems. New tracking sensors and approaches have been developed and environment sensing got into focus. Researchers included UX to evaluate and improve AR-systems. Some significant human factors include latency, depth perception, adaption, fatigue and eye strain (cf [ABB+01]).

Figure 5.2 shows an exemplary GUI as an example for AR. The application visualised in Figure

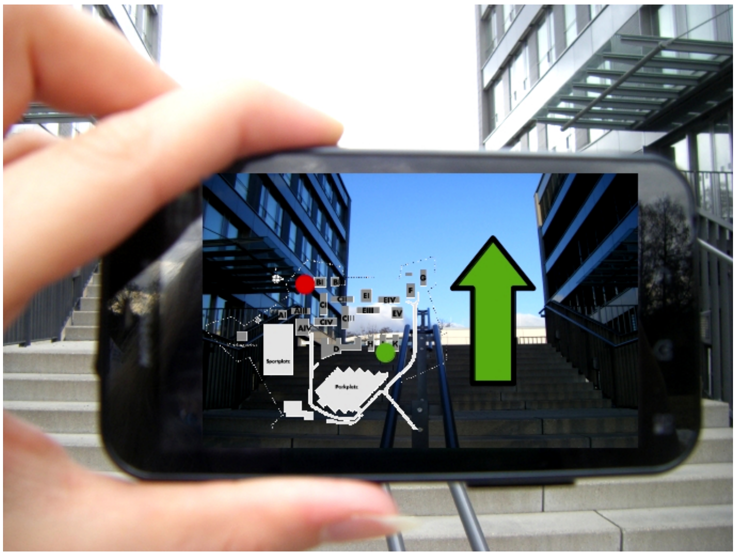


Figure 5.2: Example for AR. (Figure generated with *GIMP*.)

5.2 serves navigation. On the left hand side, a map of the surrounding is given with a green dot for the actual position of the user and a red dot for the location the user wants to reach. This dot can be moved by the user. The green arrow guides into the direction the user should follow to reach the goal-location. The arrow changes position and colour depending on the motions of the user. If the user moves into the right direction, the arrow is green-coloured, if the user moves into the wrong direction, the arrow turns red in colour gradations. In the background, the real environment can be seen through the camera of the mobile device.

One famous example for an AR-application, which is available for smartphones, is the *Wikitude World Browser*[9] (`http://www.wikitude.com` (retrieved 16/02/2014)).

[9]"*Wikitude* was the world's very first AR Browser for smartphones in 2008. It was first published when Google introduced the first *Android* device end of 2008 (the G1), which (again) was the very first mobile device carrying the hardware components required to make AR possible: GPS, accelerometer and digital compass. [...] *Wikitude World Browser* scans your surroundings for (e.g.) geo-referenced content using the camera and the device's sensors. The objects' information is displayed in the cam right where the real object is located." (`http://www.wikitude.com`

Another example is the *Project Reality View*, a *Navit* extension based on AR, which will be developed in cooperation with the *Geo-information Group of Potsdam University* based on the *Android* platform. (`http://wiki.navit-project.org/index.php/Augmented_Reality` (retrieved 16/02/2014)). The *Project Reality View* could be modified for the application for early warning and decision support to improve the comprehensibility of the navigation.
[DLL+11] concerns with the use of smartphones for the field identification of European crayfish. Therefore species identification keys, which can be extended by geo-localised multimedia data (e.g. photos, videos, audios) on each species (e.g. biology, ecology, distribution), inter alia in order to create risk maps visualising invasions of species, which can be accessed via smartphones. Therefore taxonomy and field specialists should be asked to increase the number of photos and videos and complete and improve the audio part. Finally, a multilingual version could be designed, in order to enable field specialists of all European countries to use the application. (cf [DLL+11]).
This approach could be enhanced for the identification of mosquito species (e.g. by including a pictorial key for culicidae). If e.g. anopheles mosquitoes could be identified, malaria risk zones could be tagged (e.g. via crowd-sourcing) and early warning could be delivered.

5.6 Interim Conclusion

One of the main tasks of the GUI for the application that has to be developed for digital devices is the use of risk assessment to create a risk representation tailored to the user of early warning and decision support. Accordingly, resource allocation will be optimised in order to optimise health service delivery. An important milestone in the *ReGLaN-Health and Logistics* project is the development of an application for a digital device delivering early warning and decision support tailored to different user groups using an adaptive GUI. In this thesis we are concerned with the mathematical structure of a system for producing an adaptive, comprehensible GUI tailored to different user groups which will enable the user to comprehend the spatial sense of a support proposal (AR for risk visualisation and navigation aid), for example how to escape from a hazardous area possibly unscathed or how to act in the case of an accident (using telemedicine as a resource). The system developed in this thesis is intended to adapt dynamically to each individual user, thus different types of users are supplied with different GUIs. When developing the GUI it is essential to involve the user so as to adapt the GUI to the user needs and give her/him the best GUI possible. The system developed in this thesis is adapted to the user by adjusting dynamically and flexibly to the user's mental model. If the user's mental models change, so too does the GUI. One indicator of good GUI design is that information is presented on a screen in a way that facilitates chunking and the number of chunks is limited to approximately seven. For our purposes, we have to evaluate the different GUI element objects with respect to chunking quality. Each GUI element object corresponds to a number of chunks, depending on the chunking quality of the object. The quality of the GUI in terms of the number of chunks can be

(retrieved 16/02/2014)).

represented by a membership mapping. To make the communication of the user with the system easy, appropriate electronic analogues of the traditional ways of communicating or manipulating similar information in the non-electronic domain should be created. The GUI developed for the *ReGLaN-Health and Logistics* project will include, inter alia, telemedicine, AR and speech recognition. In addition, the spatial location and user's movements can be determined by means of GPS-coordinates. Furthermore, the user can be provided with a suitable GUI using information on geo-location. An error determiner, which takes account of user skill levels and tailors error feedback, is implemented in the SAGU (Chapter 9), and further described in Section 9.7. The use of fuzzy logic, further described in Chapter 7, allows for error tolerance by considering the degree of an error, that is the degree of difference between expected user reaction and the actual user reaction. South Africa is used as the pilot region for applying the GUI that is developed. The following ideas were found to be helpful for South African users: If the concept of an "Inbox" and "Outbox" is familiar from an email GUI, this could be used in the GUI developed for this project. For example, information could be delivered to a user in an Inbox and the user could be given the option to respond in an Outbox. In South Africa, it is important to deliver the GUI in different languages, as South Africa has eleven official languages. It can also happen that there are for example illiterate and blind people among the users. Therefore an improved ASR version may be useful within the GUI; especially as it is supposed to be used for delivering early warning and decision support, as described in this thesis.

As this thesis will be followed by the implementation of the results, implementation in UML within the OOA can be concluded from an interface analysis with computer science and, thus, implemented in this thesis. If the OOA model exists, it is possible to create a prototype of the user interface, that is the interface for the future system can be designed. In the next chapter, the tools OOA and UML from computer science will be investigated in order to design a structure that describes the functionality of an implementable GUI.

6 Object Orientation for preparing the Creation of a Prototype of the GUI for an Application delivering Early Warning and Decision Support

For the development of an executable prototype of an application for digital devices, Object Orientation (OO) is a useful tool.
"Object-oriented design provides some of the flexibility for building highly integrated information systems, utilising existing software components, but supporting a high degree of customisation." ([FJ96]).
Object-Oriented Analysis (OOA) is used to design the system as software product. The functioning of the system can be displayed using UML. The SAGU is displayed using UML in CHAPTER 9. OOA is used to select the GUI element objects matching to the user's needs. Topologies described in CHAPTER 8 are connected to an object (see p.57) as information content. Objects considered in our context are the components of the SAGU, described in CHAPTER 9.
The object-oriented software development begins with the first object-oriented programming language Smalltalk-80 that was developed in the years 1970 till 1980 at the *Palo Alto Research Center* (PARC) of the company *Xerox*. The class concept was taken over and advanced by the programming language Simula-67. With the beginning of the 90s, C++ has established itself as dominant language of Object-Oriented Programming (OOP). Since 1996, Java takes a significant position in addition to C++ while Smalltalk has been pushed back to the same extent. In the late 80s and early 90s the first books were published on the practices of OOA and Object-Oriented Design (OOD), which were followed by many now. In contrast to textual notations of the programming languages graphical notations are used here. In October 1994, *Grady Booch* and *Jim Rumbaugh* have joined forces at *Rational Software Corporation* to develop their successful methods into a unified industry standard. At first it was the predecessor of the UML , published under the name "Unified Method". Since autumn 1995, *Ivar Jacobson* has also participated in the development of the UML. In October 1996, the version 0.91 of the UML has been released. Since September 1997, the version 1.1 of the UML was valid, where additional ideas of different UML partners had been included. UML 1.1 was adopted as standard by the *Object Management Group* (OMG) on 17th November 1997. ([Bal99], p. 3f).

The research of the *University of Duisburg-Essen* is focused on four areas: basic concepts, guidelines for appropriate use, integration with other languages for enterprise modelling and evaluation methods. Object-oriented concepts sometimes differ significantly from natural language concepts. This is true for generalisation relationships, specialisation relationships and the redefinition of inherited characteristics. At the same time object-oriented modelling languages and their mapping to relevant tools sometimes contain ambiguities, hindering a smooth software development. To counter such difficulties, they have developed additional language concepts such as “delegation”, which they have embedded within the design methods in a modelling language. In addition, as part of a dissertation, an object calculus for formalisation of the core language of the MEMO-OML was developed. To provide guidelines for the targeted use of object-oriented concepts, they develop alongside relevant, adaptable process models principles of good design, which might be illustrated in reference models. Static object models alone are not sufficient to guide the analysis and design of business information systems. This fact is strongly emphasised in the context of business modelling. However, the conceptual linking of object-oriented languages with other modelling languages, such as business process modelling, is frequently insufficient. For this reason, they develop in addition to their own languages for enterprise modelling conceptual interfaces, for example for the integration of UML into their business process modelling language. More about the research of the *University Duisburg-Essen* concerning OO is accessible at the following link:

`http://www.wi-inf.uni-duisburg-essen.de/FGFrank/index.php` (retrieved 16/02/2014).

This method of the *University Duisburg-Essen* of developing conceptual interfaces for the integration of UML into the own modelling language could be applied to the development of an application for a digital device delivering early warning and decision support tailored to different user groups with a GUI that adapts to the user, tailored to the different disciplines that collaborate in the *ReGLaN-Health and Logistics* project.

In the *Collaborative Research Centre 588* of the *Karlsruhe Institute of Technology* (KIT) *“Humanoid Robots - Learning and Cooperating Multimodal Robots”* concepts, methods and concrete mechatronic components for a humanoid robot are developed, that shares its workspace with humans. In order to be a helpful assistant for humans in everyday life, the robot system must have many skills and complex properties, such as cognitive skills and a consistent memory. The Subproject *M2* is processed at the *Institute of Anthropomatics*. The central goal of this project is to build an object-oriented model of the entire environment. This model contains the relevant facts for the robot in the form of objects and their relationships. It is provided with interfaces to serve as real-time information hub for all system components such as perception and inference. Information can be inserted and read. Existing objects or relations can be manipulated. New information can be used to instantiate new objects or relations. As a metaphor for the environment model a digital sandbox can serve, that contains the virtual proxy for the relevant objects in the environment and offers the opportunity to gain an overview of current events. The model complements the cognitive abilities of the robot with a memory structure that gives it a complete picture of its surroundings. More about the research of the *KIT* concerning OO is accessible at the following link:

`http://ies.anthropomatik.kit.edu/forschung_sfb588m2_umweltmodellierung.php` (retrieved 16/02/2014).

This approach could be transferred to digital devices, the application is supposed to be generated for, to improve services like telemedicine, help in an accident case or navigation by enabling the digital device to perceive a complete picture of its surroundings. Additionally, the user interface of blind users could be improved with this approach.

For the description of the mathematical structure behind an SAGU (Chapter 9), it is important to describe OO with mathematical concepts to be able to apply measure theory on topological spaces (Chapter 8), ANNs and fuzzy logic (Chapter 7). For the implementation of the application, the mathematical structure behind the system has to be transformed into OO within the OOA. This is essential as preliminary step for the development of a prototype.

A mathematising approach to describe OO and its informatics concepts, i.e., the attempt to describe OO and its concepts with mathematical concepts and structures, proves to be quite problematic. It turns out that it is not always possible to find a really appropriate and congruent mathematical description of the computer science ideas. The search for a mathematical interpretation of the object (see p.57) in the computer science sense leads to a set perspective. Objects can be viewed as elements of a set that are characterised by a similar property structure, that are definite and separable from each other by a characteristic variation in this property structure. Continuing this mathematical interpretation, a class (see p.58) in object-orientation can be interpreted as a set of objects, based on the set theory. Then, the class is the set of all its instances, i.e. the set of all the objects produced by this class with identical characteristic structure. The set-theoretic interpretation of objects and classes covers essential elements of the informatics contents from both terms, however, it denies in part if we take a closer look at the object's properties and their publication in form of the object interfaces. The methods associated with the object interface prepare interpretive problems. The related possibility of an active or passive state change of objects often is difficult to find in the mathematical context. It is also difficult to present the computerised concept of encapsulation, i.e. the information-shielding to the outside, in mathematical expressions. A content-similarity is found in mathematics in algorithmic procedures. Algorithms are characterised by initial conditions, by a sequence of elementary actions to solve a problem and by a final state. The sequence of elementary actions can now be interpreted as the information that exists, without having to or being allowed to intervene from the outside into the conduct of the proceeding. Thus that part of the whole algorithm appears encapsulated. Starting conditions and final state would then be assigned to the object interface. However, this interpretation contradicts the real meaning of the algorithmic method in mathematics, basically the elementary steps and their sequence are the real focus of the investigations and observations. Another possible interpretation of encapsulation can be found in the approach to consider two different states of a mathematical

object, and to regard the process between the beginning and end as encapsulated information and methods. The process then is viewed as a unified whole, therefore, as a capsule. The concepts of inheritance and abstract classes introduce a structure in the (computerised) OO, with that a class hierarchy can be created. According to this and the set interpretation of the classes the inheritance in mathematics can be regarded as an order relation, which leads to a hierarchy of sets. The abstract classes represent in this context of meaning the roots of the hierarchical structures. Inheritance can not be regarded as a subset relation, because the structural design of the generalised (abstract), to the specific property structures, that can be expanded to include additional properties, compared to the roots, are diametrically opposed to this view. (cf [Xyl08]).
A transfer of object-oriented thinking in the teaching of mathematics often opens a different perspective on mathematical content and expands the methodological repertoire of the pupils. Material related and material comprehensive correlations and analogies are in the fore. Structuring and systematising work practices are developed and an analytically and synthesising thinking is encouraged. OO is characterised by structuring and systematising thinking and ways of working. Characteristic also is the content and methodology transfer between similar structures. These are fundamental skills that also have a great importance in mathematics education and should be developed and acquired the same place. OO can therefore be understood as a component of mathematics education that brings the mathematical way of thinking at least a little closer to the pupils. An object-oriented perspective can enrich the teaching of mathematics and offer interesting approaches to the recognition and development of substantive contexts. On the way of the OO from the informational idea to a creative perspective in teaching mathematics, it is essential to transfer the terminological and basic conceptual ideas of computer science in the language and mindset of mathematics. The treatment of classes in mathematics from the didactic point of view leads to the systematisation of knowledge content. A second important aspect is the analogy: A comparison of two classes and the creation of a new class by inheriting transfers the content structure of a class structure to a second class. The

quadrangle	**class defined properties**	**typification**
equilateral quadrangles	$a = b = c = d$	square, rhombus
isosceles quadrangles	$a = b = c$	no special quadrangle
	$a = b$; $c = d$	dragon quadrangle
	$a = c$; $b = d$	rectangle, parallelogram
	$a = b$	no special quadrangle
	$a = c$	no special quadrangle
irregular quadrangles	$a \neq b$; $a \neq c$; $a \neq d$; $b \neq c$; $b \neq d$; $c \neq d$	no special quadrangle

Table 6.1: Systematisation by quadrangle sides. Without regard to the general public, the class defining properties are formulated beginning with side length a in the table. (cf [Xyl08].)

TABLES 6.1 and 6.2 show possible results of a lesson when the systematisation of quadrangles was the

quadrangle	**class defined properties**	**special cases**	**typification**
right-angled quadrangles	$\alpha = \beta = \gamma = \delta = 90°$		square, rhombus
	$\alpha = \beta = \gamma = 90°$		square, rhombus
	$\alpha = \beta = 90°$		rectangular trapezoid
	$\alpha = 90°$		quadrangle
obtuse-angled quadrangles	$\alpha, \beta, \gamma, \delta > 90°$		no quadrangle!
	$\alpha, \beta > 90°$		quadrangle
		$\alpha + \gamma = \beta + \delta = 180°$	chordal quadrilateral
	$\alpha, \gamma > 90°$		quadrangle
		$\alpha = \gamma$; $\beta = \delta > 90°$	rhombus, parallelogram
	$\alpha > 90°$		quadrangle
acute-angled quadrangles	$\alpha, \beta, \gamma, \delta < 90°$	$\alpha, \beta, \gamma, \delta < 90°$	no quadrangle!

Table 6.2: Systematisation by quadrangle angle sizes. Without regard to the general public, the class defining properties are formulated beginning with the angle α in the table. (cf [Xyl08].)

task. (cf [Xyl08]).
This application can be transferred to the visual properties of the GUI-elements and their positions on the screen.
In an object-oriented software development, the results of the analysis phase, design phase and implementation phase are created object-oriented. For the latter, object-oriented programming languages are used. In addition, the user interface of the software must be designed object-oriented and object-oriented databases can be used. The distribution on a network can happen object-oriented, too. ([Bal99], p. 2).

6.1 Object Oriented Analysis

One important milestone within the *ReGLaN-Health and Logictics* project is the development of an application for a digital device delivering early warning and decision support tailored to different user groups with a GUI that adapts to the user. The SAGU (see Chapter 9) is a component of this software system.
The aim of the OOA is to identify and to describe needs and requirements of a client to a new software system. There must be created a model of the functional specifications, which is consistent, complete, clear and feasible. It is important that in the process of modelling in system analysis all aspects of implementation are consciously excluded. In the analysis a perfect technique is considered, i.e. that the processor can perform any function without delay, it does not make mistakes or even fail, the memory can hold an infinite amount of information and the processor can access it without delay. System analysis is one of the most challenging activity in software development, because the requirements of the client are usually unclear, contradictory, and case-oriented and they are at different

levels of abstraction. In the OOA, we start from the objects that are located in the real world. These are not just touchable objects or people, but often words or events from the respective scope. From a real object an object of our object-oriented model results by modelling and appropriate abstraction. Objects that can be described by the same characteristics belong to the same class. In the OOA is not described how objects are displayed on the user interface or how they are stored and selected. The aim of the OOA is to understand the problem that has to be realised and to describe it in an OOA model. This model should describe the essential structure and semantic of the problem, but still no technical solution. ([Bal99], p. 8ff). [Bal99], p. 8ff, mentions, that the following products should be created in the analysis phase:

- The Functional Specification Document (FSD) :
  The first step of a system analysis should be to create an FSD that serves the basis for a systematic modelling. The FSD is a textual description of what the system, that should be realised, should achieve. It should fulfil two objectives in this procedure:

  1. It is the initial document for the project for everyone who has to maintain the system later.
  2. It should enable the system analyst to build the OOA model.

  It is not the aim to implement the system by the FSD.
  An FSD is done in Chapter 9.

- The OOA model:
  The OOA model is the technical solution of the system that has to be realised. It consists of a static and a dynamic model. Which one of these two models has the greater weight in the system analysis depends largely on the application. The static model is particularly important for typical database applications. The dynamic model is particularly important for highly interactive systems.

  - The static model:
    The static model describes in particular the classes of the system, the associations between classes and inheritance structures. It also contains the data of the system (attributes). The packages are designed to form subsystems to allow a better overview over large systems.
  - The dynamic model:
    The dynamic model shows function processes. Business processes describe the tasks that have to be performed at a very high level of abstraction. Scenarios show how objects communicate with each other to accomplish a specific task. State machines describe the life cycles of objects in the analysis, i.e. the reactions of an object on different events (messages).

  The OOA model must contain all the information to derive a prototype of the user interface. Often only the existence of this prototype allows to clarify with the future users or with the

client, whether the system really is specified as desired. In this prototype, of course, besides the information from the OOA model and also information on the best ergonomic design are included. ([Bal99], p. 8ff).
OOA models for the SAGU are illustrated in Chapter 9 and Chapter 10.

- The prototype:
  The prototype of the user interface is an executable program that displays all the attributes of the OOA model on the interface. It does neither achieve application functions, nor it has the ability to store data. The prototype consists of windows, dialogues, menus, etc. The purpose of the prototype is to evaluate the created OOA model with the future user. The objective should be to express the full user interface by this prototype, if possible. Where this is not possible, an additional documentation must be created. The separation of the OOA model and the user interface is a fundamental principle of the development. The OOA model defines which information is visible on the screen, the user interface determines the format in which it is presented. ([Bal99], p. 8ff).
  A prototype is not developed in this thesis.

In the following, some basic concepts and notations of OOA are addressed.

Object. In everyday language an object is an item of interest, in particular of an observation, inspection or measurement. Objects can be things (e.g. drugs, hospital room), persons (e.g. employee, patient) or concepts (e.g. programming language, illness). In object-oriented software development, an object has a particular state and reacts with a defined behaviour on its environment. In addition, each object has an identity that distinguishes it from all other objects. An object can know one or several other objects. We are talking about links between objects. The state of an object includes the attributes and their current values and the respective links to other objects. Attributes are inherent, unchangeable characteristics of the object, while the attribute values are subject to change. The behaviour of an object is described by a set of operations. A change or query of the state is only possible by means of the operations. The object is represented in UML as a rectangle, which can be

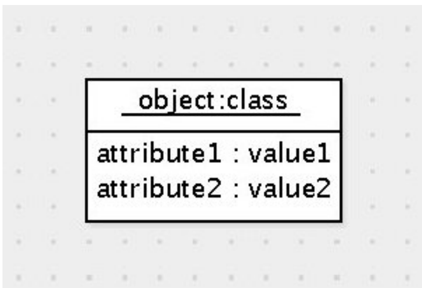


Figure 6.1: Notation of an object. (Figure generated with *ArgoUML*.)

divided into two fields, shown in Figure 6.1. In the upper field, the object is designated as follows:

If we have an anonymous object, only the class name is specified. Object and class can be specified when the object should be addressed by a name. If the object name is sufficient to identify the object and the name of the class from the context, you can only specify the object name. Anonymous objects are used, if it is any object of the class. The object name is used to designate a specific object of the class for the system analyst. In the lower field the in the context relevant attributes of the object can be registered. If the value of an attribute is not of interest, only the attribute name is denoted. The operations that an object can perform are not specified in the UML. Objects and their links with one another are specified in the object diagram, see FIGURE 6.2. It describes objects, attribute

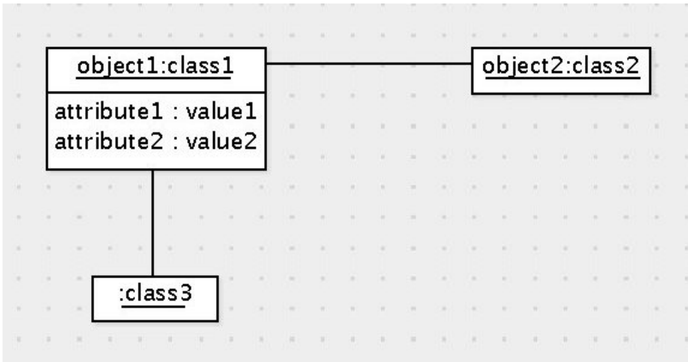
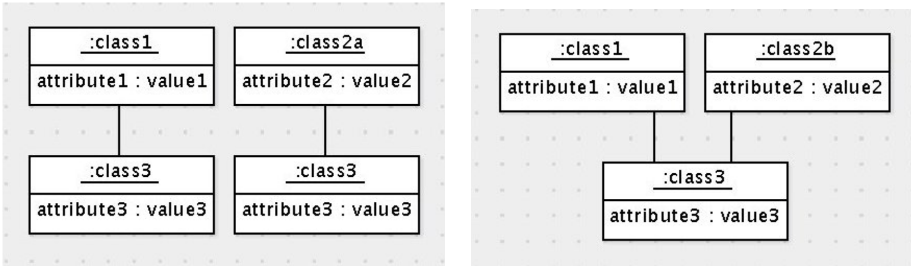


Figure 6.2: Notation of an object diagram. (Figure generated with *ArgoUML*.)

values and links between objects at a particular time. Object diagrams are snapshots of the system, so to speak. In most cases, anonymous objects are used. Concrete objects are interesting only in exceptional cases. State and behaviour of an object are the same. We say: an object encapsulates status (data) and behaviour (operations). The data of an object can only be read and modified by means of the operations. This means that the representation of the data should be hidden. We say: an object implements the principle of secrecy. The object identity is the property that distinguishes an object from all other objects. This means that all objects are distinguishable because of their existence, even if they possess identical attribute values by chance. The identity of an object can not be changed. No two objects can have the same identity. If two objects have different identities and the same attribute values, we speak of the equality of objects. We distinguish between identical and equal objects, see FIGURE 6.3. The object name identifies an object in the object diagram. In contrast to the object identity it only has to be unique in the viewed context, i.e. within the diagram. It is important to distinguish between external and internal objects. External objects exist in the real world, whereas internal objects are relevant for a software system.

Class. A class is defined for a collection of objects, their structure (attributes), behaviour (operations) and relationships. It has a mechanism to create new objects (object factory). Each created object belongs to exactly one class. The relationships are associations and inheritance structures. The behaviour of a class is described by the messages, on which the class or its objects can react. Each message activates an operation of the same name. In FIGURE 6.4 the notation of a class is



(a) Equality of objects. **(b)** Identity of objects.

Figure 6.3: Notation of equal and identical objects. (Figure generated with *ArgoUML*.)

illustrated. The class symbols are entered in the class diagram with other symbols, e.g. association

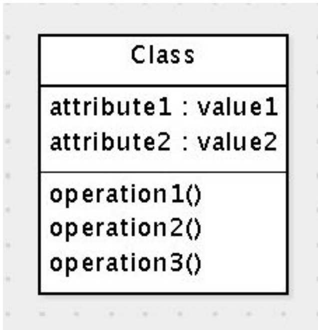


Figure 6.4: Notation of a class. (Figure generated with *ArgoUML*.)

and inheritance. The class diagram describes the static model of the system. For large systems, it is generally appropriate or necessary to create more class diagrams. The class name is always a noun in the singular, which can be supplemented by an adjective. It thus describes a single object of the class. The class name must be unique inside a package, but better within the entire system. Each object knows its class. Conversely, a class does not know what objects it owns or which objects were created by it. Since this knowledge would be very useful, however, we assume in the system analysis that a class knows its objects, i.e. the class keeps a record of the creation and deletion of objects. We call this property object management. This gives the class a chance to perform queries and manipulations on the set of objects of a class.

Attribute. The attributes (see FIGURE 6.4) describe the data that can be adopted by the objects of a class. All objects of a class possess the same attributes but different attribute values. Attributes are described by their name and their type. Optionally, an initial value and a list of features can be specified. The initial value determines the value for the attribute of a newly created object. In the list of features the characteristics or properties of the attribute can be specified. The notation of an attribute with initial value and list of features is illustrated in FIGURE 6.5. The attribute name must be unique within the context of the class. It describes the stored data. For this generally a noun is used. The attributes may be altered and read only by the operations of the respective class. Although

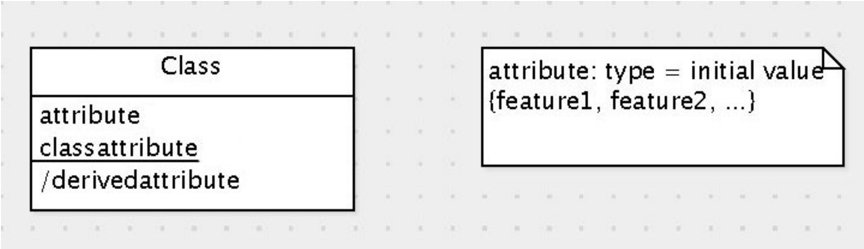


Figure 6.5: Notation of an attribute with initial value and list of features. (Figure generated with *ArgoUML*.)

they are visible to the analyst, but not visible to other classes and their objects.

Operation. An operation (see FIGURE 6.4) is an executable activity. All objects of a class use the same operations. Each operation has directly access to all attributes of an object of this class. The set of all operations is called the behaviour of the class or the interface of the class. Operations are mentioned analogous to the attributes in the class symbol. Also, for each operation, a list of features can be specified. We distinguish three types of operations:

- object operations (short: operations)
- constructor operations
- class operations.

This categorisation allows a systematic mapping of operations to the classes while creating the analysis model. Object operations are always applied to a single (already existing) object. A constructor operation creates a new object and performs appropriate initialisation and data collection. A class operation is an operation that is assigned to the particular class and can not be applied to a single object of the class. In the system analysis, we use class operations in the event that the operation manipulates class attributes without the involvement of a single object or if the operation relates to all or several objects of the class. Operations can be classified according to their functions:

- Operations with read access to attributes of the same class (accessor operation).
- Operations with write access to attributes of the same class (update operation).
- Operations to perform calculations.
- Operations for creating and deleting objects (constructor operation and destructor operation).
- Operations, that select the objects of a class according to specific criteria (query operation, select operation).
- Operations to establish connections between objects (connect operation).
- Operations that activate operations of other classes.

An operation is called external if it is directly activated by the user interface. An external operation can call other internal operations. An internal operation is always activated by another operation within the system. The goal of systems analysis is to identify all external operations. Internal operations are only entered in the class diagram, if it is necessary for understanding. The operation name should express what the operation does. It must, therefore, generally contain a verb. The name of the operation must be unique within the context of the class. Any operation that's operation is not apparent from the name, is described from a user perspective. A colloquial phrase has proven to be useful here. ([Bal99], p. 17ff).

Use Case Diagram. A use case diagram describes the relation between actors and use cases in a system. Additionally, the relations between use cases can be entered. A use case diagram delivers a good overview over the system and its interfaces to the environment on a high level of abstraction. ([Bal99], p.540).
An exemplary use case diagram is illustrated in Figure 6.6. Use case diagrams are used in Chapter

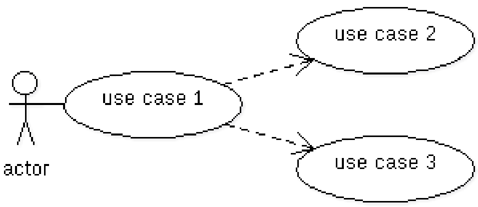


Figure 6.6: Exemplary use case diagram. (Figure generated with *ArgoUML*.)

10.

State Diagram. A state diagram is a graphical representation of the finite state machine. A finite state machine consists of states and transitions. It has an initial state and it can have a final state. ([Bal99], p.554).
An exemplary state diagram is illustrated in Figure 6.7. A state diagram is used in Chapter 10.

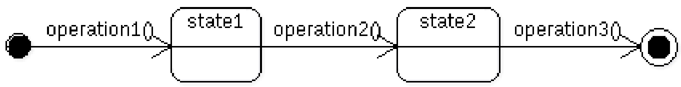


Figure 6.7: Exemplary state diagram. (Figure generated with *ArgoUML*.)

Sequence Diagram. A sequence diagram possesses two dimensions. The vertical represents the time and on the horizontal the objects are located. The messages to activate the operations are entered into the diagram. ([Bal99], p.550).
An exemplary sequence diagram is illustrated in Figure 6.8. In Chapter 10 sequence diagrams are used.

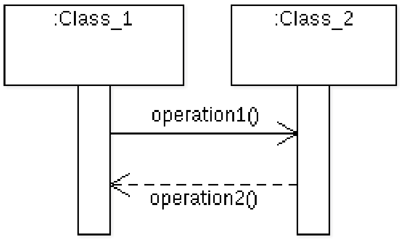


Figure 6.8: Exemplary sequence diagram. (Figure generated with *ArgoUML*.)

6.2 Design of User Interfaces

If the OOA model exists, we create a prototype of the user interface, i.e. we design the interface of the future system. For this task, a basic knowledge of software ergonomics is needed. The software ergonomics is concerned with the human-oriented design of software systems. It pursues the goal to adapt the software to the characteristics and needs of the user. Specifically, this was mentioned in Chapter 5; Chapter 9 and Chapter 10 dwell on this in more detail. A prototype of the user interface implements windows, menus, dialog management and global data, but does not contain data and dialogues within data fields. In addition, of course, no functions are realised. A GUI consists of a dialog component and an I/O (input/output) component (design of information presentation). The GUI system is the software system that manages the GUI and handles the communication with the applications. For the creation of the prototype, ideally, the same GUI system is used, which is used in the design for the realisation of the user interface. In this way, the prototype can be evolutionary developed. ([Bal99], p. 193ff).
A prototype of the user interface is not developed in this thesis, but this will be done in the future by computer scientists.

6.3 Object Oriented Design

In the analysis we have assumed an ideal environment in the modelling of the system. Task of the design is now to realise the specified application on a platform below the required technical conditions, which is not done in this thesis. In the design we are still at a higher abstraction level than in the implementation. In the design phase, the OOD model is conceived from the viewpoint of efficiency and standardisation. The object-oriented design is considerably simplified, because no paradigm shift does take place from analysis to design. Design phase and implementation phase are very much interlinked with each other. This means that each designed class can be implemented directly. Design goal is to couple OOA model, user interface and data storage as much as possible. How the user interface looks like depends crucially on the used GUI. The data management is largely determined by the used (relational or object-oriented) database. Alternatively, the data storage can be realised using flat files. From the design goal the use of a three-layered architecture can be derived, i.e. we separate the layers of user interface, OOA model and data preservation. The existing OOA model is revised from the viewpoint of efficiency and reuse. This includes consideration of existing class libraries and interfaces to other software. From the prototype of the user interface the layer of the user interface is created. This is a prototype that will be systematically developed. The component of data management has to provide the connection to object-oriented or relational databases or create its own data storage. In client-server applications, the software must be distributed on multiple computers. The following products should be created in the design phase:

- The OOD model:
  The OOD model is documented similar to the OOA model. In contrast to the OOA model the OOD model should be a replica of the later program. Every class, every attribute and every operation of the OOD model is also found in the programs. There are exactly the same names used as in the program. In contrast to an object-oriented program code the OOD model shows the system on an abstract level and, above all, it makes the interaction of individual elements clear. The OOD model consists of a static model and a dynamic model, too.
  - The static model:
    The static model of the OOD model is much more comprehensive compared with the static model of the OOA model. It should include all classes of the program, describing the architecture of the system. Classes that describe only types, e.g. string, are not registered. The packages are not only used for the modelling of subsystems, but also to demonstrate the different layers.
  - The dynamic model:
    The dynamic model is in the design for all areas of particular importance. It provides a clear description of the complex communication between the objects that is only difficult comprehensible using the program code. ([Bal99], p. 11ff).

An OOD model is not developed in this thesis, but this will be done in the future.

6.4 Interim Conclusion

The development of an application for a digital device delivering early warning and decision support tailored to different user groups using a GUI adapted to the user is an important milestone for the *ReGLaN-Health and Logictics* project. The SAGU (see Chapter 9) is a component of this software system. For a description of the mathematical structure of an SAGU, it is important to describe OO using mathematical concepts in order to be able to apply measure theory on topological spaces (Chapter 8) ANNs and fuzzy logic (Chapter 7). For the implementation of the application, the mathematical structure behind the system has to be transformed into OO within the OOA. This is essential as a preliminary step for the development of a prototype.

To be able to develop the application needed here, fuzzy logic and ANNs for implementing adaptivity play an important role. An error determiner, that takes account of user skill levels and tailors error feedback is therefore implemented in the SAGU (described in more detail in SECTION 9.7). The use of fuzzy logic (described in more detail in CHAPTER 7), allows for error tolerance by considering the degree of an error, that is, the degree of difference between the expected user reaction and the actual user reaction.

For our purposes, we have to evaluate the different GUI element objects with respect to chunking quality. Each GUI element object corresponds to a number of chunks depending on its chunking quality. Accordingly, the quality of the GUI depends on the number of chunks. This can be represented by a membership mapping. To enable the SAGU to solve new, previously unknown, problems, we include ANNs. It is important that the system that is developed is able to deal with the challenge of being flexible and being able to change its properties and behaviour in order to deliver the best results to the user; in other words, a GUI that is understood by the user and can be adapted to the user's needs. This is done with, among other things, the help of ANNs which are applied in the system (described in SECTIONS 9.3 and 9.5).

Therefore, fuzzy logic and ANNs for implementing adaptivity are addressed in the next chapter.

7 Fuzzy Logic and Artificial Neural Networks for implementing Adaptivity

Fuzzy logic is used to represent uncertainties and ambiguities of linguistic descriptions. It is used within the SAGU, described in Chapter 9. Fuzzy logic is used to determine the membership mappings, which are used within each component of the developed system: within the ANNs (Section 7.2), described in Section 9.3 and Section 9.5, by the initial state determiner, described in Section 9.1, within the TCIU, described in Section 9.4, for the database with information about other users Section 9.2, and for the error determination, described in Section 9.7. Additionally, an FLC is used within the SAGU, described in Chapter 9. Figure 7.1 shows the architecture of an FLC. Besides,

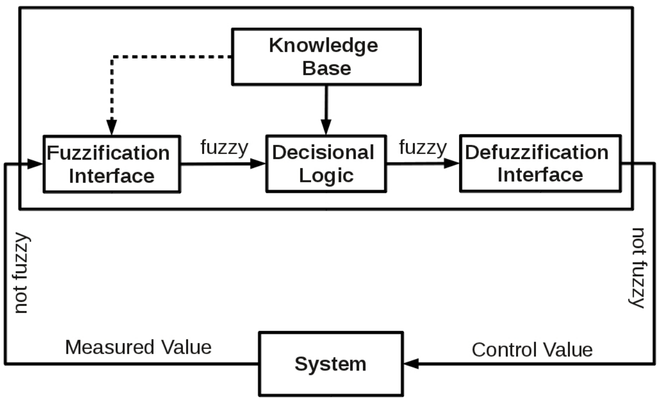


Figure 7.1: Architecture of an FLC. (Figure generated with *Libre Office Draw.* Compare: [NKK94], p.246.)

fuzzy logic is used in a similar way for the generation of risk and resource maps and thus decision support maps, see Section 10.1. The SAGU consists inter alia of a combination of an FLC and an ANN. The benefit of this combination is the learning ability of the ANNs, which makes it possible for a system to "learn" linguistic rules and/or membership functions or to optimise the ones that already exist, further described in Section 7.3. This ability is important for a system which has the aim to adapt a GUI to the user, more precise for the adaption process and for the flexibility of the system. In the following, we will have a closer look on fuzzy logic, ANNs and the combination of both.

7.1 Fuzzy Logic

Fuzzy logic is a logic system that allows more than two or even infinitely many truth values. Fuzzy logic is based on the theory of fuzzy sets. One can say with what grade of membership any elements belong to them. This grade of membership can take any value between zero and one. The currently most important use of Fuzzy logic is control technology. Fuzzy controllers are used in many areas of daily life, for example for cameras, washing machines and rail vehicles. ([Pau00], p.310).
Fuzzy set theory came from *Lotfi Asker Zadeh* a professor of systems theory and electrical engineering who was born in 1921 in the Soviet Union and since 1944 living in the USA. For the first time it was published in 1965, with a contribution of *Zadeh* titled *"Fuzzy Sets"* for the magazine titled *"Information and Control"*. *Zadeh's* article was met with little positive response at that time. However, precisely this idea helped the Japanese industry in the early 90's in the form of various consumer goods and means of production on the world market to gain "material force". Because of the technological and economic development methods for modelling of "imperfect"(i.e., fuzzy and uncertain) knowledge, as they have been developed as part of the fuzzy set theory and its branch, the possibility theory, become increasingly important. The reduction of complexity is necessary for the management of highly complex issues that brings inevitably blurs and uncertainties, and the use of computer technology experience of imperfect knowledge is necessary for perfecting automated processes. ([Bie97], p.V).
With the help of fuzzy mathematics not only the classic, sharp states like yes/ no, true/ false or 0/ 1 can be calculated[1], but also many intermediate steps. The fuzzy logic allows computers to deal with vague statements of different tolerances what was never difficult for the human brain. ([Tra93], p.2).
The fuzzy mathematics is an extension of classical mathematics. It is not spongy or inaccurate, but rather bases on logical rules of structure as the usual, classical mathematics, which is contained in it - any “sharp“ precise value is a special form of the ”fuzzy” case. Fuzzy mathematics is no degradation or coarsening of classical mathematics, it is a real extension. It allows us to handle everyday human experience and terms like "very", "somewhat" or "old", "young", "fat", "thin" mathematically precise and elegant. *Zadeh* developed fuzzy mathematics to make human-logical thinking and decision-making strategies commutable for the computer. This does obviously not take effect for emotional decisions - they are for people themselves often not understandable. ([Tra93], p.6f).
Probability describes, in contrast to fuzzy logic, the occurrence of random events with a crisp result. E.g., if there was a toxic substance which is deadly with the probability of 0.5, approximately each second person who is exposed to this substance is killed. If there is a toxic substance which is deadly with the degree of 0.5, a person who is exposed to this substance is weakened (to a degree of 0.5), but not killed.
Fuzzy logic is used inter alia within the GUI-development inter alia for the representation and inter-

[1] Calculate, because in the end of operations with fuzzy sets results an exact value - as examples show in the regime of technology.

pretation of vague statements of the user and for the generation of risk, resource and decision support maps (SECTION 10.1).

One way to determine sets is to define a mapping that is named indicator mapping, according to its purpose.

Definition 7.1.1 *Indicator Mapping ([Bie97], p. 50)*
The indicator mapping of a set A,

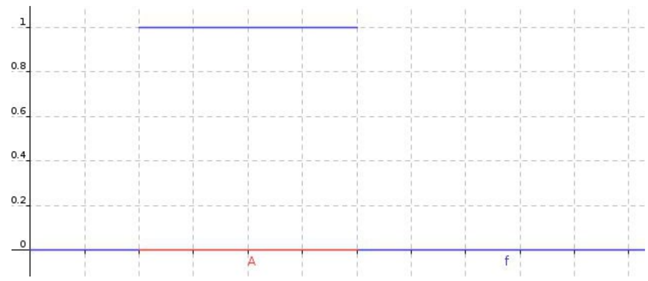
$$f_A : \Omega \rightarrow \{0, 1\}$$

delivers for each element of the underlying set Ω the value one, if it belongs to the set A, and the value zero, if it does not belong to A:

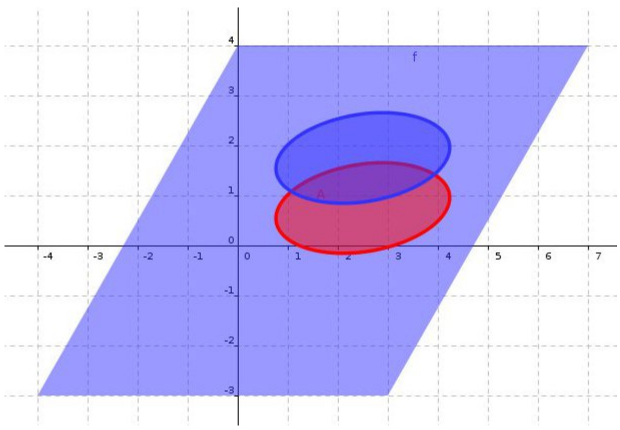
$$\forall \ x \in \Omega : f_A(x) = \begin{cases} 1, \ \textit{if } x \in A \\ 0, \ \textit{otherwise.} \end{cases}$$

.

Exemplary graphs of indicator mappings are visualised in FIGURE 7.2. Each classical set $A \in \Omega$ can



(a) Graph of the indicator mapping f of the set A in the two-dimensional.



(b) Graph of the indicator mapping f of the set A in the three-dimensional.

Figure 7.2: Graphs of indicator mappings. (Figure generated with *GeoGebra*.)

be characterised by an indicator mapping

$$m_A : \Omega \to \{0, 1\}.$$

This assignment is bijective. A membership mapping can be developed by an extension of the classical logic. Each fuzzy set can be characterised by a mapping

$$m_A : \Omega \to [0, 1].$$

Definition 7.1.2 *Fuzzy Set ([Bie97], p. 56)*
A fuzzy set is characterised by a mapping m from a underlying set Ω to the real unit interval:

$$m : \Omega \to [0, 1].$$

Note 7.1.3: *The indicator mapping is called membership mapping, its values are called grades of membership. Usually, as an abbreviation for "the grade of membership" the Greek letter μ is used ([Tra93], p. 9).*
In the following, we use m as an abbreviation for "the grade of membership", because the Greek letter μ will be used from SUBSECTION *8.2.1 on as an abbreviation for "measure".*

Example 7.1.4: *If we want to describe the term "high fever" as a set of numbers in the classic sense, there is the problem to define a limit. If we choose the temperature 39°C for this limit, the following mapping results (*FIGURE *7.3):*

$$f : \{x \mid x \in \mathbb{R} \wedge 32 \leq x \leq 44\} \to \{0, 1\}$$

$$x \mapsto \begin{cases} 1, \ \textit{if } x \geq 39 \\ 0, \ \textit{otherwise.} \end{cases}$$

Accordingly, a temperature of 42°C is also classified as "high fever" as a temperature of 39°C, while

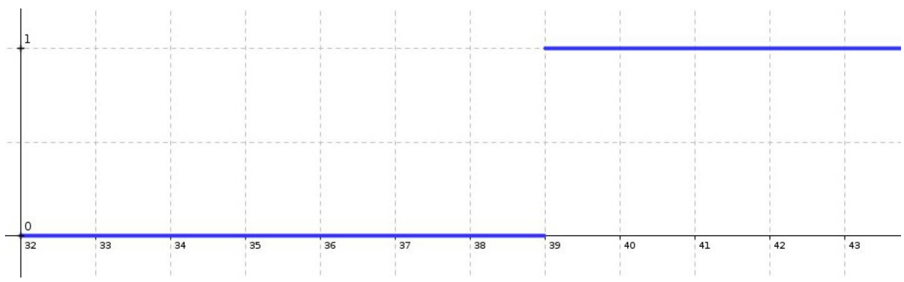


Figure 7.3: "High fever" as classic set. (Figure generated with *GeoGebra*.)

38.9°C has to be categorised as "not strong fever". Sliding transitions appear more realistic. They

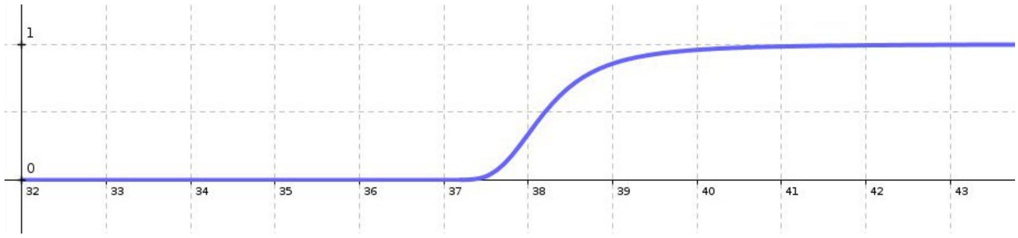


Figure 7.4: "High fever" as fuzzy set. (Figure generated with *GeoGebra*.)

can be achieved if one uses a continuous mapping as in FIGURE *7.4 instead of the step mapping in* FIGURE *7.3. Thus, a fuzzy set differs from a classic set therein that for the image set of the indicator mapping not only the values* 0 *and* 1 *are allowed, but all real figures between them ([Bie97], p. 55f). In this case the following mapping resulted:*

$$m : \{x \mid x \in \mathrm{I\!R} \wedge 32 \leq x \leq 44\} \rightarrow [0,1]$$

$$x \mapsto \begin{cases} \dfrac{1}{1+((x-37.2)^{0.8})^{-3.8}}, \; \mathit{if}\ x \geq 37.2 \\ 0, \; \mathit{if}\ x < 37.2. \end{cases}$$

Definition 7.1.5 *Cardinality of a Fuzzy Set ([Bie97], p. 66)*
The cardinality of a finite fuzzy set A on Ω, written $|A|$, is equal to the arithmetic sum of the grades of membership of its elements:

$$|A| = \sum_{x \in \Omega} m_A(x).$$

Definition 7.1.6 *Equality of Fuzzy Sets ([Bie97], p. 64)*
Two fuzzy sets A and B on Ω are equal, if and only if their membership mappings are equal:

$$A = B \Leftrightarrow \;\forall\; x \in \Omega : m_A(x) = m_B(x).$$

Definition 7.1.7 *Inequality of Fuzzy Sets ([Bie97], p. 64)*
Two fuzzy sets A and B are unequal, if and only if there exists one element of the underlying set Ω, that's grades of membership concerning A and B are unequal:

$$A \neq B \Leftrightarrow \;\exists\; x \in \Omega : m_A(x) \neq m_B(x).$$

The visibility of GUI elements can be controlled with regard to e.g. a risk at the geo-location of the user. Therefore a risk map has to be developed via a membership mapping with the fuzzy set B as domain. At those geo-locations in B, where the risk is higher than a certain value, e.g. 0.5, a certain GUI element warning the user is displayed. Those geo-locations are elements of the fuzzy set A, which is a subset of B.

Definition 7.1.8 *Subset of a Fuzzy Set ([Bie97], p. 65)*
A fuzzy set A is subset of the fuzzy set B, if and only if it is for each element of Ω that its grade of membership concerning A is not greater than its grade of membership concerning B:

$$A \subseteq B \Leftrightarrow \forall\ x \in \Omega : m_A(x) \leq m_B(x).$$

Definition 7.1.9 *Proper Subset of Fuzzy Sets ([Bie97], p. 65)*
A fuzzy set A is proper subset of the fuzzy set B, if and only if A is subset of B and A is unequal B:

$$A \subset B \Leftrightarrow A \subseteq B \wedge A \neq B.$$

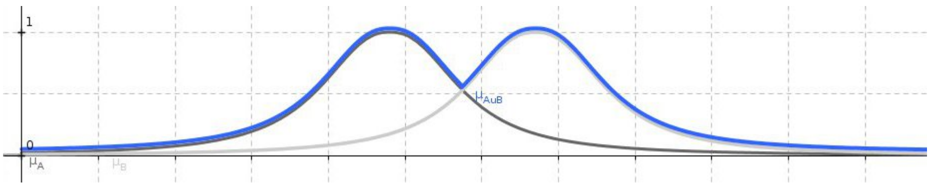
In the following, the union of fuzzy sets is defined. The union of fuzzy sets can be used for the generation of risk, resource and decision support maps. With the union of fuzzy sets, the logical OR can be expressed. I.e., if we consider two parameters for the generation of a risk, resource or decision support maps represented by membership mappings, they can be combined by using the union of fuzzy sets. I.e., if we consider two parameters contributing to the risk of a disease, i.e "1" indicates high risk and "0" indicates low risk and we want to find the locations with low risk, e.g. for finding possibly save tracks to pass a risk location i.e. locations, where the both membership mappings of the parameters contributing to the risk have a low value, the logical OR can be used.

Definition 7.1.10 *Union of Fuzzy Sets ([Bie97], p. 76 and [Tra93], p.38)*
The union of two fuzzy sets A and B, characterised by the indicator mappings $m_A : \Omega \rightarrow [0,1]$ and $m_B : \Omega \rightarrow [0,1]$ is a fuzzy set with the indicator mapping $m_{A \cup B} : \Omega \rightarrow [0,1]$, that it is for all $x \in \Omega$:

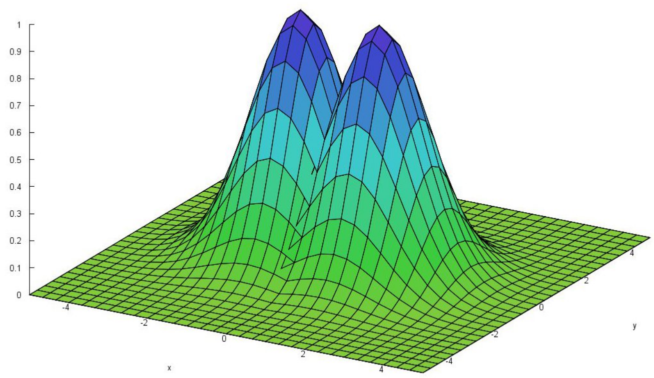
$$m_{A \cup B}(x) = \max\{m_A(x), m_B(x)\}.$$

Or:

$$m_{A \cup B}(x) = m_A(x) + m_B(x) - m_A(x) \cdot m_B(x).$$



(a) Union of fuzzy sets with a one-dimensional Ω. (Figure generated with *GeoGebra*.)



(b) Union of fuzzy sets with a two-dimensional Ω. (Figure generated with *Maxima*.)

Figure 7.5: Union of fuzzy sets. (Figure generated with *GeoGebra* and *Maxima*, respectively.)

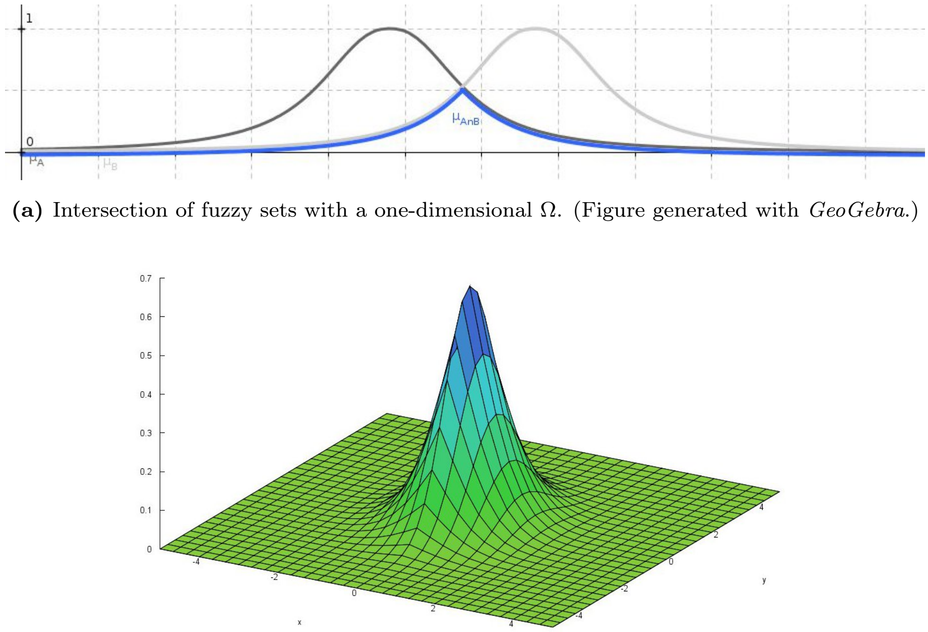
The intersection of fuzzy sets can be used for the generation of risk, resource and decision support maps, as well. With the intersection of fuzzy sets, the logical AND can be expressed. I.e., if we consider two parameters for the generation of risk, resource or decision support maps represented by membership mappings, they can be combined by using the intersection of fuzzy sets. I.e., if we consider two parameters contributing to the risk of a disease, i.e "1" indicates high risk and "0" indicates low risk and we want to find the locations with high risk, e.g. to intervene in those locations (e.g. via early warning or resource provision), i.e. locations, where the both membership mappings of the parameters contributing to the risk have a high value, the logical AND can be used.

Definition 7.1.11 *Intersection of Fuzzy Sets (cf [Bie97], p. 76f and [Tra93], p.37)*
The intersection of two fuzzy sets A and B, characterised by the indicator mappings $m_A : \Omega \to [0,1]$ *and* $m_B : \Omega \to [0,1]$ *is a fuzzy set with the indicator mapping* $m_{A\cap B} : \Omega \to [0,1]$*, that it is for all* $x \in \Omega$*:*

$$m_{A\cap B}(x) = \min\{m_A(x), m_B(x)\}.$$

Or:

$$m_{A\cap B}(x) = m_A(x) \cdot m_B(x).$$



(a) Intersection of fuzzy sets with a one-dimensional Ω. (Figure generated with *GeoGebra*.)

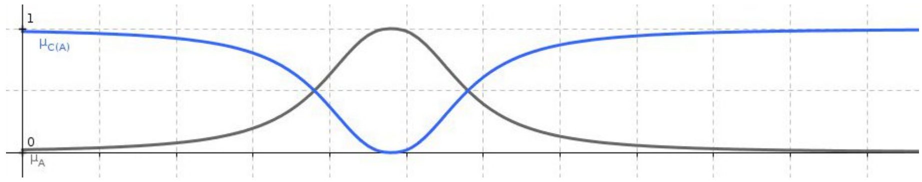
(b) Intersection of fuzzy sets with a two-dimensional Ω. (Figure generated with *Maxima*.)

Figure 7.6: Intersection of fuzzy sets. (Figure generated with *GeoGebra* and *Maxima*, respectively.)

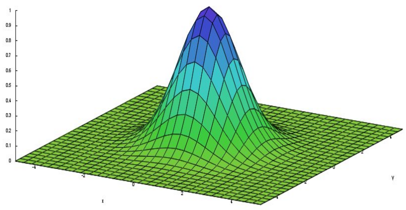
The complement of fuzzy sets can be used for the generation of risk, resource and decision support maps, as well. With the complement of a fuzzy set, the logical NOT can be expressed. E.g. if a risk map is given and the user wants to save a person located in a risk area. Then a membership mapping to support the decision, if a person in a risk area can be saved, can be generated. The distance is represented by this membership mapping, whereby the value "1" stands for "near" and "0" stands for "far away". If we want to combine the risk map with the distance map, the complement of the distance map has to be considered. When combining the two mappings with the logical AND, the value "1" stands for high risk and far away from the person, who is supposed to be saved, and the value "0" stands for low risk and near to the person who is supposed to be saved.

Definition 7.1.12 *Complement of a Fuzzy Set ([Bie97], p. 77)*
The complement of a fuzzy set A characterised by the membership mapping $m_A : \Omega \to [0,1]$, is a fuzzy set $\bar{A}$ with the membership mapping $m_{\bar{A}} : \Omega \to [0,1]$ that it is for all $x \in \Omega$:

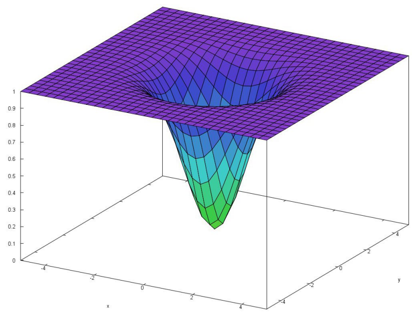
$$m_{\bar{A}}(x) = 1 - m_A(x).$$



(a) Complement of a fuzzy set with a one-dimensional Ω. (Figure generated with *GeoGebra*.)



(b) Fuzzy set with a two-dimensional Ω. (Figure generated with *Maxima*.)



(c) Complement of the fuzzy set visualised in (b). (Figure generated with *Maxima*.)

Figure 7.7: Complement of a fuzzy set. (Figure generated with *GeoGebra* and *Maxima*, respectively.)

The Cartesian product of fuzzy sets is used within SECTION 10.1, FIGURE 10.9.

Definition 7.1.13 *Cartesian Product of Fuzzy Sets ([Bie97], p. 81)*
The Cartesian product of n fuzzy sets $A_1, A_2, ..., A_n$*, on the underlying sets* $\Omega_1, \Omega_2, ..., \Omega_n$*, written* $A_1 \times A_2 \times ... \times A_n$*, is the fuzzy set of all ordered n-tuples* $(x_1, x_2, ..., x_n)$ *in the product space* $\Omega_1 \times \Omega_2 \times ... \times \Omega_n$ *with the membership mapping:*

$$m_{A_1 \times A_2 \times ... \times A_n}(x) = \min_i \{m_{A_i}(x_i) \mid x = (x_1, x_2, ..., x_n), x_i \in \Omega\}.$$

Definition 7.1.14 *Support of a Fuzzy Set ([Bie97], p. 69)*
The support of a fuzzy set A on Ω*, written supp(A), is the set of all elements of* Ω*, whose grade of membership concerning A is greater than zero:*

$$supp(A) = \{x \in \Omega \mid m_A(x) \geq 0\}.$$

Theorem 7.1.15 *Laws of the Fuzzy Set Algebra (cf [Bie97], p. 78).*

algebraic laws	underlying set: Ω; $m_i : \Omega \to [0,1], x \mapsto m_i(x)$; $i \in \{A, B, C\}$	
	of the union	**of the intersection**
commutative laws	$m_{A\cup B}(x) = \max\{m_A(x), m_B(x)\} = \max\{m_B(x), m_A(x)\} = m_{B\cup A}(x)$	$m_{A\cap B}(x) = \min\{m_A(x), m_B(x)\} = \min\{m_B(x), m_A(x)\} = m_{B\cap A}(x)$
associative laws	$m_{(A\cup B)\cup C}(x) = \max\{m_{A\cup B}(x), m_C(x)\} = \max\{\max\{m_A(x), m_B(x)\}, m_C(x)\} = \max\{m_A(x), m_B(x), m_C(x)\} = \max\{m_A(x), \max\{m_B(x), m_C(x)\}\} = \max\{m_A(x), m_{B\cup C}(x)\} = m_{A\cup(B\cup C)}(x)$	$m_{(A\cap B)\cap C}(x) = \min\{m_{A\cap B}(x), m_C(x)\} = \min\{\min\{m_A(x), m_B(x)\}, m_C(x)\} = \min\{m_A(x), m_B(x), m_C(x)\} = \min\{m_A(x), \min\{m_B(x), m_C(x)\}\} = \min\{m_A(x), m_{B\cap C}(x)\} = m_{A\cap(B\cap C)}(x)$
distributive laws	$m_{A\cup(B\cap C)}(x) = \max\{m_A(x), m_{B\cap C}(x)\} = \max\{m_A(x), \min\{m_B(x), m_C(x)\}\} = \min\{\max\{m_A(x), m_B(x)\}, \max\{m_A(x), m_C(x)\}\} = \min\{m_{A\cup B}(x), m_{A\cup C}(x)\} = m_{(A\cup B)\cap(A\cup C)}(x)$	$m_{A\cap(B\cup C)}(x) = \min\{m_A(x), m_{B\cup C}(x)\} = \min\{m_A(x), \max\{m_B(x), m_C(x)\}\} = \max\{\min\{m_A(x), m_B(x)\}, \min\{m_A(x), m_C(x)\}\} = \max\{m_{A\cap B}(x), m_{A\cap C}(x)\} = m_{(A\cap B)\cup(A\cap C)}(x)$
idempotent laws	$m_{A\cup A} = \max\{m_A(x), m_A(x)\} = \max\{m_A(x)\} = m_A(x)$	$m_{A\cap A} = \min\{m_A(x), m_A(x)\} = \min\{m_A(x)\} = m_A(x)$
identity laws	$m_{A\cup\emptyset}(x) = \max\{m_A(x), m_\emptyset(x)\} = \max\{m_A(x), 0\} = m_A(x)$	$m_{A\cap\Omega}(x) = \min\{m_A(x), m_\Omega(x)\} = \min\{m_A(x), 1\} = m_A(x)$
absorption laws	$m_{A\cup\Omega}(x) = \max\{m_A(x), m_\Omega(x)\} = \max\{m_A(x), 1\} = 1 = m_\Omega(x)$	$m_{A\cap\emptyset}(x) = \min\{m_A(x), m_\emptyset(x)\} = \min\{m_A(x), 0\} = 0 = m_\emptyset(x)$
	$m_{A\cup(A\cap B)}(x) = \max\{m_A(x), m_{A\cap B}(x)\} = \max\{m_A(x), \min\{m_A(x), m_B(x)\}\} = m_A(x)$	$m_{A\cap(A\cup B)}(x) = \min\{m_A(x), m_{A\cup B}(x)\} = \min\{m_A(x), \max\{m_A(x), m_B(x)\}\} = m_A(x)$
laws of DeMorgan	$m_{\overline{A\cup B}}(x) = 1 - m_{A\cup B}(x) = 1 - \max\{m_A(x), m_B(x)\} = \min\{1 - m_A(x), 1 - m_B(x)\} = \min\{m_{\bar{A}}(x), m_{\bar{B}}(x)\} = m_{\bar{A}\cap\bar{B}}(x)$	$m_{\overline{A\cap B}}(x) = 1 - m_{A\cap B}(x) = 1 - \min\{m_A(x), m_B(x)\} = \max\{1 - m_A(x), 1 - m_B(x)\} = \max\{m_{\bar{A}}(x), m_{\bar{B}}(x)\} = m_{\bar{A}\cup\bar{B}}(x)$
contradiction laws	theorem of the excluded third: counterexample: $m_{A\cup\bar{A}}(x) = \max\{m_A(x), m_{\bar{A}}(x)\} = \max\{m_A(x), 1 - m_A(x)\} \overset{m_A(x):=0.6}{=} \max\{0.6, 0.4\} = 0.6 \neq 1 = m_\Omega(x)$	theorem of contradiction: counterexample: $m_{A\cap\bar{A}}(x) = \min\{m_A(x), m_{\bar{A}}(x)\} = \min\{m_A(x), 1 - m_A(x)\} \overset{m_A(x):=0.6}{=} \min\{0.6, 0.4\} = 0.4 \neq 0 = m_\emptyset(x)$
involution law	$m_{\bar{\bar{A}}}(x) = 1 - m_{\bar{A}}(x) = 1 - (1 - m_A(x)) = (1-1) + m_A(x) = m_A(x)$	

Table 7.1: Laws of the fuzzy set algebra. (Compare: [Bie97], p. 78).

7.2 Artificial Neural Networks

The SAGU consists inter alia of a combination of an FLC and an ANN. The benefit of this combination is the learning ability of the ANNs, which makes it possible for a system to "learn" linguistic rules and/or membership functions or to optimise the ones that already exist. This ability is important for a system which has the aim to adapt a GUI to the user, more precise for the adaption process and for the flexibility of the system. In the following, we will have a closer look on ANNs. The exploration of ANNs began already in 1940, motivated by the interest in the neurophysiological basics of the human brain. The brain consists of neurons, that are linked with each other and influence each other with electrical signals.
Below, the mathematical-formal aspect of ANNs is investigated. An ANN can be understood as a formal structure, that can be described by a set and some mappings. ANNs can be used to solve optimisation problems. Learning processes for an ANN minimise an error function. ANNs and fuzzy-systems can be linked. (cf [NKK94], p.11ff).

Definition 7.2.1 *Artificial Neural Network (ANN) ([NKK94], p. 19f)*
An ANN is a tuple (U, W, A, O, NET, ex) with:

1. *U is a finite set of processing units (neurons),*
2. *W, the ANN structure, is a mapping from the Cartesian product $U \times U$ to $\mathbb{R}$,*
3. *A is a mapping assigning an activation function $A_u : \mathbb{R}^3 \rightarrow \mathbb{R}$ to each $u \in U$,*
4. *O is a mapping assigning an output function $O_u : \mathbb{R} \rightarrow \mathbb{R}$ to each $u \in U$,*
5. *NET is a mapping, assigning a net input function $NET_u : (\mathbb{R} \times \mathbb{R})^U \rightarrow \mathbb{R}$ to each $u \in U$, and*
6. *ex is an extern input function $ex : U \rightarrow \mathbb{R}$, assigning an extern input in form of a real number $ex_u = ex(u) \in \mathbb{R}$ to each $u \in U$.*

Note 7.2.2: *This definition describes the static properties of ANNs, but it does not say anything about their dynamic ([NKK94], p. 20).*

Note 7.2.3: *An essential component of the model of an ANN are algorithms that enable an adaptive change of the mapping W. The aim of this "learning process" is to determine W in a way, that the ANN reacts on certain inputs with certain outputs and thus answers on unfamiliar inputs with an appropriate reaction.*

Definition 7.2.4 *Input Set, Output Set ([NKK94], p. 29)*
Let NN be a random ANN with a set of processing units U, the extern input function ex and an output function O_v valid for an output unit v. Let $U_I = \{u_1, ..., u_n\}$ be the set of input units and $U_O = \{v_1, ..., v_m\}$ be the set of output units. dom denotes the domain of a mapping.
An input i of NN is an element of the set

$$\mathcal{I} = \{(x_1, ..., x_n) \mid x_i \in dom(ex)\} \subseteq \mathrm{IR}^{U_I}.$$

An output t of NN is an element of the set

$$\mathcal{T} = \{(y_{v_1}, ..., y_{v_m}) \mid y_i \in dom(O_{v_i})\} \subseteq \mathrm{IR}^{U_O}.$$

For the SAGU described in Chapter 9, we use a free learning task. A fixed learning task does not fit to our claims, because achieving a task is fuzzy. The quality of the output and thus the achievement of the learning task is measured via measure theory on topological spaces (Chapter 8).

Definition 7.2.5 *Free Learning Task ([NKK94], p. 29)*
A free learning task $\mathcal{L} \subseteq \mathcal{I}$ of an ANN is a set of inputs. The ANN is supposed to determine an output $t \in \mathcal{T}$ for each input $i \in \mathcal{L}$ of the learning task. An ANN accomplished a free learning task, if it determined an output for each input, that similar inputs produce similar outputs in the sense of a suitable distance measure.

Definition 7.2.6 *Fixed Learning Task ([NKK94], p. 29)*
A fixed learning task $\mathcal{L} \subseteq \mathcal{I} \times \mathcal{T}$ of an ANN is a set of input/ output pairs. The ANN is supposed to react with the appropriate output t on each input i of a pair $(i, t) \in \mathcal{L}$. An ANN accomplished a fixed learning task, if it produces the appropriate output for each input, according to the learning task.

Definition 7.2.7 *Learning Algorithm ([NKK94], p. 30)*
A learning algorithm is a process, that changes the ANN structure W of an ANN by a learning task. If the ANN fulfils the learning task after application of the process or if it comes below a previously determined approximation error, the learning algorithm was successful. Otherwise the learning algorithm was unsuccessful.

As we use a free learning task for the SAGU described in Chapter 9, a not monitored learning algorithm is used.

Definition 7.2.8 *(not) Monitored Learning Algorithm ([NKK94], p. 30)*
A learning algorithm that uses a free learning task is called not monitored learning algorithm.
A learning algorithm that uses a fixed learning task is called monitored learning algorithm.

Note 7.2.9: *An input is denoted as (input) pattern that describes the current state of an environment an ANN reacts on with an output. The ability of the ANNs to generalise enables the ANNs to react suitable on disturbed or incomplete patterns. In that sense, an ANN can be denoted as fault tolerant. ([NKK94], p. 30).*

A sigmoid function can be used as activation or output function.

Definition 7.2.10 *Sigmoid Function*
A sigmoid curve is produced by a mathematical function having an "S"-shape. Often, the sigmoid function refers to the special case of the logistic function and is defined by the formula:

$$\begin{aligned} \mathcal{S} :& \mathbb{R} \to [0,1] \\ x \mapsto& \frac{1}{1+e^{-x}} \end{aligned}$$

Note 7.2.11: *"A fully connected recurrent ANN can potentially be a very powerful system for temporal information processing. It can be shown, as a corollary to the universal approximation theorem for three-layered networks, that a recurrent network can, with enough units, approximate any vector field or map. [...] Convergence of learning depends critically on the choice of network topology, initial weights, and the choice of training samples." ([Doy95]).*

7.2.1 Adaptive Resonance Theory

Adaptive Resonance Theory (ART) was originally developed to solve the Stability-Plasticity Dilemma of ANNs, i.e. the question "How can new associations be learned in an ANN without forgetting old associations?". (cf [Zel94], p.251). ART "has evolved as a series of real-time ANN models that perform unsupervised and supervised learning, pattern recognition, and prediction. Models of unsupervised learning include ART 1 for binary input patterns and fuzzy ART for analogue input patterns. ARTMAP models combine two unsupervised modules to carry out supervised learning." ([CG03]).
Within the SAGU, described in Chapter 9, the fuzzy ART is used. Inter alia the different user groups can be regarded as the different clusters. A user can be assigned to a user group with the method described above by observing the membership mapping of the user and comparing it to membership mappings of other users. This is applied within the empirical study, as well, see subsubsection

11.2.1. The ANN used in the system is further described in SECTION 9.3. To define fuzzy ART, the definition of ART 1 is required:

Definition 7.2.12 *Adaptive Resonance Theory (ART) 1 ([Roj96], p.414f)*
The ART ANNs classify a stochastic sequence of vectors in clusters. The dynamics of the ANN consists of a series of automatic steps that resemble learning in humans. The weight vectors $w_1, w_2, ..., w_m$ *represent categories in input space. All vectors located inside the cone around each weight vector are considered members of a specific cluster. The weight vectors are associated with computing units in an ANN. Each unit fires a 1 only for vectors located inside its associated cone of radius* r*. The value* r *is inversely proportional to the "attention" parameter of the unit. If* r *is small, the classification of input space is fine grained. A large* r *produces a classification with low granularity, that is, with large clusters. The ANN is trained to find the weight vectors appropriate for a given data set. Once the weight vectors have been found, the ANN computes whether new data can be classified in the existing clusters. If this is not the case*[2] *a new cluster is created, with a new associated weight vector. The ANN preserves its plasticity, because it can always react to unknown inputs, and its stability, because already existing clusters are not washed out by new information. For the scheme to work, enough potential weight vectors, i.e., computing units, must be provided.*

Example 7.2.13: *In* FIGURE *7.8 the task to be solved by the ANN is illustrated.* FIGURE *7.8 shows the weight vectors* w_1 *and* w_2 *with their associated cones of radius* r*. If the new vector* w_3 *is presented to the ANN, it is not recognised and a new cluster is created, with this vector in the middle.*

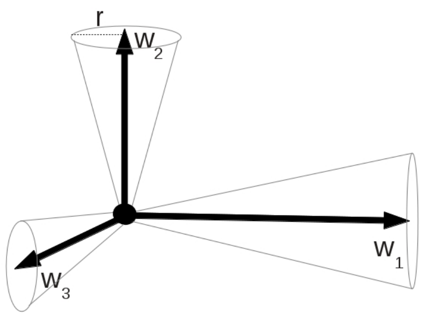


Figure 7.8: Vector clusters and attention parameters (r). (Figure generated with *Libre Office Draw.* Compare: [Roj96], p.415.)

[2]if, for example, a new vector is far away from any of the existing weight vectors

Definition 7.2.14 *Fuzzy Adaptive Resonance Theory (ART) (cf [Roj96], p.415)*
Fuzzy ART incorporates computations from fuzzy set theory into the ART 1 ANN. Fuzzy ART enables gradual assignment of data to different categories at the same time.

Note 7.2.15: *A fuzzy ART ANN works as follows:*
$w_1, ..., w_n$ *are weight vectors and* v *is a new vector (see* FIGURE *7.9).*

1. *The distances of* v *to the weight vectors has to be determined with e.g.*

$$\delta_1 = d(w_1, v), ..., \delta_n = d(w_n, v),$$

with $d(x, y) = \sqrt{(x_1 - y_1)^2 + ... + (x_n - y_n)^2}$*,* x, y *vectors,* $x = (x_1, ..., x_n)$ *and* $y = (y_1, ..., y_n)$*. In this case, the Euclidean metric was chosen, but a more general metric can be chosen, as well. If each* δ *exceeds a certain value* c*, a new weight vector* $w_{n+1} := v$ *is created. If the value* c *is not exceeded, the weight vectors are changed. The value* c *is determined by an FLC.* v *is clustered gradually to each weight vector* $w_1, ..., w_n$*, as described in 2..*

2. w_k *is changed to* $\widetilde{w_k}$ *in the following way:*

$$\widetilde{w_k} = \lambda_k \cdot w_k + (1 - \lambda_k) \cdot v,$$

with $\lambda_k \in [0, 1]$*.* λ_k *is dependent on the degree of membership of* v *to* w_k*,* $\mu_k \in [0, 1]$*, with* $\mu_k = \frac{\delta_k}{\sum_{i=1}^{n} \delta_i}$ *and on the number of vectors clustered to* w_k*. The relative amount of vectors clustered to* w_k *is* $\alpha_k = \frac{\beta_i}{\sum_{i=1}^{n} \beta_i}$*,* β_k *is the absolute amount of vectors clustered to the weight vector* w_k*,* $\alpha_k \in [0, 1]$*.* $\lambda_k = \gamma_k \cdot \mu_k + (1 - \gamma_k) \cdot \alpha_k$ *with* $\gamma_k \in [0, 1]$*.* γ_k *is controlled by an FLC.*

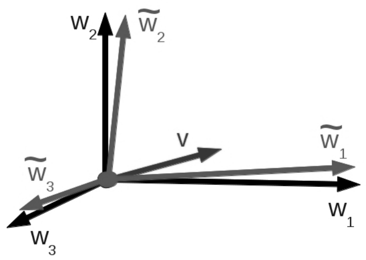


Figure 7.9: Sample iteration step of the fuzzy ART ANN in the two-dimensional. (Figure generated with *Libre Office Draw.*)

For the fuzzy ART ANN, a metric space is sufficient.

7.2.2 Backpropagation

Backpropagation (BP) is used within the SAGU (CHAPTER 9) to adjust the fuzzy membership mappings. Each fuzzy membership mapping used within the SAGU may contain a BP ANN. The learning procedure BP is a gradient descent method. FIGURE 7.10 shows an exemplary error function

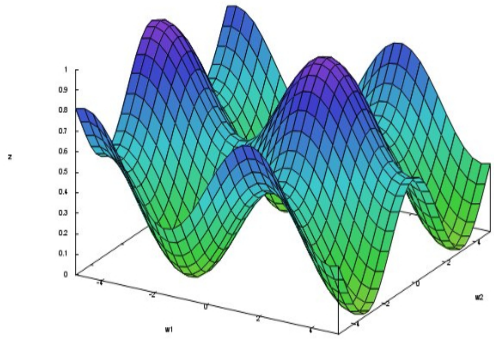


Figure 7.10: Exemplary error surface of an ANN as mapping of the weights w_1 and w_2: $E : \mathbb{R}^2 \to [0,1], (w_1, w_2) \mapsto E(w_1, w_2) = z$. (Figure generated with *Maxima*.)

$E(W) = E(w_1, ..., w_n)$ indicating the error the ANN has summing over all training patterns with the weights $w_1, ..., w_n$. By using a gradient descent method a global minimum shall be found as fast as possible, i.e. a configuration of the weights with that the error sum over all training patterns is minimal. (cf [Zel94], p.105f).

In recurrent ANNs, usually the input (generally a sequence) is created, and the ANN runs a number of cycles. At a certain time, the output of the ANN is compared with the target output and the error signals are generated for all output neurons. These error signals are then backpropagated by the ANN for the same number of cycles. For each iteration, the weight changes are calculated and the sum of the changes are stored for each weight. Afterwards all the weights are changed by that sum. The

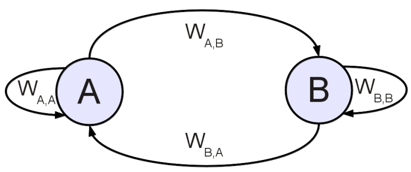


Figure 7.11: Simple recurrent ANN with two cells, A and B. (Figure generated with *Libre Office Draw*. Compare: [Zel94], p.145)

propagation for the ANN shown in FIGURE 7.11 is as follows:

$$o_j(t+1) = f(NET_j(t)) = f(\sum_i o_i(t)w_{ij} + in_j(t)),$$

where $o_j(t)$ is the output of neuron j at the time t and $in_j(t)$ is the input in neuron j at the time t. It is $in_j(t) = 0$, if no input exists at the time t. (cf [Zel94], p.145).

7.2.3 Overfitting

Within the SAGU overfitting would result in a GUI that is not adapted in an optimal way to the user's needs. Therefore, we have to stay consequently in contact with the user and observe the user's needs. If the distance of the calculated data about the user by the system to the observed data about the user is larger than a certain value, the GUI has to be reset to a time, when the distance was minor. Then the system should be improved and adjusted to the user.

Definition 7.2.16 *Overfitting ([Roj96], p.144)*
If a polynomial of arbitrary degree is fitted to a set of points, it can well happen that the function is overfitted, that is, it learns the training set perfectly but interpolates unknown points erroneously. More points do not reduce the error as long as our class of concepts consists of all polynomials of arbitrary degree. The only possibility of profiting from more examples is to reduce the size of the search space by, for example, putting a limit on the degree of the acceptable polynomial approximations.

In FIGURE 7.12 the exemplary overfitting of a polynomial approximation is illustrated.

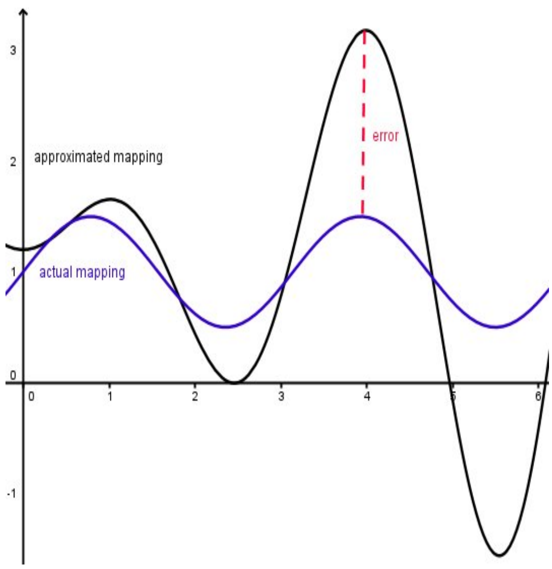


Figure 7.12: Overfitting a polynomial approximation. (Figure generated with *GeoGebra*. Compare: [Roj96], p.144.)

7.3 ANNs and Fuzzy Systems

In TABLE 7.2, neural control and fuzzy control are compared by listing the advantages and disadvantages of both. The main argument for combining fuzzy controllers with ANNs is the learning ability of the ANNs. It should be possible for a system to "learn" linguistic rules and/or membership functions or to optimise the ones that already exist. "Learn" means in this context the complete generation of

	Advantages	**Disadvantages**
Neural Control	• no mathematical process model necessary • no rule knowledge necessary • different learning algorithms	• Black-Box behaviour • no rule knowledge extractable • heuristic choice of network parameters • adaption to changed parameters is eventually difficult and it may require repetition of the learning process • learning from scratch • convergence of learning process not guaranteed
Fuzzy Control	• no mathematical process model necessary • a-priori knowledge usable • easy interpretation and implementation	• rule knowledge has to be available • no learning ability • no formal methods for tuning • semantic problems in the interpretation of tuned rules • adaption to changed parameters eventually difficult • a tuning attempt may not succeed

Table 7.2: Comparison neural control and fuzzy control. (Compare: [NKK94], p.257.)

a rule base or of membership functions, that model the corresponding linguistic terms, on the base of sample data. They can form a free or fix learning task for an ANN. The generation of a rule base implies a (preliminary) definition of a membership function. For our concerns, the following method for the generation of a rule base is used: The system starts with a rule base (that is eventually chosen by chance), that consists of a fix number of rules. In the course of the learning process, the rules are exchanged. At each exchange process the consistence of the rule base has to be rechecked. The disadvantage of this procedure is the fix number of rules. Furthermore, an evaluation scheme for removing rules and a data analysis for adding rules has to be implemented. If this is not the case, the learning process corresponds a stochastic search. If the performance becomes worse, exchange processes have to be reversed. The membership mappings can easily be described by parameters. The adaption of parameters is a standard task of ANNs. There are two approaches for learning or optimising membership functions:

- Parametric forms are assumed for the membership functions. Their parameters are optimised in a learning process.
- An ANN learns to generate a membership value for an input, based on sample data.

For our concerns, the following neuro-fuzzy-system could be used: The ANN determines online, i.e. while the fuzzy controller is running, parameters for the adaption of the membership functions. For this model, the fuzzy rules and an initialisation of the parameterised fuzzy sets have to be defined.

Furthermore, an error measure for leading the learning process has to be provided. If a fixed learning task is available, it is possible to let the model learn offline. (cf [NKK94], p.257ff).

FIGURE 7.13 visualises a first draft of the SAGU including a neuro-fuzzy-system. A detailed description

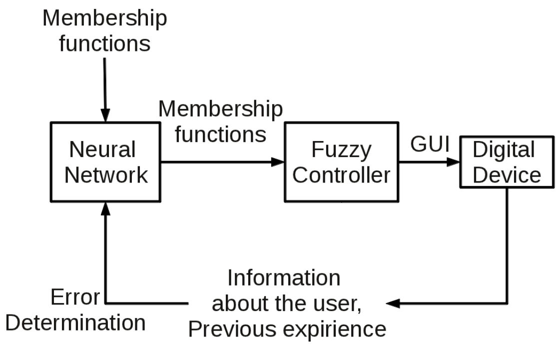


Figure 7.13: Schematic representation of a neuro-fuzzy-system. (Figure generated with *Libre Office Draw.*)

of the system is given in CHAPTER 9.

Neuro-fuzzy-systems are applied in many fields:

[KLRA06] describes the application of a neuro-fuzzy technique in the health sector for the classification of heart abnormalities in ten different cardiac states with results indicating a high level of efficiency.

[PA02] contributes to the improvement of health issues by developing a neuro-fuzzy based alarm system for septic shock patients to help physicians recognising the critical state of their patients as early as possible. A neuro-fuzzy algorithm was used to generate rules derived from septic shock patient measurement data. The developed alarm system produced reliable alarm and a multicentre study was started to investigate the clinical usefulness of the alarm system. This approach can be included into the application for a digital device delivering early warning and decision support tailored to different user groups with a GUI that adapts to the user to improve health care.

[PSGB10] uses a neuro-fuzzy approach using remote sensing data and GIS for landslide susceptibility mapping in the Cameron Highlands, Malaysia, a landslide-prone area. From a spatial database eight landslide conditioning factors, e.g. altitude, distance from drainage, distance from the road, normalised difference vegetation index, where extracted and analysed using the adaptive neuro-fuzzy inference system. Reasonable results were achieved using the neuro-fuzzy model which hence can be used for preliminary land-use planning purposes.

[Dix05] carries out a sensitivity analysis for investigating the applicability of neuro-fuzzy techniques in predicting ground-water vulnerability in a spatial context by integrating GIS. The neuro-fuzzy models turned out to be sensitive to the shape and number of the fuzzy sets, the nature of the rule weights and the validation techniques used during the learning processes. [Dix05] recommends the use of neuro-fuzzy models "only as tool within a broader framework of GIS, remote sensing and solute transport modelling to assess groundwater vulnerability along with functional, mechanistic and stochastic models."([Dix05]).

Adaptive neuro-fuzzy based modelling used for the prediction of air pollution daily levels in the city

of Zonguldak, Turkey, by estimating the impact of meteorological factors on sulfur dioxide and the total suspended particular matter pollution levels over an urban area is described in [YB06]. The developed adaptive neuro-fuzzy model forecasted satisfactorily the trends in sulfur dioxide and the total suspended particular matter concentration levels.
The approach described above can be included into the application for a digital device delivering early warning and decision support tailored to different user groups with a GUI that adapts to the user to improve risk determination and early warning.
[SDJ12] developed a flexible *Neuro-Fuzzy Pedagogical Recommender*, which has been implemented and tested with simulated data. The *Neuro-Fuzzy Pedagogical Recommender* is an adaptive recommender based on neuro-fuzzy inference, that can be used to create pedagogical rules in technology enhanced learning systems, whereby it is possible for each teacher to create her/his own set of pedagogical rules. Such an e-learning is comparable to the SAGU (Chapter 9). The initial state determiner can be seen as the teacher in this method and the pupils can be seen as the user. Differences are inter alia the purpose of the system: e-learning systems want to teach the user (pupil), our system is developed for early warning and decision support, not to teach the user. Additionally, besides an ANN and an FLC some more components are included into the SAGU.

7.4 Adaptivity

"The primary motivation for adapting user interfaces is to improve users' performance and satisfaction.“ ([GCTW06]).
Purely mechanical properties of an adaptive interface, i.e. the theoretical benefit due purely to an adaptive interface's mechanical properties, are a poor predictor of a user's performance or satisfaction or the adaptation's success or failure in practice. What other factors influence user acceptance of adaptive user interface design? In [GCTW06] it is observed that ...

- ... the predictive accuracy of an adaptive interface has a significant impact on user performance.
- ... the more useful the resulting adaptations, the more likely it is that users will take advantage of the adaptive nature of the interface.
- ... the frequency of interaction with the interface and the cognitive complexity of the task influence what aspects of the adaptive interface users find relevant.
- ... in addition to a possible benefit, users perceive adaptations as incurring a cost.
- ... *Greenberg's* ([GW85]) design that restructures the interface after each adaptation proved successful.[3]

[3]This result might be explained by the very high complexity of the interface (a hierarchical menu with over a thousand leaf elements), which prevented the users from developing strong motor memory for the location of different elements.

- ... split interfaces, which duplicate (rather than move) frequently used (but hard to access) functionality to a convenient place tend to improve users' performance and satisfaction, offer medium to high benefits while causing minimal confusion.

Before we mention some methods of improving the effectiveness and efficiency of human-computer interactions, some definitions are useful:

Definition 7.4.1 *User Model ([GW85])*
A user model is defined as a set of rules which a computer system follows to determine its reaction to a user.

Definition 7.4.2 *Personalised User Model ([GW85])*
A system is personalised when part of the user model is unique for each user or group of users.

Definition 7.4.3 *Adaptive ([GW85])*
A system is adaptive when the user model is altered during interaction with the user, to reflect a changed view of her/him.

Three different strategies for user modelling are common:

1. The system designer listens to feedback from the user population and adjusts the system to fit the current need.
2. Each user modifies her/his working environment her/himself.
3. The system monitors the user's activity and tries to adapt automatically to her/his model. ([GW85]).

In this thesis, inter alia the third strategy is implemented: The system monitors the user: What is the user's (re)action? Which GUI elements does the user use? How does the user change the GUI on her/his own? Thus, conclusions for the adaptivity can be drawn: The system responses to the user with automatic adaptivity and adaption suggestions. Each user is, depending on the geo-location and the situation in which she/he is located (in CHAPTER 10 several use cases are listed), member of one certain user group. The user can switch to another group, if the user needs change. The adaptivity process can be related to a user profile, a group profile and conclusions for an initial user profile for new users can be derived. The second strategy is always available to the user, e.g. if the user needs change too fast for the system to adapt.
In the following some applications for implementing adaptivity are presented. In SUBSECTION 7.4.1, the GIS Tailored (GIST) Questionnaire and in SUBSECTION 7.4.2 Intelligent Tutoring System (ITS)s

are addressed.

[BEK13] implements algorithms to learn about the user's usage of a digital device to improve the ease of use and productivity of the end-user. Therefore, [BEK13] presents systems, methods and non-transitory computer-readable storage media for an adaptive communication user interface, i.e. for implementing a set of algorithms via a communication device, e.g. cellphone or smartphone.

In this thesis, we implement algorithms, as well, to learn about the user's usage of a digital device to improve the ease of use and productivity of the end-user. [BEK13] does not elucidate the kind of algorithms, which were used for the application. Maybe, [BEK13] uses a similar algorithm as described in Chapter 9.

[CCA+13] developed a system providing a GUI which is configured to display automatically generated user profiles categorised based on given expertise or context associated with the end-user via robust knowledge-based management and sharing system organised by context or context-based searching and retrieval of relevant information.

This application pursues a similar goal as the SAGU (Chapter 9) presented in this thesis, whereby the "user profiles" are splitted into GUI elements appearing in a certain design dependent on the expertise of the user. Additionally, we include not only the expertise of the user, but consider the user's needs, as well.

Another approach for improving the effectiveness and efficiency of human-computer interactions through an intelligent GUI is presented in [AT00], where the developed system can determine the intents of the user, transforming the deduced intentions into system actions by monitoring the user's actions. [AT00] describes a GUI enhanced with an intelligent control module:

Definition 7.4.4 *Intelligent Control Modul ([AT00])*
The intelligent component of the GUI is composed of a group of software agents that cooperate and compete to control the system. A computer mouse is used as the input device, through which the user performs a number of clicks and drag-and-drop tasks, while selecting and moving objects on the computer screen. The intelligent control module monitors the movements and states of the mouse and the cursor, and aids the user based on the multiple agents' determinations of the user's sub-goals.

[AT00] implements an agent in the contention controller using fuzzy logic control, because the agent's perceptions and actions cannot be quantified in exact numerical values corresponding to physical measurements.

Definition 7.4.5 *Membership Mappings for the Agents ([AT00])*
Fuzzy linguistic terms are defined for each parameter, normalised in the range of $[-1, 1]$*, with the lowest possible value mapped to* -1 *and the highest possible value mapped to* 1*. Seven fuzzy linguistics terms are used for all parameters: negative-high, negative-medium, negative-low, zero, positive-low,*

positive-medium, and positive-high. The membership functions are defined to be exponential:

$$f_k : [-1,1] \to [0,1]$$
$$x \mapsto \frac{1}{e^{(\frac{4}{w_k})^2 \cdot (x-d_k)^2}},$$

where d_k and w_k are parameters specifying the fuzzy functions, $k \in \{0,1,...,n\}$, and $d_k - \frac{w_k}{2} \leq x \leq d_k + \frac{w_k}{2}$.

7.4.1 The GIS Tailored Questionnaire

The concept of a GIST Questionnaire is used to implement tutorial components into the SDSS. This concept is used within the SAGU.

A tailored questionnaire focuses on dynamic generation of items and questions according to the geo-location where the questionnaire is used. Based on the responses of the respondent, requirements and constraints can be derived for the observed geo-location. The approach is connected to the management of resources and the support of decision making processes which is based on collecting data for a certain geographical region. A GIS can be used to tailor a questionnaire to match the risk and demands of specific areas where the questionnaire will be applied. The GIS based generation of a questionnaire is used to provide tailored information for informed decision making in health care systems. Mathematical modelling can be applied to an appropriate temporal and spatial generation of questionnaires for deployment of human and medical resources according to risk and demand in rural health care. Many ICT related solutions promise to harness development in the sector of health care in South Africa, but some of these projects have already failed. The consideration of tailored data acquisition via questionnaire can result in a tailored distribution of resources via RMS that will facilitate informed decision making to the Health Care Management (HCM) by making resource related decisions. The mathematical concept of a GIST questionnaire derives the priority to ask certain questions based on GIS information. Whether the best solution is to make use of a human resource, an eHealth solution and/or medical equipment deployment at a rural health care centre in times of need can be provided by the questionnaire results. The main objective is to deploy inefficiently used or idle resources from one health care centre to another with an existing need. Risk and resources have spatial and temporal aspects for the improvement of a health care system. The time for the questionnaires is limited, the questionnaires depend on risks and availability of resources and a GIST questionnaire determines the set of questions according to priorities derived from risk and resource layers in the GIS. Since the HCM is faced with decision making challenges, a RMS becomes vital, as it aids informed decision making for resource distribution. Further it provides monitoring and control, as well as forecasting to plan for times of disaster. ([HNR+10]).

The TCIU (Section 9.4) within the SAGU, works similar to a GIST questionnaire.

7.4.2 Intelligent Tutoring Systems

Another approach implementing adaptivity are tutoring systems. Tutoring systems are programs that practice the mental or manual skills, and they are among the most important computer applications ([Lus92], blurb).

Definition 7.4.6 *Intelligent Tutoring System (ITS) ([ESTL+08])*
ITSs are computer-based instructional systems with models of instructional content that specify what to teach, and teaching strategies that specify how to teach. They make inferences about a student's mastery of topics or tasks in order to dynamically adapt the content or style of instruction. ITSs support a style of learning best categorised as "learning by doing".

ITSs claim so far to demonstrate "intelligent" behaviour, as they deal with unexpected and unpredictable behaviour of individual learners, and that they can adapt accordingly flexibly to situational conditions in the teaching-learning events. Consequently, the learning material presentation has to be adjusted gradually to the user's knowledge level and skill level. This assumes that the individual events or interactions in the instruction process are continuously registered and evaluated. Ideally, the system should "learn" of the respective dialog experience and modify its own diagnostic skills, instructional skills and problem solving skills itself in terms of a continuous optimisation. In the design of ITSs it is also intended to diagnose and classify entries and system interactions of the learner with the computer directly during the training events. By this a requirement for contingent and differential learning support in the form of no-nonsense corrective, instrumental and conducive motivational feedback shall be created. A fundamental objective for many ITS applications is the intention to convey the user with a more intrusive supervision and individualised care atmosphere, the instruction process should match to the procedure of qualified human tutors. The following claim is often formulated for the conceptual and technical implementation of ITSs: The computer system should have the ability to implement itself the intended task accomplishment or problem solving and to use not only stored solutions.
Essentially, an ITS consists of the following constitutive elements:

- The expert module, which contains the factual specific knowledge basis of the syllabus that the user should be taught,
- the user module, which represents above all the process-dependent knowledge status of the user,
- the tutoring module, which realises certain teaching methods, controls the instruction process and is responsible for the activation of adaptive educational interventions and
- the communication module, which is responsible for the information exchange between system and user. It should ideally have the ability to understand natural-language questions and to

generate meaningful responses according to the language-specific semantic and syntactic criteria. ([KS87], p.40f).

An ITS differs to our application, because we do not want to teach anybody anything but deliver decision support adapted to the user's needs. For our application the monitoring of the learning state of the user is important, as well, but it is not important to deliver learning success.

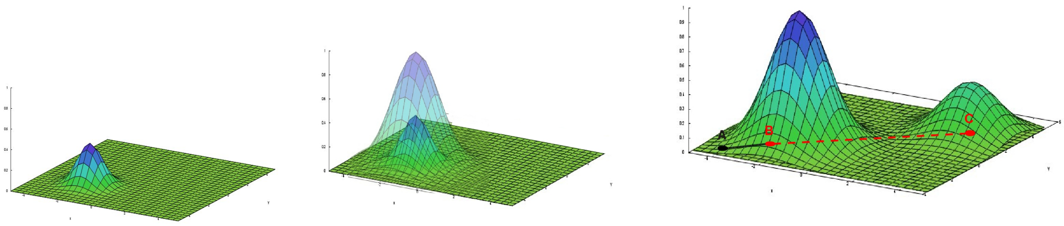
7.5 Interim Conclusion

Fuzzy logic is used to represent uncertainties and ambiguities in linguistic descriptions. It is for this reason used in the SAGU described in Chapter 9. Moreover, fuzzy logic is used to determine the membership mappings, used in each component of the developed system: in the ANNs (Section 9.3 and 9.5), by the initial state determiner (Section 9.1), in the TCIU (Section 9.4), for information on other users (Section 9.2), and for error determination (Section 9.7). An FLC is also used in the SAGU. Moreover, fuzzy logic is used in a similar way for the generation of risk and resource maps and, thus, decision support maps (see Section 10.1). Therefore, the SAGU consists of, among other things, a combination of an FLC and an ANN. The benefit of this combination is the learning ability of the ANNs, which makes it possible for a system to "learn" linguistic rules and/or membership functions or to optimise the ones that already exist. This ability is important for a system which has the aim of adapting a GUI to the user, more precisely, for the adaption process and for the flexibility of the system. For the SAGU described in Chapter 9, we use a free learning task. This is preferred to a fixed learning task, which does not fit our claims because achieving a task is fuzzy. The quality of the output and thus the achievement of the learning task, is measured by means of measure theory on topological spaces, which is addressed in the next chapter. BP is used in the SAGU (Chapter 9) to adjust the fuzzy membership mappings. Each fuzzy membership mapping used within the SAGU may contain a BP ANN. In the SAGU the fuzzy ART is used. The different user groups can be regarded as, inter alia, the different clusters. A user can thus be assigned to a user group using the method described above by observing the membership mapping of the user and comparing it to the membership mapping of other users. This is applied in the empirical pilot study as well (see subsubsection 11.2.1). The ANNs used in the system are further described in section 9.3 and section 9.5. In the SAGU, overfitting would result in a GUI that is not optimally adapted to the user's needs. For this reason, we have to stay constantly in contact with the user and observe the user's needs. If the distance between the data on the user calculated by the system and the observed data on the user is larger than a certain value, the GUI has to be reset to a time when the distance was less or a switch to manual mode is required. Another challenge that was identified is the different speeds at which the user needs can change; thus BP is important for reducing or enlarging the step-size of the system in order to adjust the GUI to the user in the best possible way. In the SAGU, an FLC is combined with an ANN. The main argument for combining fuzzy controllers with

ANNs is the learning ability of the ANNs. When the system monitors the user it asks: What is the user's (re)action? Which GUI elements does the user use? How does the user change the GUI on her/his own? Thus, conclusions about the system's adaptivity can be drawn: The system responds to the user with automatic adaptivity and suggestions for adaption. Depending on the situation in which the user is located (in CHAPTER 10 several use cases are listed), she/he is a member of a certain user group. Hence, if the user's needs change, the user can switch to another group. The adaptivity process can be related to a user profile and a group profile, and consequently conclusions for an initial user profile for new users can be derived. The option to change GUI properties manually is always available to the user if, for example, the user's needs change too quickly for the system to adapt. The concept of a GIST Questionnaire (SUBSECTION 7.4.1) is used to implement tutorial components into the SDSS. This concept is also used in the SAGU. Accordingly, the SAGU is GIS tailored and data can be collected, which is then processed in the ANNs.

In CHAPTER 9 measure theory on topological spaces is used for the development of an SAGU. General measure theory on topological spaces was previously found mainly in intra-mathematical fields and was not relevant for adapting a GUI to a user. Topologies are used to represent information about a user. Accordingly, a coarse topology can represent little information about a user, however, the more refined the topology becomes, the more the information about the user that is represented increases, in other words, the topology changes over time. Topologies are assigned to an object in OOA (CHAPTER 6) as information content by means of a Cartesian product of the status spaces of object classes. On neighbourhoods in $(\mathbb{R}, \mathfrak{T}_{\mathbb{R}})$, continuous mappings $f : (X, \mathfrak{T}_X) \overset{\mathfrak{T}}{\to} (\mathbb{R}, \mathfrak{T}_{\mathbb{R}})$ behave in a similar fashion. For the evaluation of certain elements of the system, for example error determination, measure theory is used; that is, it is used to observe the extend to which the user's (re)action complies with a (re)action expected by the system, or just to evaluate user (re)actions in general (see SECTION 9.7). Measure theory is also used in some components of the system, for example in the ANNs, where distance measures are needed to determine the clusters (see SUBSECTION 7.2.1 and SECTION 9.3). Each ANN can be represented by a mapping, that can be measured; in other words, the quality of the ANN can be determined. Based on this information, the ANN can be improved, if necessary, for example if the quality is too low or if the GUI identified for the user by the SAGU does not fit to the user's needs. Further, the ANNs also require a topology in the free learning task. An ANN has fulfilled a task if similar outputs are generated for similar inputs. This similarity is characterised by neighbourhoods (DEFINITION 8.1.14). In some parts of the SAGU described in CHAPTER 9, the property symmetry (DEFINITION 8.1.1 3.) is not fulfilled among other things, e.g. within the TCIU, (see SECTION 9.4); (this is discussed in more detail in SECTION 9.4). Spatial distances cannot always be used in the Euclidean sense, for example in epidemiological distances. FIGURE 7.14 presents an example of epidemiological distances. Even if there is a shorter Euclidean distance between the points A and B in image (c), the risk is higher at point C.

This occurs in the case of "airport malaria" ([Mai02]), for example, where infected vectors or infected



(a) Simplified risk map visualising the risk for the infection with a disease.

(b) Expected development of the risk when the disease keeps spreading (concentrically around the hot zone).

(c) Actual development of the risk when the disease keeps spreading, e.g. "airport malaria".

Figure 7.14: Exemplary visualisation of epidemiological distances. (Figure generated with *Maxima* and *Libre Office Draw*.)

people who infect local mosquitoes, are brought in by plane.

Airports are situated at points B and C, which makes it possible for the disease to be transmitted by transporting infected people or infected vectors from B to C. This exemplary space should no longer be metric, because the symmetry-property is not fulfilled if, inter alia, there are more flights from B to C than from C to B. Thus measure theory on topological spaces needs to be applied. Fuzzy mappings operate on these general topological spaces. (Adaptive) Fuzzy logic operations determine the GUI structure. Therefore, measure theory on topological spaces plays an important role in the development of an operating process for adaption and, thus, for the SAGU. Measure theory on topological spaces is addressed in the following chapter.

8 Measure Theory on Topological Spaces for Evaluating Fuzzy Membership Mappings on non-Euclidean Spaces

Some principles of measure theory on topological spaces are elucidated in the following sections.

8.1 Topology and Topological Spaces

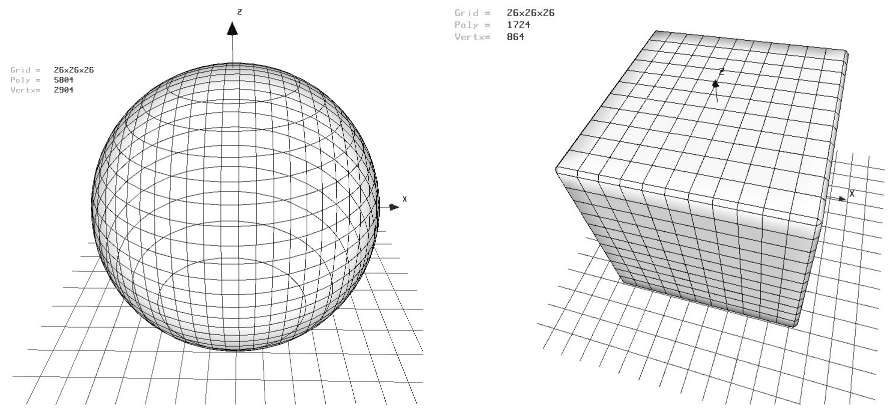
The term system of set topology exists since the release of *Felix Hausdorff's* book *"Outline of Set Theory"*, Leipzig 1914. In chapter seven of this book: *"Point Sets in General Spaces"*, the most important basic concepts of set topology are defined. The origins of the set topology, however, lie deeper. The 19th Century caused a monumental shift in the view of geometry, from which the set topology arose. At the beginning of the 19th Century, there was still the classical setting, that the geometry of the mathematical theory of physical reality was surrounding us, and its axioms were regarded as self-evident elementary things. At the end of the century men had come off of this narrow conception of geometry as a theory of space, it had become clear that the geometry of the future will have more far-reaching goals for the sake of which there must be operated in "abstract spaces". For the generation of set topology also a key contribution of a mathematician was added: The words

> "To the creator of set theory, Mr Georg Cantor, dedicated in grateful admiration"

are to see on *Hausdorff's* book. (cf [Jae99], p. 3f.).

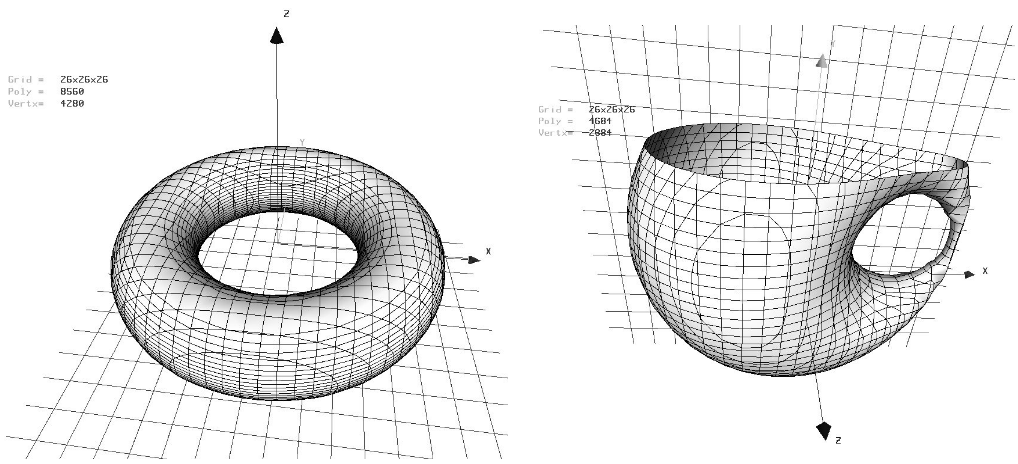
The topology is concerned with the properties of geometric solids and surfaces, that means topological spaces with homeomorphisms by deformation, that means at topological mappings, that means reversible and irreversible unique continuous mappings remain unchanged. Imagine a body made of rubber, which can be deformed in such a way that there do not disappear any existing holes and no new holes are drilled. For example, the number of holes in a Swiss cheese is a topological property of this body, but not that the outside looks round or square. A sphere is topologically indistinguishable from a cube, but by a car tire or a beer mug with handle. ([Kne77], p.393).

In FIGURE 8.1 and FIGURE 8.2, homeomorphic topological bodies are illustrated.



(a) Sphere. **(b)** Cubus.

Figure 8.1: Homeomorphic topological bodies. (Figure generated with *K3DSurf*.)



(a) Torus (car tire). **(b)** Dupin cyclide (beer mug with handle).

Figure 8.2: Homeomorphic topological bodies. (Figure generated with *K3DSurf*.)

The set topology includes everything that can be said about general concepts that have something to do with “proximity“, “neighbourhood and ”convergence“. For many first completely abstract and non-intuitive problems, set topology allows a connection to our spatial awareness. ([Jae99], p. 2).
The invariants of the set-theoretic topology can hasten away in the four following categories ([vQ01] p.328):

1. countability properties
2. connection properties
3. separation properties
4. compactness properties

Some of those terms will be defined and explained in the following.

8.1.1 Topological Basics

Isolation, openness, convergence and continuity are concepts in the analysis on a notion of distance, amount, or more generally based on a norm.

8.1.1.1 Metric and Norm

A Euclidean space induces a normed space and a normed space induces a metric space. Thus, if a space is not metrisable, we know, that the considered space is not a Euclidean space.

Definition 8.1.1 *Metric, Pseudo Metric (cf [vQ01], p.7 and [Lut76], p.16)*
A metric on a set X is a mapping $d : X \times X \to [0, \infty)$ with the following properties:

1. $\forall\ x, y \in X : d(x, y) \geq 0$
2. $\forall\ x, y \in X : d(x, y) = 0 \Leftrightarrow x = y$
3. $\forall\ x, y \in X : d(x, y) = d(y, x)$
4. $\forall\ x, y, z \in X : d(x, z) \leq d(x, y) + d(y, z)$

If it is only "$\Leftarrow$" in 2., d is called a pseudo metric.

Definition 8.1.2 *Metric Space ([vQ01], p.7)*
The couple (X, d) is called metric space.

Definition 8.1.3 *Normed Vector Space, Semi-normed Vector Space (cf [vQ01], p.7)*
A vector space over $(K, +, \cdot)$[1] *is called normalised, if there is a function*

$$\begin{aligned} \|\cdot\| : & V \to [0, \infty) \\ & x \mapsto \|x\|, \end{aligned}$$

with the following characteristics:

1. $\forall\ x \in V : \|x\| \geq 0$
2. $\forall\ x \in V : \|x\| = 0 \Leftrightarrow x = 0$
3. $\forall\ x \in V : \|\lambda x\| = |\lambda| \cdot \|x\|$
4. $\forall\ x, y \in V : \|x + y\| \leq \|x\| + \|y\|$

[1] $(K, +, \cdot)$ solid, $K = \mathbb{R}$ or $\mathbb{C}$

If it is only "$\Leftarrow$" in 2., $\|\cdot\|$ is called a semi-norm.

Definition 8.1.4 *Normed Space (cf [Jae99], p.33)*
$\|\cdot\| : X \to [0, \infty)$ *is a norm on* X. $(X, \|\cdot\|)$ *is called normed space.*

8.1.1.2 Topology

Topological spaces have to be considered for the following reason: In contributing to the optimisation of health service delivery, epidemiological distances are, inter alia, identified as being an issue. However, these distances in the public health domain do not fulfil the symmetry property of the metric, thus the spaces being considered may not be Euclidean spaces. In the SAGU described in Chapter 9, fuzzy membership mappings are included, which are elements of a function space, which is infinite dimensional.

Definition 8.1.5 *Topology (cf [Her86], p.3)*
A topology $\mathfrak{T}$ *on a set* $X \neq \emptyset$ *is a system of subsets of* X *with the following characteristics:*

1. $\emptyset \in \mathfrak{T}, \quad X \in \mathfrak{T}$
2. $U, V \in \mathfrak{T} \Rightarrow U \cap V \in \mathfrak{T}$
3. $\Upsilon \subseteq \mathfrak{T} \Rightarrow \bigcup_{U \in \Upsilon} U \in \mathfrak{T}$

Each set $U \in \mathfrak{T}$ *is called open.*

Definition 8.1.6 *Topological Space ([Her86], p.3)*
Let $\mathfrak{T}$ *be a topology on* X. *The couple* $(X, \mathfrak{T})$ *is called topological space.*

Note 8.1.7: *Every normed space is a metric space and a topological space. (cf [vQ01], p. 8 and [Jae99], p.11).*

We will have a closer look on the topologies $\mathfrak{T}_{ind}$ and $\mathfrak{T}_{nat}$ within Chapter 9. The mentioned topologies are illustrated in the following example:

Example 8.1.8:

- *X is a set,* $\mathfrak{T}_{ind} = \{\emptyset, X\}$ *is called indiscreet or chaotic or trivial topology on X.*
- *X is a set,* $\mathfrak{T}_{dis}$ *is the power set of X.* $\mathfrak{T}_{dis}$ *is called discrete topology on X and* $(X, \mathfrak{T}_{dis})$ *discrete topological space.*

- $X = \mathbb{R}$; $\mathfrak{T}_{nat}$ *produced by the unions of open intervals* $(a,b), a,b \in \mathbb{R}$. *This topology is called natural topology on* $\mathbb{R}$.
- *Analog is:* (X,d) *is a metric space. The union of open balls form a topology on* (X,d)*, the topology of the metric space* (X,d)*. Especially one gets then the natural Topologies on the n-dimensional Euclidean space* $\mathbb{R}^n$*, on the unitary spaces* $\mathbb{C}$ *and* $\mathbb{C}^n$ *or on the pre-Hilbert spaces* $\mathbb{Q}$ *and* $\mathbb{Q}^n$*. (cf [vQ01], p.22).*

With finer topologies fuzzy functions can be measured more precisely. With coarse topologies one can measure fuzzy functions only coarsely. Additionally, with coarse topologies, we have less requirements to convergence. CHAPTER 9 dwells on this again.

Definition 8.1.9 *Coarser (cf [vQ01], p.31)*
A topology $\mathfrak{T}_1$ *is called coarser than a topology* $\mathfrak{T}_2$*, if* $\mathfrak{T}_1 \subset \mathfrak{T}_2$.

Definition 8.1.10 *Finer (cf [vQ01], p.31)*
A topology $\mathfrak{T}_2$ *is called finer than a topology* $\mathfrak{T}_1$*, if* $\mathfrak{T}_1 \subset \mathfrak{T}_2$.[2]

Note 8.1.11: *For the Topologies defined on* $\mathbb{R}$ *is:*

$$\mathfrak{T}_{ind} \subset \mathfrak{T}_{nat} \subset \mathfrak{T}_{dis}.$$

Measures will be initially defined on continuous mappings with compact carrier (see SUBSECTION 8.2.1). By measuring on compact sets we can get information about a mapping we do not know. We need open sets to define compactness. Furthermore, open sets are used for the learning task of the ANN to identify the similarity of outputs to similar inputs. See SECTION 9.3. Additionally, open sets are required to separate GUI properties (e.g. GUI elements or GUI element objects) (separation axiom T_2, DEFINITION 8.1.45). This is necessary to enable the assignment of one certain GUI adjusted to the user.

[2]I.e. if $\mathfrak{T}_2$ contains at least all open sets of $\mathfrak{T}_1$.

Definition 8.1.12 *Open (cf [vQ01], p.21)*

$(X, \mathfrak{T}_X)$ *is a topological space.*

The sets $O_X \in \mathfrak{T}$ *are the open subsets of* X*.*

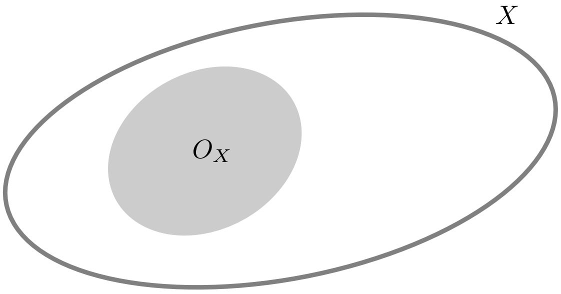


Figure 8.3: Open set (O_X). (Figure generated with *LaTeX*.)

Definition 8.1.13 *Closed (cf [vQ01], p.21)*

$(X, \mathfrak{T}_X)$ *is a topological space.*

The complements $X \setminus O_X = A_X$ *of* O_X *in* X *are called closed.*

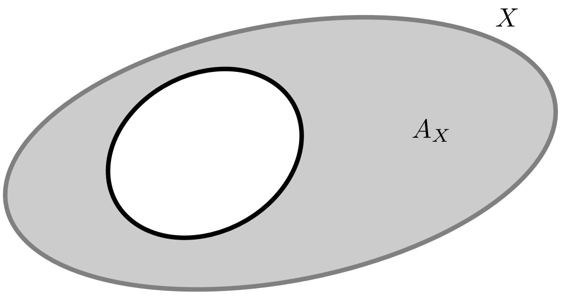


Figure 8.4: Closed set (A_X). (Figure generated with *LaTeX*.)

Definition 8.1.14 *Neighbourhood (cf [vQ01], p.25)*

$(X, \mathfrak{T}_X)$ *is a topological space.*

It is $x \in X$*. A set* $U_x \subset X$ *is called neighbourhood of* x*, when there is an open set* $O_x \subset X$ *with* $x \in O_x \subset U_x$*.*

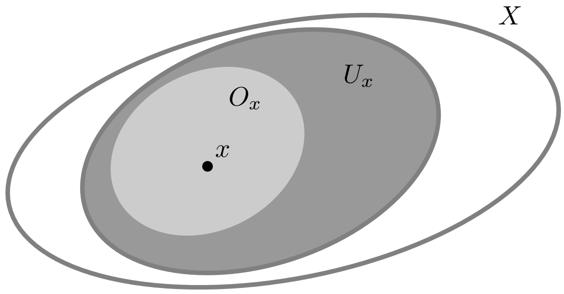


Figure 8.5: Neighbourhood of x (U_x). (Figure generated with *LaTeX*.)

Note 8.1.15:

- *Neighbourhoods must not be open.*
- *Every open set is a neighbourhood of each of its points. (cf [vQ01], p.25)*

Note 8.1.16: *$(X, \mathfrak{T}_X)$ is a topological space. The set system of the neighbourhoods of a point $x \in X$ is denoted with $\mathfrak{F}_U(x)$ ($\mathfrak{F}_U(x)$ is a Filter (*Definition *8.1.61), see* Example *8.1.62).*

For the adaptivity of the GUI we have to proceed in the following way: $(X, \mathfrak{T}_X)$ is a topological space. Information about a subspace of X is collected, e.g. via a compactum $K \subset X$ (Definition 8.1.81), e.g. in $\mathbb{R}$ for an interval $[a, b] \subset \mathbb{R}$. Afterwards, we have to narrow our observations down to the topology which is induced by X to K, the relative topology ($\mathfrak{T}_{X,X|_K}$) on K, even if X is the fundamental space for the measures.

Definition 8.1.17 *Relative Topology (cf [Els09], p.411)*
Let $(X, \mathfrak{T}_X)$ be a topological space and $\emptyset \neq M \subseteq X$.
$\mathfrak{T}_{rel} = \mathfrak{T}_{X,X|_M} := \{O_X \cap M \mid O_X \in \mathfrak{T}_X\}$ is called relative topology, $\mathfrak{T}_{rel}$, on M.

We will apply the product topology $\mathfrak{T}_P$ within Chapter 9 within the definition of a membership mapping with which the quality of a GUI for a user can be determined.

Definition 8.1.18 *Product Topology (cf [Jae99], p.14)*
Let $(X, \mathfrak{T}_X)$ and $(Y, \mathfrak{T}_Y)$ be topological spaces. A subset $M \subset X \times Y$ is called open in the product topology $\mathfrak{T}_P = \mathfrak{T}_X \otimes \mathfrak{T}_Y$, if there exist for each point $(x, y) \in M$ neighbourhoods U_x of x and U_y of y, with $U_x \times U_y \subset M$. Therewith, $(X \times Y, \mathfrak{T}_P)$ is a topological space.

For the SAGU, a topology has to be generated on our basic space X_Γ, see p.148. The following will explain how a topology can be generated.

Theorem 8.1.19 *$E \subset \mathcal{P}(X)$ is an arbitrary set and $\mathfrak{T}$ are topologies on X with $E \subseteq \mathfrak{T}$. Then $\mathfrak{T}_E := \bigcap\limits_{\mathfrak{T} \supseteq E} \mathfrak{T}$ is a topology.*

Proof (8.1.19): *$\mathfrak{T}$ are topologies on X with $E \subseteq \mathfrak{T}$. We will show, that $\mathfrak{T}_E = \bigcap\limits_{\mathfrak{T} \supseteq E} \mathfrak{T}$ fulfils the properties 1.-3. of* Definition *8.1.5 with a direct proof.*
It is:

1. $\forall\ \mathfrak{T} \supseteq E : (\emptyset \in \mathfrak{T}) \wedge (X \in \mathfrak{T}) \overset{Def.\cap}{\Rightarrow} (\emptyset \in \bigcap\limits_{\mathfrak{T} \supseteq E} \mathfrak{T}) \wedge (X \in \bigcap\limits_{\mathfrak{T} \supseteq E} \mathfrak{T})$
   $\overset{Def.\mathfrak{T}_E}{\Rightarrow} (\emptyset \in \mathfrak{T}_E) \wedge (X \in \mathfrak{T}_E)$
2. *Let* $O_1, O_2 \in \mathfrak{T}_E \Rightarrow (O_1 \in \bigcap\limits_{\mathfrak{T} \supseteq E} \mathfrak{T}) \wedge (O_2 \in \bigcap\limits_{\mathfrak{T} \supseteq E} \mathfrak{T})$
   $\Rightarrow (\ \forall\ \mathfrak{T} \supseteq E : O_1 \in \mathfrak{T}) \wedge (\ \forall\ \mathfrak{T} \supseteq E : O_2 \in \mathfrak{T}) \Rightarrow\ \forall\ \mathfrak{T} \supseteq E : (O_1 \in \mathfrak{T}) \wedge (O_2 \in \mathfrak{T})$
   $\overset{\text{Def. } 8.1.5\ 2.}{\Rightarrow}\ \forall\ \mathfrak{T} \supseteq E : O_1 \cap O_2 \in \mathfrak{T} \Rightarrow O_1 \cap O_2 \in \bigcap\limits_{\mathfrak{T} \supseteq E} \mathfrak{T} = \mathfrak{T}_E$
3. *Let J be an arbitrary index set, $J \neq \emptyset$ with* $\forall\ j \in J : O_j \in \mathfrak{T}_E$
   $\Rightarrow\ \forall\ j \in J : O_j \in \bigcap\limits_{\mathfrak{T} \supseteq E} \mathfrak{T} \Rightarrow\ \forall\ j \in J\ \forall\ \mathfrak{T} \supseteq E : O_j \in \mathfrak{T}$
   $\Rightarrow\ \forall\ \mathfrak{T} \supseteq E\ \forall\ j \in J : O_j \in \mathfrak{T} \overset{\text{Def. } 8.1.5\ 3.}{\Rightarrow}\ \forall\ \mathfrak{T} \supseteq E : \bigcup\limits_{j \in J} O_j \in \mathfrak{T}_i$
   $\Rightarrow \bigcup\limits_{j \in J} O_j \in \bigcap\limits_{\mathfrak{T}_i \supseteq E} \mathfrak{T} = \mathfrak{T}_E$

$\mathfrak{T}_E$ fulfils the properties 1.-3. of Definition *8.1.5 and is therefore a topology on X.*

□

Definition 8.1.20 *Generator*
$\mathfrak{T}$ are topologies on X.
If $\mathfrak{T}_E := \bigcap\limits_{\mathfrak{T} \supseteq E} \mathfrak{T}$ is a topology and $E \subset \mathcal{P}(X)$, E is called generator of $\mathfrak{T}_E$.

Theorem 8.1.21 *$\mathfrak{T}_E := \bigcap\limits_{\mathfrak{T} \supseteq E} \mathfrak{T}$ is the coarsest topology of the topologies $\mathfrak{T}$ with $\mathfrak{T} \supseteq E$.*

Proof (8.1.21): *Let $\mathfrak{T}_E$ be generated by E. $\mathfrak{T}_E$ is a topology with* Theorem *8.1.19.*
We will show that $\mathfrak{T}_E$ is the coarsest topology of the topologies $\mathfrak{T}$ with $E \subseteq \mathfrak{T}$ using a proof by contradiction.
Assumption: $\exists\ \mathfrak{T}_k \supseteq E : \mathfrak{T}_k \subset \mathfrak{T}_E$.
Precondition: $\mathfrak{T}_E = \bigcap\limits_{\mathfrak{T} \supseteq E} \mathfrak{T}$, $E \subseteq \mathfrak{T}_k$.
Let $\mathfrak{T}_k$ be a topology with $\mathfrak{T}_k \supseteq E \Rightarrow \mathfrak{T}_k \supseteq \bigcap\limits_{\mathfrak{T} \supseteq E} \mathfrak{T} \Rightarrow \mathfrak{T}_k \supseteq \mathfrak{T}_E$
and this is a contradiction to the assumption.

□

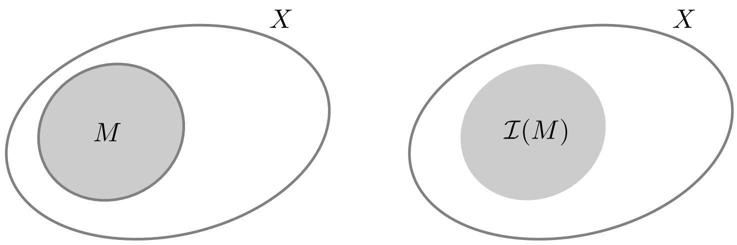
The GPS-blocks described on p.156 can be represented by fuzzy sets. The GUI assigned to a user is GIS-tailored, that is based on the geo-location of the user and thus on the GPS-block the user is located in. When a user moves towards the boundary of the GPS-block she/he is located in, the GUI proposes to change to another GUI adjusted for the GPS-block towards that the user is moving, if the user is near to the boundary of the GPS-block. For the definition of the boundary, the definitions of the interior and the closure are necessary.

Definition 8.1.22 *Interior in* $(X, \mathfrak{T})$ *(cf [vQ01], p.28)*
$(X, \mathfrak{T}_X)$ *is a topological space.*
For each subset $M \subset X$*, there is a largest open set contained in it, the interior*

$$\mathcal{I}(M) = \bigcup_{O_M \subset M, O_M \in \mathfrak{T}} O_M.$$

The interior of a set $M \subset X$ *can be characterised as neighbourhoods with:*

$$\mathcal{I}(M) = \{x \in X \mid \ \exists \ U_x : U_x \in \mathfrak{F}_U(x) \text{ and } U_x \subset M\}.$$



(a) Set M. **(b)** Interior of the set M ($\mathcal{I}(M)$).

Figure 8.6: Interior in $(X, \mathfrak{T})$ ($\mathcal{I}(M)$). (Figure generated with *LaTeX*.)

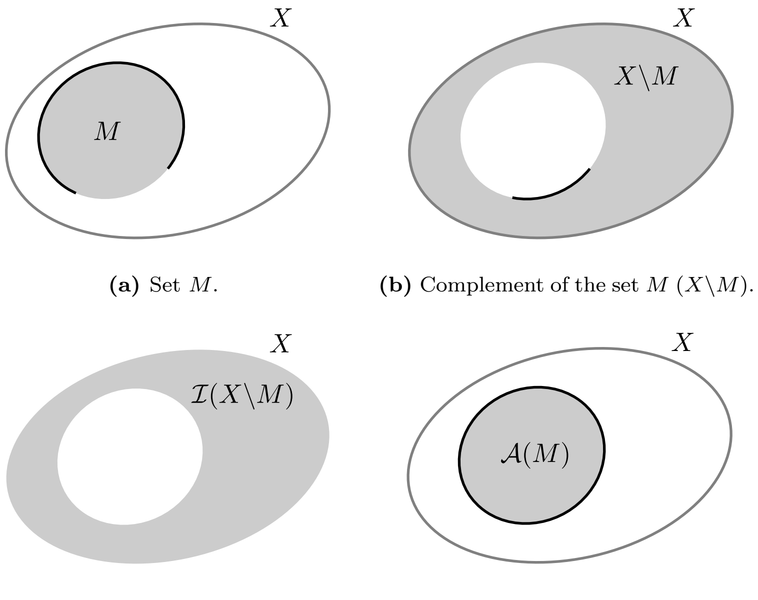
We need the closure of a GPS block, if the GUI is supposed to behave consistently on the boundary of the GPS block.

Definition 8.1.23 *Closure in* $(X, \mathfrak{T})$ *(cf [vQ01], p.28)*
$(X, \mathfrak{T}_X)$ *is a topological space.*
For each subset $M \subset X$*, there is a smallest closed set that includes M, the closure*

$$\mathcal{A}(M) = X \setminus \mathcal{I}(X \setminus M)$$

of M. The closure of a set $M \subset X$ *can be characterised as neighbourhoods with:*

$$\mathcal{A}(M) = \{x \in X \mid \ \forall \ U_x : U_x \in \mathfrak{F}_U(x) \Rightarrow U_x \cap M \neq \emptyset\}.$$



(a) Set M.

(b) Complement of the set M ($X \backslash M$).

(c) Interior of the set $X \backslash M$ ($\mathcal{I}(X \backslash M)$).

(d) Closure of the set M ($\mathcal{A}(M) = X \backslash \mathcal{I}(X \backslash M)$).

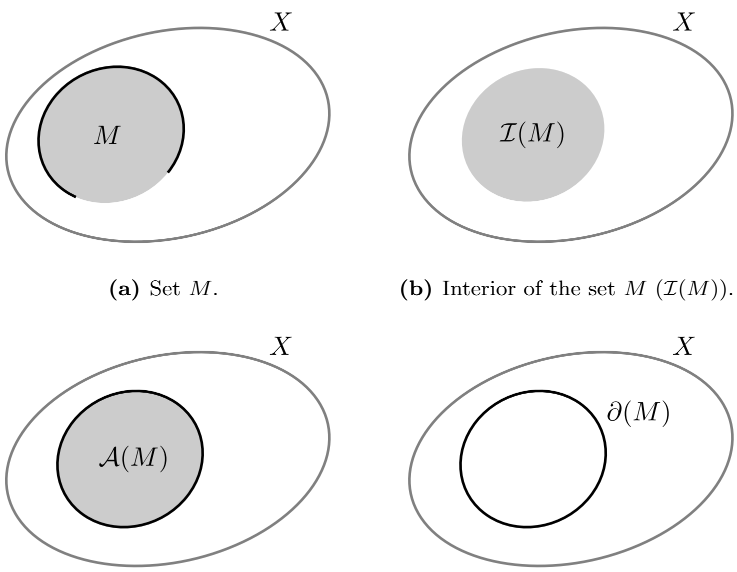
Figure 8.7: Closure in $(X, \mathfrak{T})$ ($\mathcal{A}(M)$). (Figure generated with *LaTeX*.)

Definition 8.1.24 *Boundary in* $(X, \mathfrak{T})$ *(cf [vQ01], p.28)*

$(X, \mathfrak{T}_X)$ *is a topological space.*

For each subset $M \subset X$ *the set of boundarypoints of* M *is called boundary of* M *and is denoted by* $\partial(M)$.

$$\partial(M) = \mathcal{A}(M) \setminus \mathcal{I}(M).$$



(a) Set M. **(b)** Interior of the set M ($\mathcal{I}(M)$).

(c) Closure of the set M ($\mathcal{A}(M)$). **(d)** Boundary of the set M ($\partial(M)$).

Figure 8.8: Boundary in $(X, \mathfrak{T})$ ($\partial(M)$). (Figure generated with *LaTeX*.)

Definition 8.1.25 *Neighbourhood Base (cf [vQ01], p.26)*

A set system $\mathfrak{D}_X$ *in* X *is called a neighbourhood base of the point* $x \in X$, *if the neighbourhoods of* x *are precisely the including sets of the sets from* $\mathfrak{D}_X$.

Note 8.1.26: *In the neighbourhoods*[3] *of all points of* X *a topology on* X *can be reconstructed:*

$$O_X \subset X \text{ open } \Leftrightarrow \forall\ x \in O_X\ \exists\ U_x \subset O_X : U_x \in \mathfrak{F}_U(x).$$

8.1.1.3 Topological homeomorphisms

Relations between topological spaces are mediated by continuous mappings. With homeomorphisms neighbourhood-"information" (about a user) can be transferred from a topological space $(X, \mathfrak{T}_X)$ to a topological space $(Y, \mathfrak{T}_Y)$.

[3] and hence also from neighbourhood bases

Note 8.1.27: *The notation*

$$f : (X, \mathfrak{T}_X) \overset{\mathfrak{T}}{\to} (Y, \mathfrak{T}_Y)$$

means that f is a mapping $f : X \to Y$ *and* $(X, \mathfrak{T}_X)$ *and* $(Y, \mathfrak{T}_Y)$ *are topological spaces.*

Definition 8.1.28 *Continuous in x (cf [Oss09], p.44)*
$(X, \mathfrak{T}_X), (Y, \mathfrak{T}_Y)$ *are topological spaces,* $x \in X$ *and* $f : (X, \mathfrak{T}_X) \overset{\mathfrak{T}}{\to} (Y, \mathfrak{T}_Y)$ *is a mapping.*
f *is called continuous in* x*, if the inverse image* $f^{-1}(U_{f(x)})$ *of every neighbourhood* $U_{f(x)}$ *of* $f(x)$ *is a neighbourhood* U_x *of* x*.*

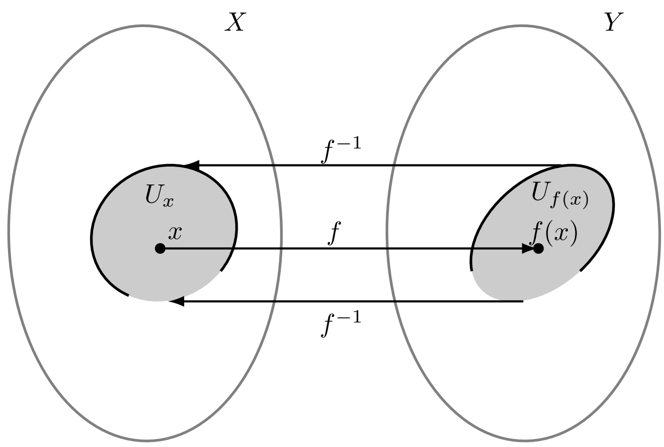


Figure 8.9: Continuous mapping (f) at the place x. (Figure generated with *LaTeX*.)

The following THEOREM 8.1.29 shows how information (about a user) can be transferred from $(X, \mathfrak{T}_X)$ to $(Y, \mathfrak{T}_Y)$. Topologies are assigned to an object in OOA (CHAPTER 6) as information content via Cartesian Product of status-spaces of object-classes. On neighbourhoods in $(\mathbb{R}, \mathfrak{T}_{\mathbb{R}})$, continuous mappings $f : (X, \mathfrak{T}_X) \overset{\mathfrak{T}}{\to} (\mathbb{R}, \mathfrak{T}_{\mathbb{R}})$ behave similar.

Theorem 8.1.29 *Continuous in x (cf [vQ01], p.27)*
A mapping $f : (X, \mathfrak{T}_X) \overset{\mathfrak{T}}{\to} (Y, \mathfrak{T}_Y)$ *between topological spaces is continuous in* $x \in X$*, if and only if for every neighbourhood* $U_{f(x)}$ *of* $f(x)$ *there is a neighbourhood* U_x *of* x *with* $f(U_x) \subset U_{f(x)}$*.*

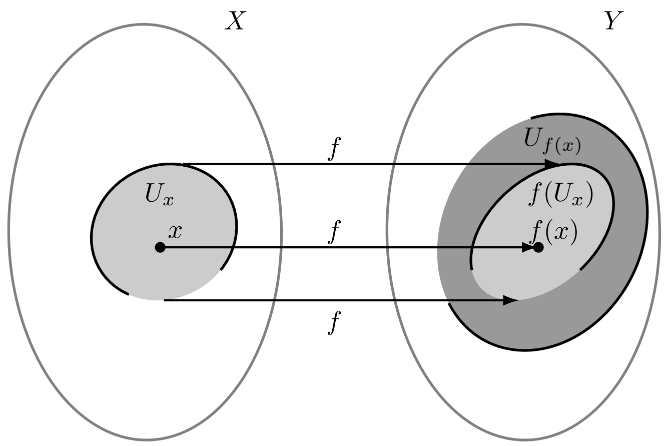


Figure 8.10: Continuous mapping (f) at the place x. (Figure generated with *LaTeX*.)

PROOF (8.1.29): *Direct proof:*

"$\Rightarrow$"

Precondition: f is continuous in x, i.e. the inverse image of every neighbourhood of $f(x)$ is a neighbourhood of x.

$U_{f(x)}$ is a neighbourhood of $f(x)$, i.e. $U_{f(x)} \in \mathfrak{F}_U(f(x))$, hence:

$\forall\ U_{f(x)} \in \mathfrak{F}_U(f(x))\ \exists\ O_{f(x)} \in \mathfrak{T}_Y : f(x) \in O_{f(x)} \subset U_{f(x)}$

$\overset{f\ continuous}{\Rightarrow}\ \forall\ U_{f(x)} \in \mathfrak{F}_U(f(x))\ \exists\ U_x \in \mathfrak{T}_X : U_x := O_x := f^{-1}(O_{f(x)}) \in \mathfrak{T}_X \wedge x \in U_x \in \mathfrak{F}_U(x)$

$\Rightarrow\ \forall\ U_{f(x)} \in \mathfrak{F}_U(f(x))\ \exists\ U_x \in \mathfrak{F}_U(x) :$

$f(U_x) = f(O_x) = f(f^{-1}(O_{f(x)})) = O_{f(x)} \subset U_{f(x)} \in \mathfrak{F}_U(f(x))$

"$\Leftarrow$"

Precondition: For every neighbourhood $U_{f(x)}$ of $f(x)$ there exists a neighbourhood U_x of x with $f(U_x) \subset U_{f(x)}$.

Then it is: for every neighbourhood $U_{f(x)}$ of $f(x)$ there exists $U_x \subset f^{-1}(U_{(f(x)})$ and $f^{-1}(U_{f(x)})$ is a neighbourhood of x.

□

Definition 8.1.30 *Continuous (cf [vQ01], p.31)*

f is a mapping $f : (X, \mathfrak{T}_X) \overset{\mathfrak{T}}{\to} (Y, \mathfrak{T}_Y)$ between topological spaces.

f is called continuous if f is continuous in each $x \in X$.

Lemma 8.1.31 *$(X, \mathfrak{T}_X)$ is a topological space.*

$$\Big(\ \forall\ x \in U_X \subset X\ \exists\ O_x \in \mathfrak{T}_X : x \in O_x \subset U_X\Big) \Rightarrow U_X \in \mathfrak{T}_X.$$

PROOF (8.1.31): *Direct proof:*

Precondition: $x \in U_X, O_x \subset U_X, O_x \in \mathfrak{T}_X$.

$\forall\ x \in U_X \subset X\ \exists\ O_x \in \mathfrak{T}_X : U_X = \bigcup_{x \in U_X} \{x\} \subseteq \bigcup_{x \in U_X} O_x \subseteq \bigcup_{x \in U_X} U_X = U_X$

$\overset{\text{Def. } 8.1.5\ 3.}{\Rightarrow} \left(\forall\ x \in U_X\ \exists\ O_x \in \mathfrak{T}_X : U_X = \bigcup_{x \in U_X} O_x \in \mathfrak{T}_X \right)$

$\Rightarrow U_X \in \mathfrak{T}_X$.

□

Note 8.1.32: LEMMA *8.1.31 is used within the proof of* THEOREM *8.1.33.*

Hereinafter, the term continuity will often be used in the following way:

Theorem 8.1.33 *Continuous (cf [Kow61], p.118)*

f is a mapping $f : (X, \mathfrak{T}_X) \overset{\mathfrak{T}}{\to} (Y, \mathfrak{T}_Y)$ between topological spaces. The following statements are equivalent:

1. *f is continuous.*
2. *For every open set $O_Y \subset Y$ the inverse image $f^{-1}(O_Y) \subset X$ is open.*
3. *For every closed set $A_Y \subset Y$ the inverse image $f^{-1}(A_Y) \subset X$ is closed.*

PROOF (8.1.33): *Direct proof:*

1. $\Leftrightarrow$ 2.*:*

"$\Rightarrow$":

Precondition: Let f be continuous and $O_Y \in \mathfrak{T}_Y$ an arbitrary open set.

It is $\forall\ x \in f^{-1}(O_Y) : f(x) \in O_Y$.

$\overset{\text{Def. } 8.1.14}{\Rightarrow} \forall\ O_Y \in \mathfrak{T}_Y : O_Y$ *is a neighbourhood of* $f(x)$

$\overset{\text{Percond.}}{\Rightarrow} \forall\ O_Y \in \mathfrak{T}_Y\ \exists\ U_x \in \mathfrak{F}_U(x) : f(U_x) \subseteq O_Y$

$\Rightarrow\ \forall\ O_Y \in \mathfrak{T}_Y\ \exists\ U_x \in \mathfrak{F}_U(x) : U_x \subseteq f^{-1}(O_Y)$.

Thus, $f^{-1}(O_Y)$ contains with each point a neighbourhood of this point and is therefore with LEMMA *8.1.31 open.*

"$\Leftarrow$":

Precondition: Let f be a mapping with $\forall\ O_Y \in \mathfrak{T}_Y : f^{-1}(O_Y) \in \mathfrak{T}_X$.

It is $x \in X, U_{f(x)}$ is a neighbourhood of $f(x)$.

$\overset{\text{Def. } 8.1.14}{\Rightarrow} \forall\ U_{f(x)} \in \mathfrak{F}_U(f(x))\ \exists\ O_Y \in \mathfrak{T}_Y : f(x) \in O_Y \subseteq U_{f(x)}$

$\overset{\text{Precond.}}{\Rightarrow} \forall\ U_{f(x)} \in \mathfrak{F}_U(f(x))\ \exists\ O_Y \in \mathfrak{T}_Y, O_Y \subseteq U_{f(x)} : f^{-1}(O_Y) \in \mathfrak{T}_X$

$\Rightarrow\ \forall\ U_{f(x)} \in \mathfrak{F}_U(f(x))\ \exists\ O_Y \in \mathfrak{T}_Y, O_Y \subseteq U_{f(x)} : f^{-1}(O_Y) := U_x$ *is a neighbourhood of x with*

$f(x) \in f(\underbrace{f^{-1}(O_Y)}_{=U_x}) \subseteq O_Y \subseteq U_{f(x)}$

$\Rightarrow f$ *continuous.*

1. $\Leftrightarrow$ 3.*:*

"$\Rightarrow$:"

Precondition: Let f be continuous, $A_Y \subset Y$ is an arbitrary closed set.

$\forall\ A_Y \subset Y, A_Y$ *closed:* $(Y \backslash A_Y)$ *open*

$\overset{1.\Rightarrow 2.}{\Rightarrow}\ \forall\ A_Y \subset Y, A_Y$ *closed:* $f^{-1}(Y \backslash A_Y)$ *open*

$\Rightarrow\ \forall\ A_Y \subset Y, A_Y$ *closed:* $f^{-1}(Y \cap A_Y^C)$ *open*

$\Rightarrow\ \forall\ A_Y \subset Y, A_Y$ *closed:* $f^{-1}(Y) \cap f^{-1}(A_Y^C)$ *open*

$\Rightarrow\ \forall\ A_Y \subset Y, A_Y$ *closed:* $f^{-1}(Y) \cap (f^{-1}(A_Y))^C$ *open*

$\Rightarrow\ \forall\ A_Y \subset Y, A_Y$ *closed:* $X \backslash f^{-1}(A_Y)$ *open*

$\Rightarrow\ \forall\ A_Y \subset Y, A_Y$ *closed:* $f^{-1}(A_Y)$ *closed.*

"$\Leftarrow$":

Precondition: Let f be a mapping with $\forall\ A_Y$ closed$: f^{-1}(A_Y)$ closed.

It is $a \in X$. $U_{f(a)} \subset Y$ is a neighbourhood of $f(a)$.

$\Rightarrow\ \forall\ U_{f(a)} \in \mathfrak{F}_U(f(a))\ \exists\ O_Y \in \mathfrak{T}_Y : f(a) \in O_Y \subset U_{f(a)}$.

$\Rightarrow\ \forall\ U_{f(a)} \in \mathfrak{F}_U(f(a))\ \exists\ O_Y \in \mathfrak{T}_Y : Y \backslash O_Y$ *closed.*

$\overset{Precond.}{\Rightarrow}\ \forall\ U_{f(a)} \in \mathfrak{F}_U(f(a))\ \exists\ O_Y \in \mathfrak{T}_Y : f^{-1}(Y \backslash O_Y)$ *closed.*

$\Rightarrow\ \forall\ U_{f(a)} \in \mathfrak{F}_U(f(a))\ \exists\ O_Y \in \mathfrak{T}_Y : X \backslash f^{-1}(O_Y)$ *closed.*

$\Rightarrow\ \forall\ U_{f(a)} \in \mathfrak{F}_U(f(a))\ \exists\ O_Y \in \mathfrak{T}_Y : f^{-1}(O_Y)$ *open.*

$\overset{2.\Rightarrow 1.}{\Rightarrow} f$ *continuous.*

□

Definition 8.1.34 $\mathfrak{C}(X, \mathfrak{T}_X)$

$\mathfrak{C}(X, \mathfrak{T}_X)$ *is the space of the continuous mappings* $f : (X, \mathfrak{T}_X) \overset{\mathfrak{T}}{\rightarrow} (Y, \mathfrak{T}_Y)$.

Within the topological space $(X, \mathfrak{T}_{dis})$ the one-pointed sets are open.

Each mapping $f : (X, \mathfrak{T}_{dis}) \overset{\mathfrak{T}}{\rightarrow} (Y, \mathfrak{T}_Y)$ is continuous and for an one-pointed set $\{y\} \subset Y$ we can not imply the behaviour of f in a larger neighbourhood $U_y \subset Y$ of y, because we can not extrapolate single point information of the codomain of a mapping to the topological neighbourhood. This is not of advantage, if we want to transfer information (about a user) from $(X, \mathfrak{T}_X)$ to $(Y, \mathfrak{T}_Y)$.

Definition 8.1.35 *Open in x (cf [vQ01], p.32)*

f is a mapping $f : (X, \mathfrak{T}_X) \overset{\mathfrak{T}}{\rightarrow} (Y, \mathfrak{T}_Y)$ *between topological spaces.*

f is called open in $x \in X$, if the image $f(U_x)$ of each neighbourhood U_x of x is a neighbourhood of $f(x)$.

Definition 8.1.36 *Open (cf [vQ01], p.32)*
f is a mapping $f : (X, \mathfrak{T}_X) \overset{\mathfrak{T}}{\to} (Y, \mathfrak{T}_Y)$ *between topological spaces.*
f is called open, if it is open in each $x \in X$*.*

Note 8.1.37: *The difference between open and continuous mappings is that if a mapping is continuous, the inverse-image set of open sets is open and if a mapping is open, the image of open sets is open.*

Two topological spaces, $(X, \mathfrak{T}_X)$ and $(Y, \mathfrak{T}_Y)$, have exactly the same topological properties. With homeomorphisms neighbourhood-"information" can be transferred from the topological space $(X, \mathfrak{T}_X)$ to the topological space $(Y, \mathfrak{T}_Y)$.

Definition 8.1.38 *Topological Homeomorphism (cf [vQ01], p.32)*
A continuous, open and bijective mapping is called topological homeomorphism.

Definition 8.1.39 *Homeomorphic (cf [vQ01], p.32)*
The spaces $(X, \mathfrak{T}_X), (Y, \mathfrak{T}_Y)$ *are called Homeomorphic, if there is a topological homeomorphism*

$$f : (X, \mathfrak{T}_X) \overset{\mathfrak{T}}{\to} (Y, \mathfrak{T}_Y).$$

8.1.1.4 Separation axioms

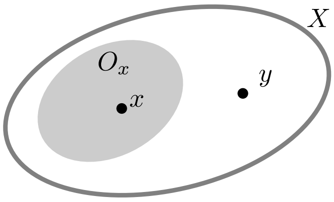
Separation axioms are axioms in the sense that one can demand, in addition to the definition of a topological space, some of these conditions. By separation axioms somebody postulates the existence of sufficiently many open sets to separate specific sets of each other. ([vQ76], p. 62).
In a data space about the objects of the GUI or a GIS, we try to adjust properties of the GUI using the methods of OOA. If there are two points, which are not separable in a topology, it is not possible to assign open sets (cluster) to the two points for the properties of the GUI. If those properties of the GUI are not separable in the topology, they are dependent.

Note 8.1.40: *The following separation axioms* $T_i, i = 0, 1, 2, 3, 3a, 4, 5$ *claim some properties from a topological space* $(X, \mathfrak{T}_X)$*, that guarantee the separation of points and sets by open sets.*

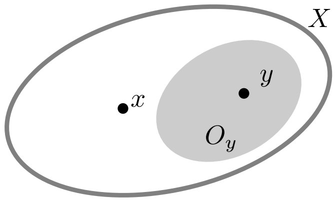
Definition 8.1.41 T_i*-Space*
If T_i *is satisfied, then* $(X, \mathfrak{T}_X)$ *is called a* T_i*- space.*

Definition 8.1.42 T_0 *(Kolmogoroff) (cf [Lut76], p.19 and [Kow61], p.68)*
$(X, \mathfrak{T}_X)$ *is a topological space. For two different points* x, y *of* X *there exists an open set in* X *that*

contains exactly one of the two points.



(a) Open set (O_x) containing x and not containing y.



(b) Open set (O_y) containing y and not containing x.

Figure 8.11: T_0-space (Kolmogoroff). (Figure generated with *LaTeX*.)

Definition 8.1.43 T_1 *(Fréchet) (cf [Lut76], p.19 and [Kow61], p.62)*
$(X, \mathfrak{T}_X)$ *is a topological space. For two different points* $x, y \in X$ *there exist neighbourhoods* U_x *of* x *and* U_y *of* y *with* $y \notin U_x$ *and* $x \notin U_y$.

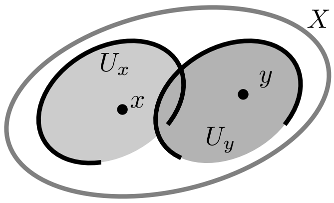


Figure 8.12: T_1-space (Fréchet). (Figure generated with *LaTeX*.)

Theorem 8.1.44 $(X, \mathfrak{T}_X)$ *is a topological space.* X *is a* T_1*-space, if and only if the one-pointed sets in* X *are closed. (cf [vQ01], p.84f).*

The proof can be found in [vQ01], p.84f.

Definition 8.1.45 T_2 *(Hausdorff) (cf [Lut76], p.19 and [vQ01], p.83)*
For $x \neq y$ *there are neighbourhoods* U_x *of* x *and* U_y *of* y *with* $U_x \cap U_y = \emptyset$.

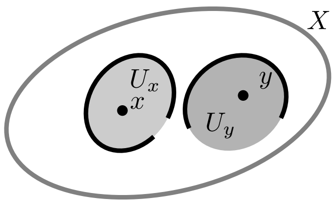


Figure 8.13: T_2-space (Hausdorff). (Figure generated with *LaTeX*.)

Note 8.1.46: *The most commonly requested separation axiom is* T_2*.*

For our concerns, it is important to separate sets and points via fuzzy-mappings. T_3 enables the separation of sets and points via open sets.

Definition 8.1.47 T_3 *(Vietories) (cf [Lut76], p.19 and cf [Kow61], p.64)*
$(X, \mathfrak{T}_X)$ *is a topological space. For* $x \in X$ *and each closed set* $A_X \subset X$ *with* $x \notin A_X$ *there are open sets* O_1, O_2 *in* X *with* $x \in O_1, A_X \subset O_2$ *and* $O_1 \cap O_2 = \emptyset$*.*

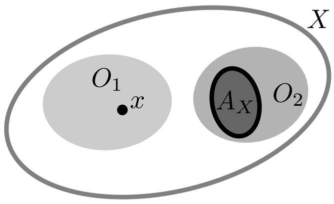


Figure 8.14: T_3-space (Vietories). (Figure generated with *LaTeX*.)

Note 8.1.48: *In the following context the separation axiom* T_3 *plays an important role especially for measure theory,* SECTION *8.2.*

Theorem 8.1.49 *A topological space* $(X, \mathfrak{T}_X)$ *is a* T_3*-space, if and only if for every point* x *and every open set* O_x *containing* x *there is an open neighbourhood* U_x *of* x*, whose closure is a subset of* O_x*:*

$$\forall \ O_x \in \mathfrak{T}_X : x \in O_x \Rightarrow \ \exists \ U_x \in \mathfrak{T}_X : x \in U_x \subset \mathcal{A}(U_x) \subset O_x.$$

(cf [Kow61], p.64).

PROOF (8.1.49): *Direct proof:*
"⇒"
Precondition: $(X, \mathfrak{T}_X)$ *is a* T_3*-space and* $O_x \in \mathfrak{T}_X$ *with* $x \in O_x$ *chosen freely.*
$A_x := X \backslash O_x$ *closed with* $x \notin A_x$

$\overset{T_3}{\Rightarrow} \ \forall \ A_x \subset X, x \in X, A_x = X \backslash O_x, x \notin A_x \ \exists \ O_1, O_2 \in \mathfrak{T}_X : O_1 \cap O_2 = \emptyset \wedge x \in O_1 \wedge A_x \subset O_2$

$\Rightarrow \ \forall \ A_x \subset X, x \in X, A_x = X \backslash O_x, x \notin A_x \ \exists \ O_1, O_2 \in \mathfrak{T}_X :$

$x \in O_1 \subset \mathcal{A}(O_1) \subseteq X \backslash O_2 \subset X \backslash A_x = O_x,$

with $O_1 = U_x$ *the assertion follows.*

"$\Leftarrow$"

Precondition: $\forall \ O_x \in \mathfrak{T}_X : x \in O_x \Rightarrow \ \exists \ U_x \in \mathfrak{T}_X : x \in U_x \subset \mathcal{A}(U_x) \subset O_x.$

We have to show: $(X, \mathfrak{T}_X)$ *is a* T_3*-space.*

For all $O_x \in \mathfrak{T}_X$ *with* $x \in O_x$*, there is* A_x*, which is closed and* $x \notin A_x$*. Since* $X \backslash A_x$ *is open,* $X \backslash A_x$ *with* $x \in X \backslash A_x$ *is an open neighbourhood* U_x *of* x *with* $x \in U_x \subset \mathcal{A}(U_x) \subset X \backslash A_x$*. Then* U_x *and* $X \backslash \mathcal{A}(U_x)$ *can function as the open sets separating* x *and* A_x*.*

□

Note 8.1.50: *Equivalent to* Theorem *8.1.49, the closed neighbourhoods form a neighbourhood base for each point* $x \in X$ *([vQ01], p. 86).*

The separation axiom T_{3a} defined below plays an important role for the development of an SAGU described in Chapter 9. T_{3a} enables the separation of a measurable set (Definition 8.2.28) and a point. This is important for the evaluation of a set of collected data (about a user) and the determination of a membership mapping representing the user' s needs. Furthermore this is important to separate the set of GUI elements form the therein contained points, the GUI element objects. With T_{3a}, measurable sets can be identified. Those sets can be fuzzy, i.e represented via a fuzzy mapping. Therewith fuzzy operations (e.g. the fuzzy-AND, p.71, the fuzzy-OR, p.70 or the fuzzy-NOT, p.72) can be conducted. Fuzzy mappings are used to represent vague statements of a user and they are used to control the GUI.

Definition 8.1.51 T_{3a} *(cf [Lut76], p.19)*

$(X, \mathfrak{T}_X)$ *is a topological space. For* $x \in X$ *and each closed set* $A_x \subset X$ *with* $x \notin A_x$ *there is always a continuous mapping*

$$f : (X, \mathfrak{T}_X) \overset{\mathfrak{T}}{\rightarrow} ([0,1], \mathfrak{T}_{rel}) = ([0,1], \mathfrak{T}_{nat, \mathbb{R}|_{[0,1]}})$$

with

$$f(x) = 0,$$

$$f(A_x) = \{1\}.$$

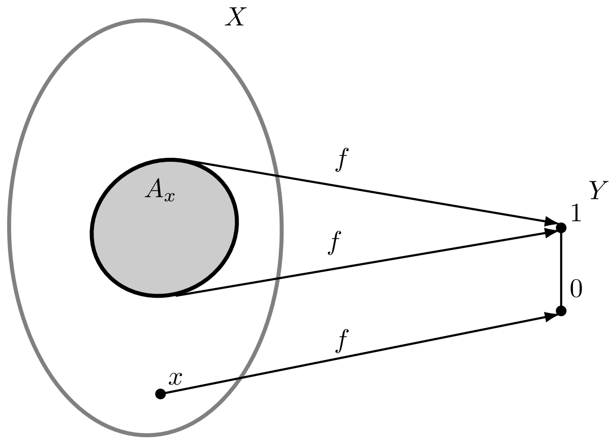


Figure 8.15: T_{3a}-space. (Figure generated with *LaTeX*.)

Theorem 8.1.52 *A T_{3a}-space ist T_3-space, as well. ([vQ01], p.84)*

PROOF (8.1.52): *Direct proof:*

Precondition: $(X, \mathfrak{T}_X)$ *is a T_{3a}-space,* $A_x \subseteq X$ *closed and* $x \in X \setminus A_x$.

$\Rightarrow\ \forall\ A_x \subseteq X, x \in X \setminus A_x\ \exists\ f : (X, \mathfrak{T}_X) \overset{\mathfrak{T}}{\rightarrow} ([0,1], \mathfrak{T}_{nat,\mathbb{R}|_{[0,1]}}) := (Y, \mathfrak{T}_Y),$

f *continuous* $: f(x) = 0 \wedge f(A_x) = \{1\}.$

$\Rightarrow\ \forall\ A_x \subseteq X, x \in X \setminus A_x\ \exists\ U_1 \subset Y, U_0 \subset Y, f(A_x) \in U_1, f(x) \in U_0 :$

$(U_1 = (\frac{2}{3}, 1]) \wedge (U_0 = [0, \frac{1}{3})) \wedge (U_1 \in \mathfrak{T}_Y) \wedge (U_0 \in \mathfrak{T}_Y)$

$\overset{f\ continuous}{\Rightarrow}\ \forall\ A_x \subseteq X, x \in X \setminus A_x\ \exists\ U_1 \subset Y, U_0 \subset Y, f(A_x) \in U_1, f(x) \in U_0 :$

$(f^{-1}(U_1) = U_A \in \mathfrak{T}_X) \wedge (f^{-1}(U_0) = U_x \in \mathfrak{T}_X) \wedge (U_1 \cap U_0 = \emptyset)$

$\Rightarrow\ \forall\ A_x \subseteq X, x \in X \setminus A_x\ \exists\ U_A, U_x, f^{-1}(U_1) = U_A \in \mathfrak{T}_X,\ f^{-1}(U_0) = U_x \in \mathfrak{T}_X:$

$(U_x \cap U_A = \emptyset) \wedge (x \in U_x) \wedge (A_x \subset U_A)$

$\Rightarrow (X, \mathfrak{T}_X)$ *is T_3-space.*

□

Definition 8.1.53 *T_4 (Tietze) (cf [Lut76], p.19)*

$(X, \mathfrak{T}_X)$ is a topological space. If $A_1, A_2 \subset X$ are closed and disjoint, then there are open sets $O_1, O_2 \subset X$ with $A_1 \subset O_1, A_2 \subset O_2, O_1 \cap O_2 = \emptyset$.

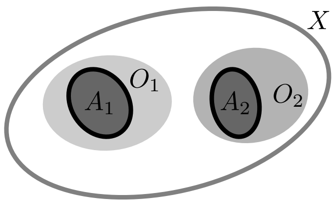
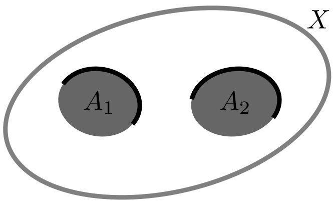


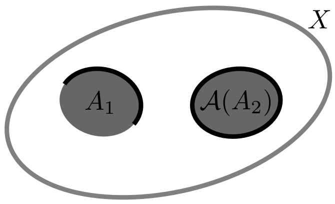
Figure 8.16: T_4-space (Tietze). (Figure generated with *LaTeX*.)

Definition 8.1.54 T_5 *(cf [Kow61], p.65)*

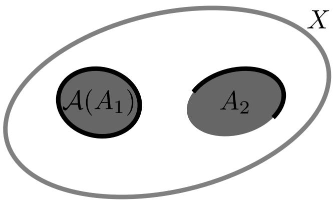
$(X, \mathfrak{T}_X)$ is a topological space. If $A_1 \cap \mathcal{A}(A_2) = \mathcal{A}(A_1) \cap A_2 = \emptyset$, then there are open sets $O_1, O_2 \subset X$ with $A_1 \subset O_1, A_2 \subset O_2, O_1 \cap O_2 = \emptyset$.



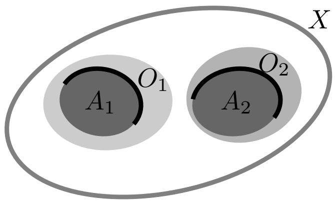
(a) Two disjoint sets (A_1 and A_2).



(b) Two disjoint sets (A_1 and $\mathcal{A}(A_2)$).



(c) Two disjoint sets ($\mathcal{A}(A_1)$ and A_2).



(d) Two disjoint open supersets of the sets A_1 and A_2 (O_1 and O_2).

Figure 8.17: T_5-space. (Figure generated with *LaTeX*.)

Definition 8.1.55 *Regular ([vQ01], p.87)*

The topological space $(X, \mathfrak{T}_X)$ is called regular, if it satisfies T_3 and T_1.

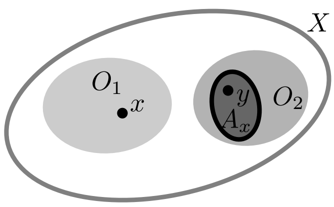


Figure 8.18: Regular space. (Figure generated with *LaTeX*.)

Definition 8.1.56 *Completely Regular ([vQ01], p.87)*

The topological space $(X, \mathfrak{T}_X)$ *is called completely regular, if it satisfies* T_{3a} *and* T_1.

Definition 8.1.57 *Normal ([vQ01], p.87)*

The topological space $(X, \mathfrak{T}_X)$ *is called normal, if it satisfies* T_4 *and* T_1.

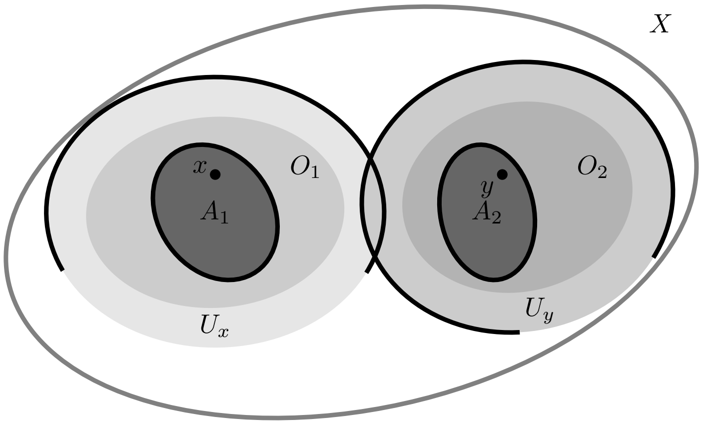


Figure 8.19: Normal space. (Figure generated with *LaTeX*.)

Definition 8.1.58 *Completely Normal ([Kow61], p.65)*

The topological space $(X, \mathfrak{T}_X)$ *is called completely normal, if it satisfies* T_5 *and* T_1.

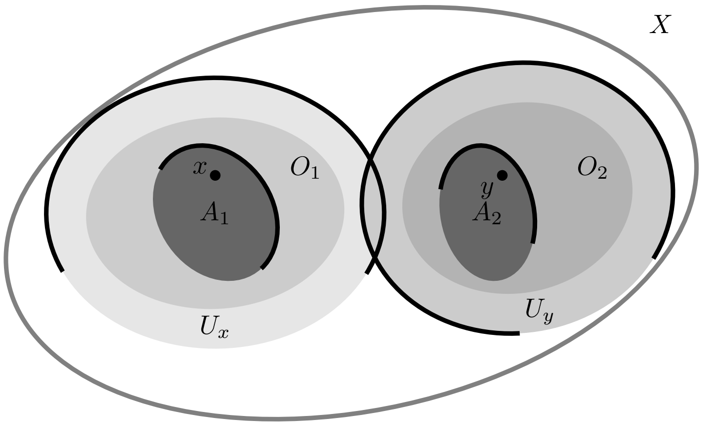


Figure 8.20: Completely normal space. (Figure generated with *LaTeX*.)

Note 8.1.59: *The spaces mentioned in* DEFINITION *8.1.55, 8.1.56, 8.1.57, 8.1.58 are Hausdorff spaces.*

Note 8.1.60: *The following relationship results between the T_i-spaces:*

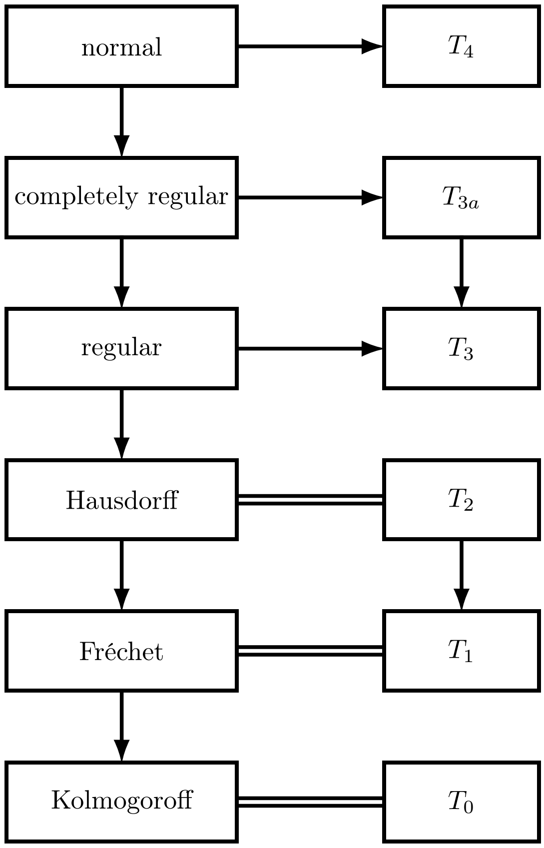


Figure 8.21: Relationship between the T_i-spaces. (Figure generated with *LaTeX*. Compare: [vQ01], p.87.)

For the SAGU, the separation axioms T_{3a} for separating a measurable set (Definition 8.2.28) from a point and T_3 to apply the measuring tools described in more detail in Section 8.2 are used. T_2 is required for separating GUI properties (e.g. GUI elements or GUI element objects) by open sets.

8.1.2 Filter

The well-known formulation of continuity through the convergence of sequences from the analysis can be modified only strongly transferred to the topology, since the sequence concept is inadequate ([Oss09], p. 44).
Therefore, hereinafter, the term of the filter is introduced. Filters are set systems. Convergence of sequences can inter alia be defined with neighbourhoods and index barriers. The coherence between neighbourhoods and sequences regarding to the convergence in metric spaces is in general replaced by the coherence between neighbourhoods and filters in topological spaces. The neighbourhoods of a point x form a filter (neighbourhood filter, $\mathfrak{F}_U(x)$). A filter is convergent to x_0, if it is finer than the neighbourhood filter. This relation is clarified in the following. Convergence and continuity are important properties which play an important role within the following, especially within the SAGU

(Chapter 9).

8.1.2.1 Filter and neighbourhoods

Definition 8.1.61 *Filter (cf [vQ01], p.77)*
$X \neq \emptyset$ is an arbitrary set.
A system $\mathfrak{F}$ of subsets of X is called filter in X, if the following four properties are satisfied:

1. *$\mathfrak{F} \neq \emptyset$ (a filter is not empty)*
2. *$F \in \mathfrak{F} \Rightarrow F \neq \emptyset$ (the sets in the filter are not empty)*
3. *$F_1, F_2 \in \mathfrak{F} \Rightarrow F_1 \cap F_2 \in \mathfrak{F}$ (intersection stability)*
4. *$F_1 \in \mathfrak{F}, F_1 \subset F_2 \subset X \Rightarrow F_2 \in \mathfrak{F}$ (superset stability)*

Example 8.1.62:

- *If it is $x \in X$,*
$$\mathfrak{F} := \{F \subset X \mid x \in F\}$$
is called principal filter generated by x.
- *The Fréchet filter[4] belongs to a sequence (x_n) in X :*
$$\mathfrak{F} := \{F \subset X \mid \exists\ n_0 \in \mathbb{N}\ \forall\ n \geq n_0 : x_n \in F\}.$$
- *If $(X, \mathfrak{T}_X)$ is a topological space, the neighbourhoods of a point $x \in X$ form the neighbourhood filter $\mathfrak{F}_U(x)$ generated by x.*
- *In $\mathbb{R}$ and $\mathbb{R}^n$ respectively*
$$\mathfrak{F} := \{F \subset \mathbb{R} \mid \exists\ c \in \mathbb{R} : [c, \infty) \subset F\}$$
and
$$\mathfrak{F} := \{F \subset \mathbb{R}^n \mid \exists\ c > 0\ \forall\ x \in \mathbb{R}^n : |x| \geq c \Rightarrow x \in F\}$$
respectively constitute filters, that are convergent "to $+\infty$" and "to $-\infty$" respectively. (cf [vQ01], p.77f and [Kow61], p.29f).

Note 8.1.63: *One can see from* Example *8.1.62 that many filters can be represented by a system of all supersets of the sets of a different set system.*

[4]This shows clearly how filters can take over the properties of sequences.

Definition 8.1.64 *Filter Base (cf [Oss09], p.44f)*
A filter base $\mathfrak{B}_{\mathfrak{F}}$ in X is a set of subsets of X with:

1. $\mathfrak{B}_{\mathfrak{F}} \neq \emptyset$
2. $B \in \mathfrak{B}_{\mathfrak{F}} \Rightarrow B \neq \emptyset$
3. *C is a set.* $B_1, B_2 \in \mathfrak{B}_{\mathfrak{F}} \Rightarrow \ \exists \ C \in \mathfrak{B}_{\mathfrak{F}} : C \subset (B_1 \cap B_2)$.

Definition 8.1.65 *The Generated Filter generated by $\mathfrak{B}_{\mathfrak{F}}$ (cf [Oss09], p.45)*
The system

$$\mathfrak{F}_{Erz}(\mathfrak{B}_{\mathfrak{F}}) := \{F \subset X \mid \ \exists \ B \in \mathfrak{B}_{\mathfrak{F}} : B \subset F\}$$

is a filter in X, the generated filter generated by $\mathfrak{B}_{\mathfrak{F}}$.

Note 8.1.66: *Properties satisfied by a filter generated by a filter base are attributed to this filter base.*

Note 8.1.67: *The image of a filter base $\mathfrak{B}_{\mathfrak{F}}$ in X under the mapping $f : X \to Y$ is a filter base*

$$f(\mathfrak{B}_{\mathfrak{F}}) = \{f(B) \mid B \in \mathfrak{B}_{\mathfrak{F}}\}$$

in Y.

Note 8.1.68: *In general, the image of a filter is not a filter.*

Definition 8.1.69 *Image Filter (cf [Oss09], p.44)*
The image of a filter $\mathfrak{F}$ in X under f must not be a filter, but it is a filter base, and the generated filter generated by this filter base $\mathfrak{F}_{Erz}(f(\mathfrak{F}))$ is called image filter generated by $\mathfrak{F}$ under f.

Definition 8.1.70 *Finer (cf [Oss09], p.44)*
A filter $\mathfrak{F}_1$ in X is called finer than the filter $\mathfrak{F}_2$ in X, if it is $\mathfrak{F}_2 \subset \mathfrak{F}_1$.

Definition 8.1.71 *Coarser (cf [Oss09], p.44)*
A filter $\mathfrak{F}_2$ in X is called coarser than a filter $\mathfrak{F}_1$ in X, if it is $\mathfrak{F}_2 \subset \mathfrak{F}_1$.

Definition 8.1.72 *Ultra Filter (cf [vQ01], p.77)*
If there is no other finer filter in X generated by $\mathfrak{F}$ different from $\mathfrak{F}$, $\mathfrak{F}$ is called ultra filter in X.

Note 8.1.73:

- *Each filter $\mathfrak{F}$ is contained in an ultra filter. ([vQ01], p.78).*
- *Each principal filter generated by a point $x \in X$ is an ultra filter. (cf [vQ01], p.78).*

8.1.2.2 Convergence

The continuity of a mapping between topological spaces can generally not be characterised with convergent sequences.[5] Filter form an elegant replacement for the sequences.[6] The SAGU described in CHAPTER 9 works like a filter. If the filter is convergent, the limit determines the GUI which is proposed as suitable for the user.

Definition 8.1.74 *Convergence (cf [vQ01], p.78 and [Jae99], p.209)*
A filter $\mathfrak{F}$ in a topological space X is convergent to a point $x \in X$, if $\mathfrak{F}$ is finer than the neighbourhood filter $\mathfrak{F}_U(x)$ (compare EXAMPLE *8.1.62) generated by x.*
Or:
If each neighbourhood of x lies in $\mathfrak{F}$.
One then writes $\mathfrak{F} \to x$.

Example 8.1.75: $(x_n)_{n\geq 1}$ *is a sequence in X. A sequence (x_n) in X belongs to its Fréchet filter:*

$$\mathfrak{F} := \{F \subset X \mid \exists\ n_0 \in \mathbb{N}\ \forall\ n \geq n_0 : x_n \in F\}.$$

The filter is convergent to α, if and only if the sequence is convergent to α. ([Jae99], p.209)

If the SAGU represented by a filter is not convergent, but a refinement of this filter, the point of adherence determines the GUI assigned to a user. In a compact space, each filter has a point of adherence (THEOREM 8.1.86). Thus the space the SAGU operates on should be a compact space, as well, to guarantee the assignment of a GUI to a user.

Definition 8.1.76 *Point of Adherence (cf [vQ01], p.78f)*
x is called point of adherence of $\mathfrak{F}$, if there is a refinement of $\mathfrak{F}$ convergent to x.
Or:
If it is $F \in \mathfrak{F}, U_x \in \mathfrak{F}_U(x) \Rightarrow F \cap U_x \neq \emptyset$.[7]

[5] This common method of differential calculus and integral calculus fails if the 1st countability axiom is not satisfied.
[6] Nets are another replacement for sequences.
[7] This condition is necessary; if it is satisfied, $\{F \cap U_x \mid F \in \mathfrak{F}, U_x \in \mathfrak{F}_U(x)\}$ is a filter base generating a filter, which is finer than $\mathfrak{F}$ and than $\mathfrak{F}_U(x)$.

The more information we have about the user, the precise the result and thus the assigned GUI is. One information, which is always available about the user is the user's geo-location, which can be determined with the digital device of the user. If we have no information about the user (except the geo-location) our situation could be comparable to having the indiscreet topology on the space containing the membership mapping describing the quality of a GUI for a user. With THEOREM 8.1.77 each GUI would be proposed as suitale for the user. Thus, we had a variety of options. In order to get a handle on this, an initial GUI will be provided to users we do not have any information about except the geo-location. The initial GUI is not a global GUI, because it is GIS-tailored. We refer to the information about users in similar situations, that is, users located at the same geo-location to assign a GUI to the user that worked out fine for users in similar situations. The GUI adapts then by replacing, adding or deleting GUI element objects. By doing this, the variety of options becomes smaller and the best fitting GUI for the user is isolated. In CHAPTER 9 this is addressed in again.

Theorem 8.1.77 *In the indiscreet topology $\mathfrak{T}_{ind}$, every filter is convergent to every point $x \in X$.*

PROOF (8.1.77): *Direct proof:*
Precondition: X is equipped with the indiscreet topology.
A Filter $\mathfrak{F}$ on X is convergent to every point in X, because X is the only neighbourhood of every point $x \in X$. Thus it is

$$\forall\ x \in X : \mathfrak{F}_U(x) = \{X\} \overset{\text{DEF. } 8.1.61\ 4.}{\Rightarrow} \forall\ \mathfrak{F}_X \text{ Filter on } X : \mathfrak{F}_U(x) \subseteq \mathfrak{F}_X$$
$$\Rightarrow\ \forall\ x \in X\ \forall\ \mathfrak{F}_X \text{ on } X : \mathfrak{F}_X \to x.$$

□

If the filter representing the SAGU was operating on a T_2-space, we knew with the following theorem, that the filter representing the system was convergent to exactly one point, i.e. exactly one GUI would be would be proposed as suitable for the user.

Theorem 8.1.78 *$(X, \mathfrak{T}_X)$ is a T_2-space $\Leftrightarrow$ every filter $\mathfrak{F}$ is convergent to one point at most. ([vQ01], p.85)*

PROOF (8.1.78): *Proof by contradiction:*
"$\Rightarrow$":
Precondition: Let $(X, \mathfrak{T}_X)$ be a T_2-space.
Hypothesis: There is a filter $\mathfrak{F}$ on X with $\mathfrak{F} \to x$ and $\mathfrak{F} \to y$, $x \neq y$, $x, y \in X$.
With DEFINITION *8.1.74 it is for the two neighbourhood systems $\mathfrak{F}_U(x)$ of x and $\mathfrak{F}_U(y)$ of y:*
$\mathfrak{F}_U(x) \subseteq \mathfrak{F} \wedge \mathfrak{F}_U(y) \subseteq \mathfrak{F}$.

$\forall\ x, y \in X, x \neq y\ \exists\ U_x, U_y : U_x \in \mathfrak{F}_U(x)$ and $U_y \in \mathfrak{F}_U(y)$ with $U_x \cap U_y = \emptyset$, because X is Hausdorff.
Then it is $U_x, U_y \in \mathfrak{F}$, but $U_x \cap U_y = \emptyset \notin \mathfrak{F}$.
And this is a contradiction to Definition *8.1.61 3. (3. filter axiom).*
"$\Leftarrow$":
Precondition: Every filter $\mathfrak{F}$ on X is convergent to one point at most.
Hypothesis: $\exists\ x_0, y_0 \in X, x_0 \neq y_0\ \forall\ U_{x_0} \in \mathfrak{F}_U(x_0), U_{y_0} \in \mathfrak{F}_U(y_0) : U_{x_0} \cap U_{y_0} \neq \emptyset$.
$\overset{\text{Def. } 8.1.61\ 3.}{\Rightarrow}\ \exists\ x_0, y_0 \in X, x_0 \neq y_0 :$
$\mathfrak{B}_{\mathfrak{F}} = \{U_{x_0} \cap U_{y_0} \mid U_{x_0} \in \mathfrak{F}_U(x_0), U_{y_0} \in \mathfrak{F}_U(y_0) \wedge U_{x_0} \cap U_{y_0} \neq \emptyset\}$ *filterbase and generates therefore a filter $\mathfrak{F} = \mathfrak{F}_{Erz}(\mathfrak{B}_{\mathfrak{F}})$.*
Because of $U_{x_0} \cap U_{y_0} \subseteq U_{x_0} \wedge U_{x_0} \cap U_{y_0} \subseteq U_{y_0}$ it is:
$\exists\ x_0, y_0 \in X, x_0 \neq y_0 : \mathfrak{F}_U(x_0) \subseteq \mathfrak{F}_{Erz}(\mathfrak{B}_{\mathfrak{F}}) \wedge \mathfrak{F}_U(y_0) \subseteq \mathfrak{F}_{Erz}(\mathfrak{B}_{\mathfrak{F}})$
$\Rightarrow\ \exists\ x_0, y_0 \in X, x_0 \neq y_0 : \mathfrak{F}_{Erz}(\mathfrak{B}_{\mathfrak{F}}) \to x_0 \wedge \mathfrak{F}_{Erz}(\mathfrak{B}_{\mathfrak{F}}) \to y_0$.
And this is a contradiction to the Precondition.

□

Theorem 8.1.79 *Continuous in x ([Oss09], p.45)*
$(X, \mathfrak{T}_X), (Y, \mathfrak{T}_Y)$ are topological spaces, $f : (X, \mathfrak{T}_X) \overset{\mathfrak{T}}{\to} (Y, \mathfrak{T}_Y)$ is a mapping, $x \in X, y := f(x)$.
f is continuous in x, if and only if the image filter $\mathfrak{F}_{Erz}(f(\mathfrak{F}))$ of each filter $\mathfrak{F}$ convergent to x is convergent to y.

Proof (8.1.79): *Direct proof:*
"$\Rightarrow$":
Precondition: f is continuous in x and $\mathfrak{F}$ is a filter with $\mathfrak{F} \to x$.
$\overset{\text{Def. } 8.1.74}{\Rightarrow} \mathfrak{F}_U(x) \subset \mathfrak{F}$ *(*)*.
Let $U_{f(x)} \in \mathfrak{F}_U(f(x))$ be arbitrary.
$\overset{\text{Def. } 8.1.28}{\Rightarrow}\ \forall\ U_{f(x)} \in \mathfrak{F}_U(f(x)) : U_x = f^{-1}(U_{f(x)}) \in \mathfrak{F}_U(x)$ *(**)*
$\Rightarrow\ \forall\ U_{f(x)} \in \mathfrak{F}_U(f(x))\ \forall\ \mathfrak{F}$ *filter*, $\mathfrak{F} \to x : U_{f(x)} \in \mathfrak{F}_{Erz}(f(\mathfrak{F}))$
$\Rightarrow\ \forall\ U_{f(x)} \in \mathfrak{F}_U(f(x))\ \forall\ \mathfrak{F}$ *filter*, $\mathfrak{F} \to x :$
$U_{f(x)} = f(f^{-1}(U_{f(x)})) \overset{(**)}{\in} \mathfrak{F}_U(f(x)) \overset{(*)}{\subseteq} f(\mathfrak{F}) \overset{\text{Def. } 8.1.65}{\subseteq} \mathfrak{F}_{Erz}(f(\mathfrak{F}))$
$\Rightarrow\ \forall\ \mathfrak{F}$ *filter*, $\mathfrak{F} \to x : \mathfrak{F}_U(f(x)) \subset \mathfrak{F}_{Erz}(f(\mathfrak{F}))$
$\Rightarrow\ \forall\ \mathfrak{F}$ *filter*, $\mathfrak{F} \to x : \mathfrak{F}_{Erz}(f(\mathfrak{F})) \to f(x)$.
"$\Leftarrow$":
Precondition: If $\mathfrak{F}$ is any filter with $\mathfrak{F} \to x$, then it is for $\mathfrak{F}_{Erz}(f(\mathfrak{F}))$: $\mathfrak{F}_{Erz}(f(\mathfrak{F})) \to f(x)$.
$\overset{\text{Precond.}}{\Rightarrow}\ \forall\ \mathfrak{F}$ *filter*, $\mathfrak{F} \to x : \mathfrak{F}_U(f(x)) \subseteq \mathfrak{F}_{Erz}(f(\mathfrak{F}_U(x)))$
$\Rightarrow\ \forall\ U_{f(x)} \in \mathfrak{F}_U(f(x)) : U_{f(x)} \in \mathfrak{F}_{Erz}(f(\mathfrak{F}_U(x)))$
$\Rightarrow\ \forall\ U_{f(x)} \in \mathfrak{F}_U(f(x)) : \mathfrak{B}_{\mathfrak{F}_{Erz}(f(\mathfrak{F}_U(x)))} = \{f(U_x) \mid U_x \in \mathfrak{F}_U(x)\}$ *filterbase of* $\mathfrak{F}_{Erz}(f(\mathfrak{F}_U(x)))$
$\Rightarrow\ \forall\ U_{f(x)} \in \mathfrak{F}_U(f(x))\ \exists\ U_x \in \mathfrak{F}_U(x) : f(U_x) \subseteq U_{f(x)}$

$\Rightarrow f$ *is continuous in* x.

□

8.1.2.3 Compactness

The expansion of the definition of the concept of compactness to general topological spaces, which is known from real analysis, first lead to various types of compact terms in the early decades of this century. Even today the concept of compactness is not completely uniform. ([Oss09], p. 58). Compactness is required to define the basics of measures in Subsection 8.2.1.

Definition 8.1.80 *Open Cover (cf [Jae99], p.24)*
$\ddot{\mathfrak{U}}$ *is called open cover of* $(X, \mathfrak{T}_X)$, *if it is* $\ddot{\mathfrak{U}} \subset \mathfrak{T}_X$ *and* $\bigcup\limits_{U \in \ddot{\mathfrak{U}}} U = X$.

Definition 8.1.81 *Compactness ([Jae99], p.24)*
A subset K *of a topological space is called compact, if each open cover has a finite sub-cover.*

Note 8.1.82: *For the compactness of* X *it is required, that there always exists a finite subset* $\ddot{\mathfrak{U}}_1 \subset \ddot{\mathfrak{U}}$, *that covers* X *as well.*[8]

The following Corollary 8.1.84 and Note 8.1.85 play an important role in the approximation process of the membership mapping of a user, see Example 9.0.5.

Theorem 8.1.83 *The closed subsets of a compact set are compact. (cf [Kow61], p.83).*

Proof (8.1.83): *Direct proof:*
Precondition: Let $(X, \mathfrak{T}_X)$ *be a topological space and let* $K \subset X$ *be a compact set and let* $A \subset K$ *be a closed set.*
Let $\bigcup\limits_{i \in I} U_i \supset A$ *be an open cover of* A.
$\Rightarrow (\bigcup\limits_{i \in I} U_i) \cup (X \backslash A) \supset K$ *is an open cover of* K *with* $U_0 := X \backslash A \in \mathfrak{T}_X$ *and* A *closed.*
Define $I_0 := I \cup \{0\}$, *thus* $0 \notin I$, *and* $T := T_0 \backslash \{0\}$ *with* $|T| \leq |T_0|$, $T \subset T_0 \subset I_0, |T| < \infty$, $|T_0| < \infty$.
Because K *is compact, there exists a finite sub-cover* $\bigcup\limits_{i \in T_0} U_i \supset K \supseteq A$.
$\Rightarrow \bigcup\limits_{i \in T} U_i \supset A$, *because* $U_0 \cap A = \emptyset$.
Thus A *is compact.*

□

Corollary 8.1.84 *The closed subsets of a compact space are compact. (cf [Kow61], p.83).*

[8] X must not be Hausdorff.

Note 8.1.85: *Each finite topological space is compact, because there exist only principal filter in a finite set. (cf [Kow61], p.81).*

If the SAGU represented by a filter is not convergent, but a refinement of this filter, the point of adherence determines the GUI assigned to a user. In a compact space, each filter has a point of adherence, as the following theorem, THEOREM 8.1.86 signifies.

Theorem 8.1.86 $(X, \mathfrak{T}_X)$ *is a topological space. The following is equivalent:*

1. *X is compact.*
2. *Each ultra filter in X converges.*
3. *Each filter in X has a point of adherence.*

PROOF (8.1.86):

- *1.⇒ 2.: Proof by contradiction:*
  Precondition: X is compact.
  Hypothesis: No $x \in X$ is the limit of the ultra filter $\mathfrak{F}$ in $(X, \mathfrak{T}_X)$.
  Then it is $\forall\ x \in X\ \exists\ U_x \in \mathfrak{F}_U(x), U_x \in \mathfrak{T}_X : U_x \notin \mathfrak{F}$. A finite number of $U_x \in \mathfrak{F}_U(x)$ then cover whole $(X, \mathfrak{T}_X)$, thus $\forall\ U_x : U_x \notin \mathfrak{F}$ and it follows from this $X \notin \mathfrak{F}$, which is a contradiction to the fact that $\mathfrak{F}$ is an ultra filter in X.

- *2.⇒ 3.: Direct proof:*
  Holds because of NOTE *8.1.73.*

- *3. ⇒ 1.: Proof by contradiction:*
  Precondition: Each filter in X has a point of adherence.
  $\ddot{\mathfrak{U}}_X$ is an open cover of X.
  Hypothesis: X is not compact, i.e. $\ddot{\mathfrak{U}}_X$ does not contain a finite sub-cover.
  *Then, the complements of the finite unifications of sets in x would form a filter base in X with sets of $\ddot{\mathfrak{U}}_X$. The generated filter $\mathfrak{F}$ generated by the latter would have a point of adherence x. There was a $\ddot{U} \in \ddot{\mathfrak{U}}_X$ for x with $x \in \ddot{U} \in \mathfrak{F}_U(x)$, and it would follow $X \setminus \ddot{U} \in \mathfrak{F}$, so $\ddot{U} \cap (X \setminus \ddot{U}) \neq \emptyset$ (*DEFINITION *8.1.76) and this is not possible.*

□

8.1.3 Locally compact spaces

In the mathematical branch of topology, locally compact spaces are a class of topological spaces that satisfy a certain local finiteness condition. Locally compactness is required to define the basics of measures in SUBSECTION 8.2.1.

Definition 8.1.87 *Carrier ([vQ01], p.101)*
If $f : (X, \mathfrak{T}_X) \stackrel{\mathfrak{T}}{\to} (\mathbb{C}, \mathfrak{T}_{nat})$ *is a continuous mapping, the closed set*

$$Tr(f) := \mathcal{A}(\{x \in X \mid f(x) \neq 0\})$$

is called carrier of f*.*

Definition 8.1.88 *Locally Compact (cf [HR63], p.11)*
A topological space $(X, \mathfrak{T}_X)$ *is locally compact, if every point* $x \in X$ *has a neighbourhood* $U_x \subset X$ *such that* $\mathcal{A}(U_x)$ *is compact.*

Lemma 8.1.89 *Let* $(X, \mathfrak{T})$ *be a locally compact* T_3*-space. Then there exists for all compact sets* $K \subset X$ *and for each neighbourhood* U_K *of* K *a continuous mapping*

$$g : (X, \mathfrak{T}_X) \stackrel{\mathfrak{T}}{\to} ([0, 1], \mathfrak{T}_{rel})$$

with $g(K) = \{1\}$ *and with* $Tr(g) \subset U_K$*,* $Tr(g)$ *compact. (cf [Kow61],p.128)*

The proof can be found in [Kow61],p.128.

Note 8.1.90: LEMMA *8.1.89 is used within the proof of* THEOREM *8.2.9.*

In the following context the separation axiom T_3 plays an important role especially for measure theory, SECTION 8.2, because there locally compact T_3 spaces are considered. THEOREM 8.1.93 is necessary, because the separation axiom T_{3a} plays an important role for the development of an SAGU described in CHAPTER 9. T_{3a} enables the separation of a measurable set (DEFINITION 8.2.28) and a point. This is important for the evaluation of a set of collected data (about a user) and the determination of a membership mapping representing the user's needs. With respect to the GUI, the measurable sets are the GUI elements and the points are the associated GUI element objects. With T_{3a}, measurable sets can be identified. Those sets can be fuzzy, i.e represented via a fuzzy mapping. Therewith fuzzy operations (e.g. the fuzzy-AND, p.71, the fuzzy-OR, p.70 or the fuzzy-NOT, p.72) can be conducted. Fuzzy mappings are used to represent vague statements of a user and they are used to control the GUI.

Theorem 8.1.91 *Each locally compact space is regular. ([vQ01], p.109)*

The proof can be found in [vQ01], p.109f.

Theorem 8.1.92 *Each closed set of a locally compact space is locally compact. (cf [Kow61], p.90).*

The proof can be found in [Kow61], p.90.

Theorem 8.1.93 $(X, \mathfrak{T}_X)$ *is a locally compact topological space.*
Then it is: $(X, \mathfrak{T}_X)$ T_3*-space* $\Leftrightarrow$ $(X, \mathfrak{T}_X)$ T_{3a}*-space. (cf [Kow61], p.128.)*

PROOF (8.1.93): *Direct proof:*
"⇐":
Precondition: Let $(X, \mathfrak{T}_X)$ *be a locally compact topological* T_{3a}*-space.*
$(X, \mathfrak{T}_X)$ T_{3a}*-space* $\overset{\text{THEOREM 8.1.52}}{\Rightarrow}$ $(X, \mathfrak{T}_X)$ T_3*-space.*
"⇒":
Precondition: Let $(X, \mathfrak{T}_X)$ *be a locally compact topological* T_3*-space.*
Let A *be a closed set and* $x \in X \backslash A$.
With THEOREM *8.1.91,* THEOREM *8.1.44 and* THEOREM *8.1.92 we know, that each one-pointed set is compact. Thus, we can set* $K = \{x\}$ *and* $U_K = X \backslash A$ *in* LEMMA *8.1.89.*
Then exists a mapping

$$g : (X, \mathfrak{T}_X) \overset{\mathfrak{T}}{\rightarrow} ([0, 1], \mathfrak{T}_{rel})$$

with $g(x) = 1$ *and* $g(A) = \{0\}$*. The mapping defined by*

$$\begin{aligned} f :& (X, \mathfrak{T}_X) \overset{\mathfrak{T}}{\rightarrow} ([0, 1], \mathfrak{T}_{rel}) \\ & x \mapsto 1 - g(x) \end{aligned}$$

fulfils the property of DEFINITION *8.1.51 (*T_{3a}*).*
(cf [Kow61],p.128)

□

8.2 Measure Theory

The terms "area of a flat or curved surface", "the volume of a body" first appear self-evident. This is probably the reason why the inherent fundamental problems of these concepts, including the question of what is meant by the surface area of any area or volume of any body, were first detected and clearly resolved relatively late. *Archimedes* succeeded in 3rd Century BC, to determine the surface and volume of the ball. Not until the 17th Century the development and perfection of the calculus as

a new method to treat a variety of specific problems was in the focus. Finally, in the 19th Century question of a clear conceptual version of the foundations of analysis occurred. (cf [Els09], p.1f).
The concept of measurement of quantities is fundamental, both in daily life (length, surface area, volume, weight) and in the natural sciences (electric charge, magnetic pole strength, etc.).[9] ([Els09], p.1).
The "measurement-problem" is the problem of the formulation of the concept of the volume of a body. An appropriate framework for formulating the question of the appropriate term of the volume of a subset of $\mathbb{R}^p$, is the set theory created by *Cantor* (1845-1918). Since the beginnings of set theory, mathematicians have been concerned with this issue, and it was the end of the 19th Century when, several, sometimes different answers were proposed. Only *É. Borel* (1871-1956) and *H. Lebesgue* (1875-1941) provided a satisfactory answer. *Lebesgue* calls this problem the measurement problem:

> "We want to map to every bounded subset E of the real axis a non-negative real figure $m(E)$, which we call the measure of E, so that the following conditions are satisfied:
>
> 1. Each two congruent sets have the same measure.
> 2. The unification of a finite or countable infinite number of sets, of that each two do not contain a common point, has the sum of the measures as measure.
> 3. The measure of the unit interval $[0, 1]$ is 1."

Lebesgue explicitly pointed out that he had solved this problem only for a certain class of sets, which he calls measurable quantities. This restriction was absolutely necessary, as a solution of this problem does not exist for all subsets of $\mathbb{R}$ (power set). Exceptional is that in his second condition *Lebesgue* allowed finite or countably infinite unifications of sets. From *Borel* came the idea to demand the additivity of the measure for countable disjoint unions, as well. This idea was of great importance for the further development of the measure theory and integration theory. The transition from finite to countable had the consequence that the *Lebesgue measure theory and integration theory* is superior crucial to the earlier theory of *Plato* and *Jordan*. From the perspective of modern mathematics, the *Lebesgue measure* is the natural concept of volume and surface area. This concept is the end product of a whole series of ideas, trying to grasp the concept of volume mathematically exact. Only with the *Lebesgue measure*, this process can be considered as completed. The *Lebesgue measure* assigns an area not only to simple geometric objects, but also to more general quantities, including all open and closed sets. $\mathscr{L}^p$-spaces are special Banach spaces, that are produced by spaces of p-integrable mappings. The *"L"* in the expression goes back to the French mathematician *Henri Léon Lebesgue*, because these spaces are defined on the *Lebesgue integral*. Sometimes they are therefore also called *Lebesgue spaces*. (cf [Els09], p. 2ff).

[9] "La notion de mesure des grandeurs est fondamentale, aussi bien dans la vie de tous les jours (longeurs, surface, volume, poids) que dans la science expérimentale (charge électrique, masse magnétique, etc.)." (*N. Bourbaki*)

8.2.1 Measures

In the following the basics for measures are developed. The measures are noted depending the kind of mappings, which are measured with the measure:

- $\mu_{\mathfrak{K}}$ is used to measure mappings in $\mathfrak{K}(X, \mathfrak{T}_X)$.
- μ^* is used to measure mappings in $\mathfrak{I}(X, \mathfrak{T}_X)$.
- μ_L, the Lebesgue-measure, is used to measure mappings in $\mathscr{L}^1((X, \mathfrak{T}_X), \mu_L)$.

Within the SAGU, Chapter 9, μ_L is inter alia used within the NN2, Section 9.5 and within the topological space $(X_\Gamma, \mathfrak{T}_\Gamma)$ used for the mathematical description of the SAGU, see p.148.

Definition 8.2.1 $\mathfrak{K}((X, \mathfrak{T}_X))$ *(cf [Die75], p.112)*
$(X, \mathfrak{T}_X)$ is a locally compact T_3-space.
The space $\mathfrak{K}((X, \mathfrak{T}_X))$ is the space of all continuous mappings $f : (X, \mathfrak{T}_X) \to (\mathbb{C}, \mathfrak{T}_{nat})$, whose carrier $Tr(f)$ is compact.

Note 8.2.2: *The space $\mathfrak{K}((X, \mathfrak{T}_X))$ forms under the point-wise declared algebraic operations a $\mathbb{C}$-vector space. (cf [Die75], p.112)*

Definition 8.2.3 $\mathfrak{K}_+((X, \mathfrak{T}_X))$, *Positive Mapping (cf [HR63], p.120)*
With $\mathfrak{K}_+((X, \mathfrak{T}_X))$ one denotes the set of all $f \in \mathfrak{K}((X, \mathfrak{T}_X))$ with $f(x) \geq 0$ for all $x \in X$. The mappings $f \in \mathfrak{K}_+((X, \mathfrak{T}_X))$ are called positive.

Definition 8.2.4 $\|f\|_\infty$ *(cf [HR63], p.119)*

$$\|f\|_\infty := \sup |f| = \sup\{|f(x)| \mid x \in X\}$$

for all $f \in \mathfrak{K}((X, \mathfrak{T}_X))$.

Note 8.2.5: *$\|f\|_\infty$ is always finite, $\|\cdot\|_\infty$ is a norm on $\mathfrak{K}((X, \mathfrak{T}_X))$. (cf [HR63], p.119).*

Note 8.2.6: *With f the mappings[10] $|f|, Re(f)$ and $Im(f)$ lie in $\mathfrak{K}((X, \mathfrak{T}_X))$, as well, they are defined by*

$$\begin{aligned} |f| :& (X, \mathfrak{T}_X) \to (\mathbb{C}_0^+, \mathfrak{T}_{nat}) \\ & x \mapsto |f(x)|, \end{aligned}$$

[10] The mappings $|\cdot|$, Re = form real part, Im = form imaginary part, are continuous.

$$|f| :(X, \mathfrak{T}_X) \to (\mathbb{R}, \mathfrak{T}_{nat})$$
$$x \mapsto Re(f(x)),$$

$$|f| :(X, \mathfrak{T}_X) \to (\mathbb{R}, \mathfrak{T}_{nat})$$
$$x \mapsto Im(f(x)).$$

Each $f \in \mathfrak{K}((X, \mathfrak{T}_X))$ is the linear combination

$$f = Re(f) + iIm(f)$$

of two real-valued mappings from $\mathfrak{K}((X, \mathfrak{T}_X))$. With f the complex conjugate mapping

$$\bar{f} = Re(f) - iIm(f)$$

lies in $\mathfrak{K}((X, \mathfrak{T}_X))$. If $f \in \mathfrak{K}((X, \mathfrak{T}_X))$ is real-valued, then

$$f^+ := \max(f, 0) = \frac{1}{2}(|f| + f), f^- := \max(-f, 0) = \frac{1}{2}(|f| - f)$$

lie in $\mathfrak{K}((X, \mathfrak{T}_X))$, and it is

$$f = f^+ - f^-, |f| = f^+ + f^-.$$

Hence, each mapping $f \in \mathfrak{K}((X, \mathfrak{T}_X))$ is a linear combination of four positive mappings. (cf [Die75], p.116f)

In the following, the (Complex Radon) Measure is introduced.

Lemma 8.2.7 *It is $f \in \mathfrak{K}((X, \mathfrak{T}_X))$.*
Let $\alpha f = g_1 + ig_2$ with real-valued $g_1, g_2 \in \mathfrak{K}((X, \mathfrak{T}_X))$ and $\alpha \in \mathbb{C}$ with $|\alpha| = 1$. Then it is $g_1 \leq |\alpha f|$.

Proof (8.2.7): *Direct proof:*
Precondition: It is $f \in \mathfrak{K}((X, \mathfrak{T}_X))$. Let $\alpha f = g_1 + ig_2$ with real-valued $g_1, g_2 \in \mathfrak{K}((X, \mathfrak{T}_X))$ and $\alpha \in \mathbb{C}$ with $|\alpha| = 1$.
We have to show the following: $\forall\ f \in \mathfrak{K}((X, \mathfrak{T}_X))\ \forall\ x \in X : g_1(x) \leq |\alpha f(x)|$.
Let $\forall\ f \in \mathfrak{K}((X, \mathfrak{T}_X))\ \forall\ x \in X : \alpha f(x) = \underbrace{g_1(x)}_{=Re(\alpha f(x))} + i \underbrace{g_2(x)}_{Im(\alpha f(x))}$
$\overset{Pythagorean\ Theorem}{\Rightarrow} \forall\ f \in \mathfrak{K}((X, \mathfrak{T}_X))\ \forall\ x \in X :$
$|\alpha f(x)| = \sqrt{g_1(x)^2 + g_2(x)^2} \geq \sqrt{g_1(x)^2} = |g_1(x)| \wedge g_2(x)^2 \geq 0$
$\Rightarrow \forall\ f \in \mathfrak{K}((X, \mathfrak{T}_X))\ \forall\ x \in X : |\alpha f(x)| \geq |g_1(x)| \geq g_1(x).$

□

Theorem 8.2.8 *$\mu_{\mathfrak{K}} : \mathfrak{K}((X, \mathfrak{T}_X)) \to \mathbb{C}$ is a positive linear form.*
For all $f \in \mathfrak{K}((X, \mathfrak{T}_X))$ it is $|\mu_{\mathfrak{K}}(f)| \le \mu_{\mathfrak{K}}(|f|)$. (cf [HR63], p.120).

PROOF (8.2.8): *Direct proof:*
Precondition: $\mu_{\mathfrak{K}} : \mathfrak{K}((X, \mathfrak{T}_X)) \to \mathbb{C}$ is a positive linear form.
With LEMMA *8.2.7 exists an $\alpha \in \mathbb{C}$ with $|\alpha| = 1$. Decompose $\alpha f = g_1 + ig_2$ with real-valued $g_1, g_2 \in \mathfrak{K}((X, \mathfrak{T}_X))$. Then $\mu_{\mathfrak{K}}(g_1), \mu_{\mathfrak{K}}(g_2)$ are real-valued, because $\mu_{\mathfrak{K}}$ is positive, and with* LEMMA *8.2.7 it is $g_1 \le |g_1| \le |\alpha f|$, so*
$0 < |\mu_{\mathfrak{K}}(f)| = \frac{|\mu_{\mathfrak{K}}(f)|^2}{|\mu_{\mathfrak{K}}(f)|} = \underbrace{\frac{\overline{\mu_{\mathfrak{K}}(f)}}{\mu_{\mathfrak{K}}(f)}}_{=\alpha} \mu_{\mathfrak{K}}(f) = \alpha \mu_{\mathfrak{K}}(f) = \underbrace{\mu_{\mathfrak{K}}(\alpha f)}_{\in \mathbb{R}_0^+} = \underbrace{\mu_{\mathfrak{K}}(g_1)}_{\ge 0} + \underbrace{i\mu_{\mathfrak{K}}(g_2)}_{=0} = \mu_{\mathfrak{K}}(g_1)$
$\overset{g_1 \le |\alpha f|}{\le} \mu_{\mathfrak{K}}(|\alpha f|) \overset{|\alpha|=1}{=} \mu_{\mathfrak{K}}(|f|)$.
And it is, if $|\mu_{\mathfrak{K}}(f)| = 0$, $0 = |\mu_{\mathfrak{K}}(f)| \le \mu_{\mathfrak{K}}(|f|)$.
(cf [HR63], p.120).

□

Theorem 8.2.9 *$\mu_{\mathfrak{K}} : \mathfrak{K}((X, \mathfrak{T}_X)) \to \mathbb{C}$ is a positive linear form.*
For all compact sets $K \subset X$ there is a $c_K > 0$ with

$$f \in \mathfrak{K}((X, \mathfrak{T}_X)), Tr(f) \subset K \Rightarrow |\mu_{\mathfrak{K}}(f)| \le c_K \|f\|_\infty.$$

(cf [HR63], p.120).

PROOF (8.2.9): *Direct proof:*
Precondition: $\mu_{\mathfrak{K}} : \mathfrak{K}((X, \mathfrak{T}_X)) \to \mathbb{C}$ is a positive linear form and
$\forall\ K \subset X, K$ *compact* $\exists\ c_K > 0 : f \in \mathfrak{K}((X, \mathfrak{T}_X)) \wedge Tr(f) \subset K$.
Choose $h \in \mathfrak{K}_+((X, \mathfrak{T}_X)), 0 \le h \le 1, h(K) = 1$, compare LEMMA *8.1.89.*
Put $c_K := \mu_{\mathfrak{K}}(h)$.
Then it is $|f| \le \|f\|_\infty h$ for all $f \in \mathfrak{K}((X, \mathfrak{T}_X))$ with $Tr(f) \subset K$, so
$|\mu_{\mathfrak{K}}(f)| \overset{\text{THEOREM 8.2.8}}{\le} \mu_{\mathfrak{K}}(|f|) \le \|f\|_\infty \mu_{\mathfrak{K}}(h) \overset{c_K := \mu_{\mathfrak{K}}(h)}{=} c_K \|f\|_\infty$.
(cf [HR63], p.120).

□

Definition 8.2.10 *(Complex Radon) Measure (cf [Die75], p.112)*
A (complex Radon) measure on $(X, \mathfrak{T})$ is a linear form

$$\mu_{\mathfrak{K}} : \mathfrak{K}((X, \mathfrak{T}_X)) \to (\mathbb{C}, \mathfrak{T}_{nat})$$

which fulfils the following property:
For each compact set $K \subset X$ there is a $c_K \ge 0$ with

$$f \in \mathfrak{K}((X, \mathfrak{T}_X)), Tr(f) \subset K \Rightarrow |\mu_{\mathfrak{K}}(f)| \le c_K \|f\|_\infty.$$

We consider $\mathbb{C}$ as codomain of a measure, i.e. $\mu_{\mathfrak{K}}(f) = c \in \mathbb{C}$, because c can be illustrated by a vector indicating the direction of the effect of a force by identifying the space $\mathbb{C}$ with the the space $\mathbb{R}^2$, i.e. with polar coordinates. This is useful e.g. for navigating a user, who is located in a risk-area, to a save location.

Definition 8.2.11 *Positive Measure (cf [Die75], p. 117)*
$\mu_{\mathfrak{K}}$ is a linear form on the vector space $\mathfrak{K}(X, \mathfrak{T}_X)$ with the property that each positive mapping f of $\mathfrak{K}(X, \mathfrak{T}_X)$ fulfils the inequality $\mu_{\mathfrak{K}}(f) \geq 0$. Then $\mu_{\mathfrak{K}}$ is a positive measure.

Definition 8.2.12 *Real Measure (cf [Die75], p.116)*
A measure $\mu_{\mathfrak{K}}$ is called real, if $\mu_{\mathfrak{K}}(f)$ is real for all real-valued mappings $f \in \mathfrak{K}((X, \mathfrak{T}_X))$.

The in the following deduced product measure is used within the TCIU, SECTION 9.4.

Note 8.2.13: *The measures on $(X, \mathfrak{T}_X)$ form a vector subspace $M((X, \mathfrak{T}_X))$ of the dual space $\mathfrak{K}((X, \mathfrak{T}_X))^* = Hom_{\mathbb{C}}(\mathfrak{K}((X, \mathfrak{T}_X)), \mathbb{C})$ of $\mathfrak{K}((X, \mathfrak{T}_X))$.*

$$\bar{\mu}_{\mathfrak{K}} : f \mapsto \overline{\mu_{\mathfrak{K}}(\bar{f})}$$

is a measure. The measures $\frac{1}{2}(\mu_{\mathfrak{K}} + \bar{\mu}_{\mathfrak{K}})$ and $\frac{1}{2i}(\mu_{\mathfrak{K}} - \bar{\mu}_{\mathfrak{K}})$ are real, and from them $\mu_{\mathfrak{K}}$ and $\bar{\mu}_{\mathfrak{K}}$ are linear combineable. (cf [Die75], p.116).

Note 8.2.14: *In the following measures on spaces that can be represented as a Cartesian product are examined. Therefore, the following notation is particularly suitable:*
For all $\mu_{\mathfrak{K}} \in M((X, \mathfrak{T}_X))$ and $f \in \mathfrak{K}((X, \mathfrak{T}_X))$ one puts

$$\mu_{\mathfrak{K}}(f) = \int_X f(x) d\mu_{\mathfrak{K}}(x).$$

Theorem 8.2.15 *X and Y are two local compact spaces, $\mu_{\mathfrak{K}_1}$ is a measure on X and $\mu_{\mathfrak{K}_2}$ is a measure on Y. Then there exists exactly one measure $\mu_{\mathfrak{K}}$ on the product space $X \times Y$, that for each pair of mappings $f \in \mathfrak{K}(X, \mathfrak{T}_X)$ and $g \in \mathfrak{K}(Y, \mathfrak{T}_Y)$ the following relation is valid:*

$$\int_{X \times Y} f(x)g(y) d\mu_{\mathfrak{K}}(x, y) = \left(\int_X (x) d\mu_{\mathfrak{K}}(x) \right) \left(\int_Y g(y) d\mu_{\mathfrak{K}}(y) \right).$$

(cf [Die75], p.228).

The proof can be found in [Die75], p.228ff.

Note 8.2.16: THEOREM *8.2.15 provides a mapping*

$$M((X,\mathfrak{T}_X)) \times M((Y,\mathfrak{T}_Y)) \to M((X \times Y, \mathfrak{T}_X \otimes \mathfrak{T}_Y)),$$
$$(\mu_{\mathfrak{K}_1}, \mu_{\mathfrak{K}_2}) \mapsto \mu_{\mathfrak{K}},$$

and this is bilinear, so there is a linear mapping

$$M((X,\mathfrak{T}_X)) \otimes M((Y,\mathfrak{T}_Y)) \to M((X \times Y, \mathfrak{T}_X \otimes \mathfrak{T}_Y)),$$
$$\mu_{\mathfrak{K}_1} \otimes \mu_{\mathfrak{K}_2} \mapsto \mu_{\mathfrak{K}}.$$

This mapping is injective and one can understand $M((X,\mathfrak{T}_X)) \otimes M((Y,\mathfrak{T}_Y))$ as subspace of $M(X \times Y, \mathfrak{T}_X \otimes \mathfrak{T}_Y))$. (cf [Die75], p.230).

Definition 8.2.17 *Product Measure (cf [Die75], p.230)*
The constructed measure $\mu_{\mathfrak{K}}$ in 8.2.15 is called the product measure of $\mu_{\mathfrak{K}_1} \otimes \mu_{\mathfrak{K}_2}$.

8.2.2 The Spaces $\mathscr{L}^p((X,\mathfrak{T}_X),\mu_L)$

The integration concerning $\mu_{\mathfrak{K}}$, where only continuous mappings with compact carrier were considered, can be expanded to a larger class of mappings. This continuation process can be done in three stages. In the first two stages, an "outer integral" for arbitrary not negative mappings can be introduced, which allows then to approximate the integrable mappings by continuous mappings with compact carrier.
We need this extension for our concerns, because we consider in CHAPTER 9 inter alia not-continuous fuzzy mappings.

Note 8.2.18: *$(X,\mathfrak{T}_X)$ is always a locally compact T_3-space and $\mu_{\mathfrak{K}}$ is a measure on $(X,\mathfrak{T}_X)$. One assumes $\mu_{\mathfrak{K}}$ as positive, until further notice.*

Definition 8.2.19 *Lower Semi-Continuous in x (cf [Die75], p.33)*
A mapping $f : X \to [-\infty,\infty]$ is called lower semi-continuous in $x \in X$, if there is for each $\alpha < f(x)$ a neighbourhood U_x of x with $y \in U_x \Rightarrow \alpha < f(y)$.

Definition 8.2.20 *Lower Semi-Continuous (cf [Die75], p.33)*
A mapping is called (generally) lower semi-continuous, if it is lower semi-continuous in each $x \in X$.

Definition 8.2.21 $\mathfrak{I}_+((X, \mathfrak{T}_X))$ *(cf [Die75], p.126)*

$\mathfrak{I}_+((X, \mathfrak{T}_X))$ *denotes the set of all lower semi-continuous mappings*

$$f : (X, \mathfrak{T}_X) \overset{\mathfrak{T}}{\to} ([0, \infty], \mathfrak{T}_{nat}).$$

Definition 8.2.22 *Outer Measure (μ^*) (cf [Die75], p.126)*

$f \in \mathfrak{I}_+((X, \mathfrak{T}_X))$. μ^* *is a mapping:*

$$\mu^* : \mathfrak{I}_+((X, \mathfrak{T}_X)) \to [0, \infty]$$
$$f \mapsto \int^* f d\mu_{\mathfrak{K}} = \int^* f(x) d\mu_{\mathfrak{K}}(x) := \sup\{\mu_{\mathfrak{K}}(h) \mid h \in \mathfrak{K}_+((X, \mathfrak{T}_X)), h \leq f\}.$$

Note 8.2.23: *The mapping*

$$\mu^* : \mathfrak{I}_+((X, \mathfrak{T}_X)) \to [0, \infty]$$

is then an extension of

$$\mu_{\mathfrak{K}} : \mathfrak{K}_+((X, \mathfrak{T}_X)) \to [0, \infty).$$

We need this extension, because e.g. the mapping $1_X \notin \mathfrak{K}_+((X, \mathfrak{T}_X)$.

Example 8.2.24: *Let*

$$\mathfrak{T}_0 = \{\emptyset, \mathbb{R}, (a, b)\}.$$

The mapping

$$f : (\mathbb{R}, \mathfrak{T}_0) \overset{\mathfrak{T}}{\to} (\mathbb{C}, \mathfrak{T}_{nat})$$
$$x \mapsto f(x) = \begin{cases} 1 \text{ for } x \in (a, b) \\ 0 \text{ for } x \in \mathbb{R} \backslash (a, b) \end{cases}$$

is not continuous, because $B_{\frac{1}{2}}(0) \subset \mathbb{C}$ *and* $B_{\frac{1}{2}}(0) \in \mathfrak{T}_{nat}$.
But $f^{-1}(B_{\frac{1}{2}}(0)) = \mathbb{R} \backslash (a, b) \notin \mathfrak{T}_0$. *Thus* $f \notin \mathfrak{K}(\mathbb{R}, \mathfrak{T}_0)$.
$\mathbb{R} \backslash (a, b)$ *is compact, because* $\mathfrak{T}_0$ *is a finite topology, thus the space* $(\mathbb{R}, \mathfrak{T}_0)$ *is compact (*NOTE 8.1.85*) and thus closed subsets of* $\mathbb{R}$ *are compact (*COROLLARY 8.1.84*). The carrier of* f *is* $Tr(f) = \mathcal{A}(\{x \in \mathbb{R} \mid f(x) \neq 0\}) = \mathcal{A}((a, b)) = [a, b]$. *This is a closed set an thus compact.*

Theorem 8.2.25 *It is* $f \in \mathfrak{I}_+((X, \mathfrak{T}_X))$. *Then is:*

$$f \text{ integrable} \Leftrightarrow \mu^*(f) < \infty.$$

The proof can be found in [Die75], p.135.

With indicator mappings we can describe crisp sets within the SAGU described in CHAPTER 9.

Definition 8.2.26 *Indicator/ Characteristic Mapping (cf [Die75], p.34)*
For each $A \subset X$, 1_A denotes the characteristic mapping of A, that is defined by

$$1_A : X \to \{0,1\}$$

$$x \mapsto 1_A(x) = \begin{cases} 1 \textit{ for } x \in A \\ 0 \textit{ for } x \in X \backslash A \end{cases}$$

Definition 8.2.27 *Integrable Set ([Die75], p.135)*
$(X, \mathfrak{T}_X)$ is a topological space.
A set $M \subset X$ is integrable, if its indicator mapping is integrable.

The separation axiom T_{3a} plays an important role for the development of an SAGU described in CHAPTER 9. T_{3a} enables the separation of a measurable set and a point. This is important for the evaluation of a set of collected data (about a user) and the approximation of a membership function representing the user‘ s needs. Furthermore this is important to separate the set of GUI elements form the therein contained points, the GUI element objects. With T_{3a}, measurable sets can be identified. Those sets can be fuzzy, i.e represented via a fuzzy mapping. Therewith fuzzy operations (e.g. the fuzzy-AND, p.71, the fuzzy-OR, p.70 or the fuzzy-NOT, p.72) can be conducted. Fuzzy mappings are used to represent vague statements of a user and they are used to control the GUI. Therefore, the term measurable set has to be defined.

Definition 8.2.28 *Measurable Set ([Die75], p.145)*
$(X, \mathfrak{T}_X)$ is a topological space.
A set $M \subset X$ is measurable, if it is

$$\forall \ K \subset X, K \textit{ compact} : M \cap K \textit{ integrable.}$$

Definition 8.2.29 *Outer Measure of Sets (cf [Die75], p.131)*
$(X, \mathfrak{T}_X)$ *is a topological space,* $A \subset X$*.*
The value

$$\mu^*(A) := \mu^*(1_A) \in [0, \infty]$$

is called outer measure of the set A *concerning* μ*.*

Definition 8.2.30 μ*-Null Set (cf [Die75], p.131)*
$(X, \mathfrak{T}_X)$ *is a topological space,* $A \subset X$*.*
A *is called* μ*-null set, if it is*

$$\mu^*(A) = 0.$$

Example 8.2.31:

- *Each subset of a* μ*-null set is a* μ*-null set.*
- *The unification of a countable number of* μ*-null sets is a* μ*-null set.*

(cf [Die75], p.131)

Definition 8.2.32 μ*-Almost Everywhere Correct (cf [Die75], p.132)*
If $P(x)$ *is an assertion for each* $x \in X$*, so one says, that* P *is* μ*-almost everywhere correct, if*

$$\{x \in X \mid P(X) \textit{ is wrong}\}$$

is a μ*-null set.*

Example 8.2.33: *"*$f = 0$ μ*-almost everywhere" means, that*

$$N = \{x \in X \mid f(x) \neq 0\}$$

is a μ*-null set, i.e.* $\mu(N) = \mu(1_N) = 0$*. (cf [Die75], p.132)*

In the following it will be deducted how to measure mappings like the mapping f from Example 8.2.24.
We use the Lebesgue-measure within the SAGU, because changes of a mapping f on a μ-null set will not have an impact on the Lebesgue-measure $\mu_L(f)$. This is important for the SAGU, described in Chapter 9, because this way it is possible to separate important measurable properties of a user from the unimportant via measures.

Definition 8.2.34 $\|f\|_p$ *(cf [HR63], p.135)*

For each $p \in [1, \infty]$ and each extended real- or complex-valued integrable mapping $f : X \to \mathbb{C}$ or $f : X \to [-\infty, \infty]$ such that $\mu^*(|f|^p) < \infty$, i.e. $f \in \mathscr{L}^p((X, \mathfrak{T}_X), \mu_L)$ (see DEFINITION *8.2.37), one defines the norm*[11] $\|f\|_p \in [0, \infty]$ *by*

$$\|f\|_p := (\mu^*(|f|^p))^{1/p} = \sqrt[p]{\int^* |f|^p d\mu_{\mathfrak{K}}}.$$

Note 8.2.35: *The norm $\|f\|_p$ has the usual properties ascribed to a norm. ([HR63], p.136)*

Definition 8.2.36 *Lebesgue Topology*

The topology induced by $\|f\|_p$ is called Lebesgue topology, $\mathfrak{T}_L$.

Definition 8.2.37 *p-fold Integrable Mapping, Square Integrable Mapping, Integrable Mapping, $\mathscr{L}^p((X, \mathfrak{T}_X), \mu_L)$ (cf [Die75], p.134 and p.170)*

$\| \cdot \|_p$ is a norm on the $\mathbb{C}$-vector space $\{f : X \to \mathbb{C} \mid \|f\|_p < \infty\}$, that contains the vector sub-space $\mathfrak{K}((X, \mathfrak{T}_X))$. Its closure $\mathscr{L}^p((X, \mathfrak{T}_X), \mu_L)$ is a vector space, again, that consists of all mappings $f : X \to \mathbb{C}$ with the property

$$\forall\ \varepsilon > 0\ \exists\ g \in \mathfrak{K}((X, \mathfrak{T}_X)) : \|f - g\|_p < \varepsilon.$$

These mappings are called p-fold integrable, in the case $p = 2$ square integrable, in the case $p = 1$ integrable.

Note 8.2.38: *If f is integrable, $|f|$, f^+ and f^- are integrable, as well. If f and g are integrable, $sup(f, g)$ and $inf(f, g)$ are integrable, as well. If f is integrable, $Re(f)$ and $Im(f)$ are integrable. (cf [Die75], p.135).*

Definition 8.2.39 *Lebesgue Measure (cf [Die75], p.135)*

If f is integrable, the Lebesgue Measure can be defined by

$$\mu_L(f) = (\mu^*(Re(f)^+) - \mu^*(Re(f)^-)) + i(\mu^*(Im(f)^+) - \mu^*(Im(f)^-)).$$

[11] One has to put $\infty^p = \infty$.

Definition 8.2.40 *Complete (cf [Mes72], p.277)*
A complex vector space E with a semi norm $\|\cdot\|$ is called complete, if each Cauchy sequence converges relevant to $\|\cdot\|$. This is equivalent to the fact, that the abelian group $(E,+)$ is complete with the topology, delivered of $\|\cdot\|$.

Note 8.2.41: *Completeness is an important term for the mapping spaces of analysis because it is common, that "interesting" mappings, "solutions" of anything, are constructed as limits of sequences of mappings. In any topological vector spaces one can speak of Cauchy sequences and therefore one can speak of complete or incomplete. (cf [Jae99], p. 69).*

Definition 8.2.42 *Banach Space (cf [Mes72], p.34)*
E is called Banach space, if E is complete and $\|\cdot\|$ is even a norm.

Definition 8.2.43 *Hilbert Space (cf [Mes72], p.277)*
A complete unitary space is a Hilbert space.

Theorem 8.2.44 *Theorem of Riesz-Fischer (cf [Els09], p.231)*
For all $p \in [1,\infty)$ it is $\mathscr{L}^p((X,\mathfrak{T}_X),\mu_L)$ complete, so $\mathscr{L}^p((X,\mathfrak{T}_X),\mu_L)$ is a Banach space.

The proof can be found in [Els09], p.231 f.

Within a Banach Space, each Cauchy sequence converges. Several adjustment steps of the system adapting GUI to a user are not necessarily representable by a Cauchy sequence, because the user needs can change and thus another GUI, represented by a membership mapping in $\mathscr{L}^1((X,\mathfrak{T}_X),\mu_L)$, is required.

Note 8.2.45: *$\mathscr{L}^2((X,\mathfrak{T}_X),\mu_L)$ is a Hilbert space. (cf [HR63], p.139)*

With Note 8.2.45 the inner product $(f,g) = \int\limits_A f(x)\cdot g(x)d\mu(A)$ with $(X,\mathfrak{T}_X)$ a topological space, $A \subset X$ compact, $x \in A$ and $f,g \in \mathscr{L}^1((X,\mathfrak{T}_X),\mu_L)$ can be defined, i.e. the fuzzy-AND, compare Definition 7.1.11.

8.3 Interim Conclusion

The mathematical fields of measure theory on topological spaces, fuzzy logic and ANNs (Chapter 7) are used as basal tools for the development of the mathematical structure of an SAGU (Chapter

9).

Topological spaces have to be considered for the following reason: In contributing to the optimisation of health service delivery, epidemiological distances are, inter alia, identified as being an issue. However, these distances in the public health domain do not fulfil the symmetry property of the metric, thus the spaces being considered may not be Euclidean spaces. In the SAGU described in CHAPTER 9, fuzzy membership mappings are included, which are elements of a function space, which is infinite dimensional. Topological spaces and fuzzy logic are, inter alia, connected in the following way: The GPS blocks described on p.156 can be represented by fuzzy sets. The GUI assigned to a user is GIS-tailored, that is based on her/his geo-location and, thus, on the GPS block she/he is located in. When a user moves towards the boundary of the GPS block she/he is located in, the GUI proposes a change to another GUI that is adjusted for the GPS block towards which the user is moving; that is, the GPS block over the border closest to where the user is situated. Topological spaces and ANNs are, inter alia, connected in the following way: Open sets are used for the ANN learning task to identify similarities between certain outputs and inputs. Convergence is applied to the ANN learning task (see SECTION 9.3), in order to investigate, whether similar inputs produce "more similar" outputs within a certain time period; in other words, whether the learning task is solved better when the system is improved.

For the SAGU described in CHAPTER 9, it is important to collect data (about a user). Thus, for the adaptivity of the GUI we have to proceed in the following way: $(X, \mathfrak{T}_X)$ is a topological space. Information about a subspace of X is collected by means of a compactum $K \subset X$ (DEFINITION 8.1.81), for example in $\mathbb{R}$ for an interval $[a, b] \subset \mathbb{R}$. Subsequently, observations have to be narrowed down to the topology, which is induced by X to K, the relative topology $(\mathfrak{T}_{rel})$ on K, even if X is the fundamental space for the measures. The separation axiom T_{3a} plays an important role in the development of an SAGU, as described in CHAPTER 9, as T_{3a} enables the separation of a measurable set and a point. This is important for the evaluation of a set of collected data (about a user) and the determination of a membership mapping, which is contained in a function space, which is infinite dimensional. We use the Lebesgue-measure within the SAGU, because changes of a mapping f on a μ-null set will not have an impact on the Lebesgue-measure $\mu_L(f)$. This is important for the SAGU, described in CHAPTER 9, because this way it is possible to separate important measurable properties of a user from the unimportant via measures.

With T_{3a}, measurable sets can be identified. Those sets can be fuzzy, i.e represented via a fuzzy mapping. Therewith fuzzy operations (e.g. the fuzzy-AND, p.71, the fuzzy-OR, p.70 or the fuzzy-NOT, p.72) can be conducted. Fuzzy mappings are used to represent vague statements of a user and they are used to control the GUI.

Relations between topological spaces are mediated by continuous mappings. With homeomorphisms, neighbourhood "information" can be transferred from a topological space $(X, \mathfrak{T}_X)$ to a topological space $(Y, \mathfrak{T}_Y)$. THEOREM 8.1.29 shows how information (about a user) can be transferred from

$(X, \mathfrak{T}_X)$ to $(Y, \mathfrak{T}_Y)$. Using a Cartesian product of the status spaces of object classes, topologies are assigned to an object in OOA (CHAPTER 6) as information content. On neighbourhoods in $(\mathbb{R}, \mathfrak{T}_{\mathbb{R}})$, continuous mappings $f : (X, \mathfrak{T}_X) \xrightarrow{\mathfrak{T}} (\mathbb{R}, \mathfrak{T}_{\mathbb{R}})$ behave similarly. Within the topological space $(X, \mathfrak{T}_{dis})$, the one-pointed sets are open. Each mapping $f : (X, \mathfrak{T}_{dis}) \xrightarrow{\mathfrak{T}} (Y, \mathfrak{T}_Y)$ is continuous and, for an one-pointed set $\{y\} \subset Y$, we cannot imply the behaviour of f in a larger neighbourhood $U_y \subset Y$ of y, because we can not extrapolate single point information of the codomain of a mapping to the topological neighbourhood. This would not, however, be to our advantage, if we wanted to transfer information (about a user) from $(X, \mathfrak{T}_X)$ to $(Y, \mathfrak{T}_Y)$. Open sets are required to separate GUI properties (e.g. GUI elements or GUI element objects) (separation axiom T_2). This is necessary to enable the assignment of a certain GUI that has been adjusted to a user. In a data space for the objects of the GUI or a GIS, we try to adjust the properties of the GUI using the OOA methods. If there are two points that are not separable in a topology, then it is not possible to assign open sets (cluster) to the two points for the properties of the GUI. If those properties of the GUI are not separable in the topology, then they are dependent. The well-known formulation of continuity through the convergence of sequences from the analysis can be modified only strongly transferred to the topology, since the sequence concept is inadequate. Therefore, the term "filter" was introduced. The SAGU described in the next chapter works like a filter. If the filter is convergent, the limit determines the GUI which is assigned to the user. If the SAGU represented by a filter is not convergent, but rather a refinement of this filter, the point of adherence determines the GUI assigned to a user. In a compact space, each filter has a point of adherence (THEOREM 8.1.86). If we have no information about the user (except the geo-location) our situation could be comparable to having the indiscreet topology on the space containing the membership mapping describing the quality of a GUI for a user. With THEOREM 8.1.77 each GUI would be proposed as suitale for the user. Thus, we had a variety of options. In order to get a handle on this, an initial GUI will be provided to users we do not have any information about except the geo-location. The initial GUI is not a global GUI, because it is GIS-tailored. We refer to the information about users in similar situations, that is, users located at the same geo-location to assign a GUI to the user that worked out fine for users in similar situations. The GUI adapts then by replacing, adding or deleting GUI element objects. By doing this, the variety of options becomes smaller and the best fitting GUI for the user is isolated. In the next chapter this is addressed again. If the space the SAGU works on was T_2, the filter representing the system was convergent to exactly one point (see THEOREM 8.1.78); in other words exactly one GUI would be proposed as suitable for a user. Each Cauchy sequence converges in a Banach Space. Several adjustment steps in the system adapting the GUI to a user cannot necessarily be represented by a Cauchy sequence, because the user needs can change and thus another GUI, which is represented by a membership mapping in $\mathscr{L}^p((X, \mathfrak{T}_X), \mu_L)$, is required.

The subject of this thesis is the mathematical link between spatial risk and spatial resource availability for use in adaptation processes for GUIs on digital devices. As spatial distances can not always be used in the Euclidean sense, measure theory on topological spaces is needed. In these general topological spaces, fuzzy mappings operate. (Adaptive) Fuzzy logic operations (CHAPTER 7), in turn, determine the GUI structure. Decisions made using non-significant data are supported by the adaption process using dynamic limit functions. Because of the generic structure of the algorithm, it can be transferred to other spatial decision support that does not operate on Euclidean spaces. However, the development of such a mathematical structure for the purposes of adaptivity in a GUI goes beyond the current research. Moreover, measure theory on topological spaces was observed only insofar as it concerns mathematics. In this thesis, measure theory on topological spaces is applied to extra-mathematical topics.

The GUI for an application for digital devices (e.g. smartphones) to deliver decision support to users in hazardous situations consists of several GUI elements, which are represented by fuzzy membership mappings

$$f_{E_i} : (X_{\Gamma_0}, \mathfrak{T}_{\Gamma_0}) \to ([0,1], \mathfrak{T}_{nat,\mathrm{I\!R}|_{[0,1]}}),$$

with $E_i \in GUI = \{E_1, ..., E_m\}$, GUI is the set of the GUI elements.

$$X_{\Gamma_0} := S^2_{R_0}(z) \times T$$

with $S^2_{R_0}(z) = \{\omega \in [0, R_0]^3 \mid \|\omega - z\|_2 = R_0\}$ with

$$\omega = \begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix} = \begin{pmatrix} R_0 \cdot \sin(\theta) \cdot \cos(\varphi) \\ R_0 \cdot \sin(\theta) \cdot \sin(\varphi) \\ R_0 \cdot \cos(\theta) \end{pmatrix}$$

with $\theta \in [-\frac{\pi}{2}, \frac{\pi}{2}]$ which corresponds to the latitude $[-90°, 90°]$ and $\varphi \in (-\pi, \pi]$ which corresponds to the longitude $(-180°, 180°]$, $z = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ is the barycentre of the three-dimensional object describing the earth, $T = \mathrm{I\!R}$ the time and R_0 the distance from the barycentre z to the earth's surface at the timepoint $t \in T$ (averaged $6371km$).

$$\|\omega - z\|_2 = \sqrt{(\omega_1 - 0)^2 + (\omega_2 - 0)^2 + (\omega_3 - 0)^2}.$$

The user's perception of the degree of the quality of a GUI element is dependent on geo-location and time, because certain infectious diseases only occur in areas with suitable environmental conditions which depend, for example, on the weather.

$\mathfrak{T}_{\Gamma_0}$ is generated by $L_1 \times L_2 \times T_0$ with

$$L_1 := \{(a_1, b_1] \mid -180 < a_1 \leq b_1 \leq 180\} \subset (-180, 180],$$

$$L_2 := \{[a_2, b_2] \mid -90 \leq a_2 \leq b_2 \leq 90\} \subset [-90, 90] \text{ and}$$

$$T_0 := [a_0, b_0] \subset T = \mathrm{I\!R}.$$

Thus,

$$\mathfrak{T}_{\Gamma_0} = \mathfrak{T}_{nat,(-180,180]}|_{(a_1,b_1]} \otimes \mathfrak{T}_{nat,[-90,90]}|_{[a_2,b_2]} \otimes \mathfrak{T}_{nat,\mathbb{R}}|_{[a_0,b_0]}$$

with $-180 < a_1 \leq b_1 \leq 180$, $90 \leq a_2 \leq b_2 \leq 90$, $a_0, b_0 \in \mathbb{R}$.

$$f_{E_i} \in \mathscr{L}^1((X_{\Gamma_0}, \mathfrak{T}_{\Gamma_0}), \mu_{L,X_{\Gamma_0}}) =: L_{\Gamma_0},$$

$\mu_{L,X_{\Gamma_0}}$ is the Lebesgue-measure restricted to X_{Γ_0}. With

$$F : (X_{\Gamma_0}, \mathfrak{T}_{\Gamma_0}) \xrightarrow{\mathfrak{T}} ([0,1]^m, \underbrace{\mathfrak{T}_{nat,\mathbb{R}}|_{[0,1]} \otimes ... \otimes \mathfrak{T}_{nat,\mathbb{R}}|_{[0,1]}}_{m \text{ times (product topology)}})$$

$$(x, y, t) \mapsto (f_{E_1}(x, y, t), ..., f_{E_m}(x, y, t)),$$

$F \in X_{GUI_0} \subset L^m_{\Gamma_0}$, we can determine the quality of each GUI element $E_i \in GUI$ at each geo-location on earth at time points $\omega = (x, y, t) \in X_{\Gamma_0}$, with $t \in T_1$ and T_1 a compact subset of T, and thus the best fitting GUI elements can be chosen for a user. X_{Γ_0} is a locally compact T_3-space, while X_{GUI_0} is a topological space with the Lebesgue topology $\mathfrak{T}_{L_0}$ induced by the $\mathscr{L}_1$-norm: $\|F\| = \sum_{i=1}^{m} \|f_{E_i}\|_1$. This topology is not sufficient to describe the space in question properly. Membership mapping of a user F that describes the quality of each GUI can be measured via

$$\mu_0(F) := \sum_{i=1}^{m} \mu_{L,X_{\Gamma_0}}(f_{E_i}).$$

$\mu_0(F)$ is a complex Radon measure, because it fulfils the properties of Definition 8.2.10. However, the measure μ_0 is not suitable for our claims, because it determines the summed degree of the quality of each GUI element at a certain geo-location at a certain point in time; no information can be derived for the quality of a GUI.

In the following part, Part IV, the concepts addressed in Part III are applied to the development of a mathematical structure for an SAGU. Accordingly, the mathematical structure of an SAGU is presented in Chapter 9. Chapter 10, meanwhile, gives an object-oriented overview of the system and addresses maps for visualisation and exemplary GUIs tailored to different user groups. In order to adapt the GUI to South Africa conditions it is important to determine an initial state for the GUI. Therefore, an empirical pilot study was carried out, which is described in Chapter 11.

IV | The System for adapting a GUI to a User

9 Mathematical Structure behind the System for adapting a GUI to a User

One of the goals of the *ReGLaN-Health and Logistics* project is the development of an adaptive GUI for an OS-application for digital devices (e.g. smartphones) in order to deliver decision support to users in hazardous situations. This GUI will be GIST with the application of spatial fuzzy logic. Therefore, the principle of the GIST questionnaire will be included. The GUI will be able to adapt to the user depending on what the user wants to accomplish with the help of the GUI. The quality of a GUI is assessed by how readable or comprehensible the GUI is to the user. This comprehensibility can be measured by the actions of the user in reaction to any decision support delivered by the GUI. This can be done by, inter alia, observing the location of the user with the aid of the GPS coordinates of the user or the IP address (SECTION 2.2 discusses digital devices in this respect). In this way, we can see for example whether a user moves in the direction proposed by the GUI (i.e. by the navigation application) or if the user has passed certain locations. The motion speed of a user can also be determined. With this information it is possible to ask questions or give advice adapted to the results of the observation of the user's location (GIST questionnaire). Accordingly, the GUI can be optimised and data can be collected to optimise the Tool for Collecting Information about the User (TCIU).

[TCK13] studied the relation between mobility, (risk-) exposure and health and presents a kernel-based algorithm for detecting stop locations and estimating stop durations of the user.

The inclusion of such algorithms improves the accuracy of our results and, thus, those of the SDSS provided by the GUI. It should be noted that information about the user is only collected in a database with the user's permission, for data protection. The GUI determines optimal tracks for the user using Ant Colony Optimisation (ACO)[1] for an appropriate application of navigation by using routes with

[1]"Ant algorithms are a class of the algorithms based on artificial swarm intelligence, which is inspired by the collective behaviour of social insects. For real ants, they have two important types of behaviour: foraging and clustering. In the ACO algorithm, an artificial ant colony simulates the pheromone trail following behaviour of real ants. Artificial ants move on a synthetic map representing a specific problem to construct solutions successively. The artificial pheromone that corresponds to the record of routes taken by the ant colony is accumulated at run-time through a learning mechanism. Individual ants concurrently collect necessary information, stochastically make their own decisions, and independently construct solutions in a stepwise procedure. The information required for making a decision at each step include pheromone concentration, problem data and heuristic function values. The pheromone laid on the

low cost, that is, routes where the user is exposed to as little risk as possible.

For a navigation using routes with low cost, possible routes from a starting point to a destination point via intermediate points are determined. The ACO minimises the cost-function which represents the relationship between cost and a benchmark. Cost is not necessarily a monetary value, it can be e.g. the time, as well. The cost (DEFINITION 9.0.2) in our case is the risk the user is exposed to when taking a route. The benchmark is the topological space $(X_B, \mathfrak{T}_B)$ with

$$X_B = S^2_{R_0}(z)$$

with $S^2_{R_0}(z) = \{\omega \in [0, R_0]^3 \mid \|\omega - z\|_2 = R_0\}$ with

$$\omega = \begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix} = \begin{pmatrix} R_0 \cdot \sin(\theta) \cdot \cos(\varphi) \\ R_0 \cdot \sin(\theta) \cdot \sin(\varphi) \\ R_0 \cdot \cos(\theta) \end{pmatrix}$$

with $\theta \in [-\frac{\pi}{2}, \frac{\pi}{2}]$ which corresponds to the latitude $[-90°, 90°]$ and $\varphi \in (-\pi, \pi]$ which corresponds to the longitude $(-180°, 180°]$, $z = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ is the barycentre of the three-dimensional object describing the earth and R_0 the distance from the barycentre z to the earth's surface (averaged $6371km$).

$$\|\omega - z\|_2 = \sqrt{(\omega_1 - 0)^2 + (\omega_2 - 0)^2 + (\omega_3 - 0)^2}.$$

$\mathfrak{T}_B$ ist generated by $L_1 \times L_2$ with

$$L_1 := \{(a_1, b_1] \mid -180 < a_1 \leq b_1 \leq 180\} \subset (-180, 180] \text{ and}$$

$$L_2 := \{[a_2, b_2] \mid -90 \leq a_2 \leq b_2 \leq 90\} \subset [-90, 90].$$

Thus,

$$\mathfrak{T}_B = \mathfrak{T}_{nat,(-180,180]}|_{(a_1,b_1]} \otimes \mathfrak{T}_{nat,[-90,90]}|_{[a_2,b_2]}$$

with $-180 < a_1 \leq b_1 \leq 180$, $90 \leq a_2 \leq b_2 \leq 90$.

Points in X_B represent the nodes defined in DEFINITION 9.0.1. As mentioned before, the cost can change at each time-step. Thus the ACO operates on a network, which can change at each time-step. The nodes are intermediate locations and edges represent the tracks, also in X_B. The edges are weighted. The weights are determined via measures and represent the cost.

Definition 9.0.1 *Random Proportional Rule ([BPD06])*

At the generic iteration h, suppose that ant k is in node i. Let $\mathcal{N}_i^k$ be the set of feasible nodes. The node $j \in \mathcal{N}_i^k$, to which ant k moves, is selected with probability:

$$p^k_{i,j,h} = \frac{[\tau_{ij,h}]^\alpha [\eta_{ij}]^\beta}{\sum\limits_{l \in \mathcal{N}_i^k} [\tau_{il,h}]^\alpha [\eta_{il}]^\beta},$$

path belonging to the iteration-best solution will be positively increased to become more attractive in the subsequent iterations."([KC06]) *where α and β are parameters, $\tau_{ij,h}$ is the pheromone value associated with arc $\langle i, j\rangle$ at iteration h, and η_{ij} represents heuristic information on the desirability of visiting node j after node i.*

Definition 9.0.2 *Pheromone Update Rule ([BPD06])*
At the generic iteration h, suppose that m ants have generated the solutions $T_h^1, T_h^2, ..., T_h^m$ of cost $C_h^1, C_h^2, ..., C_h^m$, respectively. The pheromone on each arc $\langle i, j\rangle$ is updated according to the following rule:

$$\tau_{ij,h+1} = (1-\rho)\tau_{ij,h} + \sum_{k=1}^{m} \Delta_{ij,h}^k,$$

where ρ is a parameter called evaporation rate and

$$\Delta_{ij,h}^k = \begin{cases} \frac{1}{C_h^k}, \text{ if } \langle i, j\rangle \in T_h^k \\ 0, \text{ otherwise.} \end{cases}$$

Definition 9.0.3 *Ant System ([BPD06])*
Ant system is an ACO algorithm in which solutions are constructed according to the random proportional rule given in Definition 9.0.1*, and the pheromone is updated according to the rule given in* Definition 9.0.2*. The evaporation rate ρ, the number of ants m, and the exponents α and β are parameters of the algorithm.*

Note 9.0.4: *The ACO determines cost-effective optimal ways. [BPD06] reports about the convincing invariance of the ant system.*

The considered space is a non-Euclidean space. A graph can be represented as a matrix and in our case, the weights of the edges are bounded, as we use fuzzy values in $[0, 1]$. Thus, the considered space is not a vector space.
ACO is used in a similar way within the TCIU to determine appropriate sensors for collecting useful data about the user, i.e. to generate a GIST questionnaire, to be able to adapt the GUI as fast as possible.
An external instructor is needed for determining or refining some fundamentals (e.g. certain values or membership mappings) before the procedure starts so as to enable fast convergence. However, the concept of making a GUI adaptive is supposed to work no matter who the external instructor is (a psychologist or a layperson). This is essential, because two different initial state determiners can provide different fundamentals, for example because of different cultural backgrounds. The System for Adapting a GUI to a User (SAGU) developed in this thesis is visualised in Figure 9.1. Within the system, inter alia ANNs, further described in Section 9.3 and Section 9.5, are combined with a

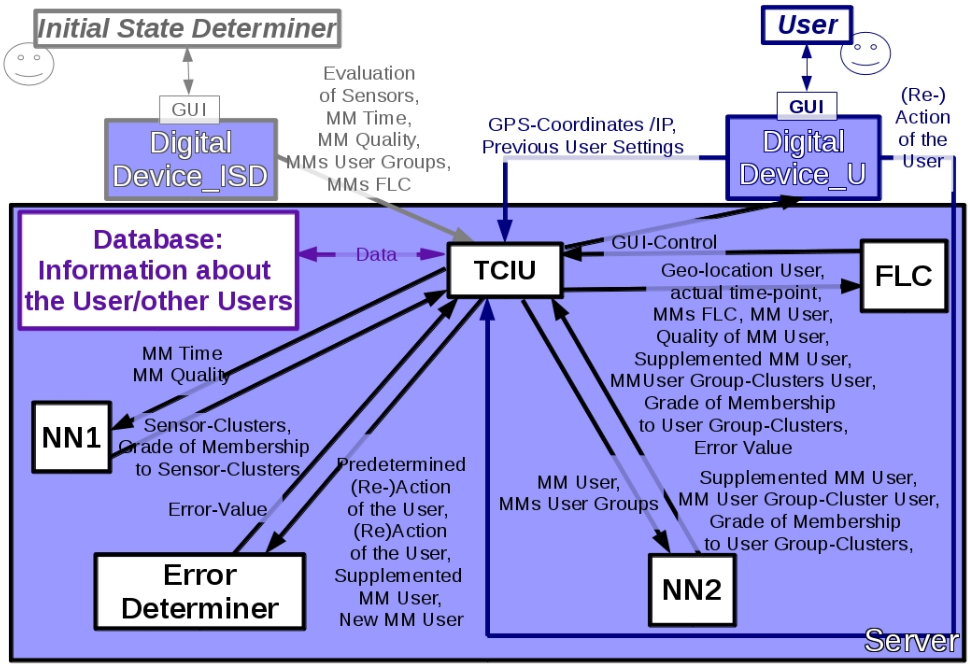


Figure 9.1: The SAGU. (Figure generated with *Libre Office Draw*.)

TCIU, further described in SECTION 9.4, and an FLC, further described in SECTION 9.6. One thing to think about is the distribution of the functions both to the digital device of the user and to a server. It is important that the application should be operational offline, because internet access cannot be guaranteed everywhere and, if the user is offline, some information may not be available. For example, the system will not be able to access information about the previous experiences of users in similar situations. Another concern is that the performance of the digital device is limited, therefore some calculations may need to be outsourced on a server. One solution to this could be to provide the TCIU, the NN1 and the FLC offline on the digital device in order to enable the collection of data. The analysis of this collected data within the NN2 could subsequently happen online when the user is able to connect to the internet.

Within the SAGU, a BP ANN may be assigned to each fuzzy membership mapping to train the mapping (see FIGURE 9.21). This works as in the following described for the approximation of the membership mappings of a user: For the adaptivity of the GUI we have to proceed in the following way: $(X_\Gamma, \mathfrak{T}_i)$, $i \in \{1, ..., n\}$ is a topological space. Information about a subspace of X_Γ is collected, e.g. via a compactum $K \subset X_\Gamma$ (DEFINITION 8.1.81), e.g. in $\mathbb{R}$ for an interval $[a, b] \subset \mathbb{R}$. Afterwards, we have to narrow our observations down to the topology which is induced by X_Γ to K, the relative topology $(\mathfrak{T}_{\Gamma, X_\Gamma|_K})$ on K, even if X_Γ is the fundamental space for the measures. Therefore, the approximation of the membership mappings of a user works as follows (see FIGURE 9.2):

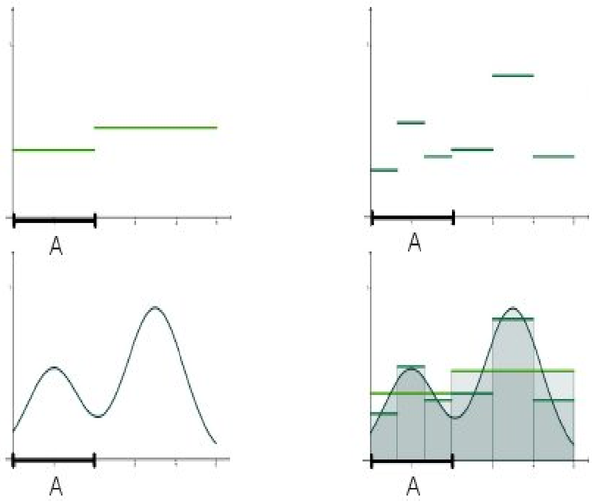


Figure 9.2: Adjusting the graphs of the membership mappings of a user to the user's needs by the availability of more information. (Figure generated with *GeoGebra* and *Libre Office Draw.*)

We collect information about the unknown mapping $f_{user} \in X_{object}$ (X_{object} is described on p.148) on the interval $A = [a, b] \in \mathfrak{T}_t \subset \mathfrak{T}_L$. $(X_\Gamma, \mathfrak{T}_\Gamma)$ and $(X_\Gamma, \mathfrak{T}_t)$ are topological spaces. f_{user} is an unknown mapping. We want to approximate f_{user} with the mapping $g_t \in X_{object}$. g_t is a dynamical mapping which changes during time. At each time $t \in \mathbb{N}$, we have a topology $\mathfrak{T}_t$ and it is $\forall\ t \in \mathbb{N} : \mathfrak{T}_t \subset \mathfrak{T}_{t+1} \subset \mathfrak{T}_\Gamma$ or $\forall\ t_1, t_2 \in \mathbb{N}, t_1 \leq t_2 : \mathfrak{T}_{t_1} \subset \mathfrak{T}_{t_2} \subset \mathfrak{T}_\Gamma$. The topology $\mathfrak{T}_t \subset \mathfrak{T}_L$ is not necessarily metrisable for all $t \in \mathbb{N}$. The information content about the unknown mapping f_{user} increases during time. We know that $\mu_{entire}(f_{user}) = \mu_{entire}(g_t) \pm \varepsilon$ with $\varepsilon > 0$ (μ_{entire} is explained on p.149). With finer topologies fuzzy functions can be measured more precisely. Coarse topologies can measure fuzzy functions only coarsely. Additionally, with coarse topologies, we have less requirements to convergence.

Example 9.0.5: *With* $\mathfrak{T}_1 = \{\emptyset, X_\Gamma, A, B\}$ *(*FIGURE *9.3, left image),*

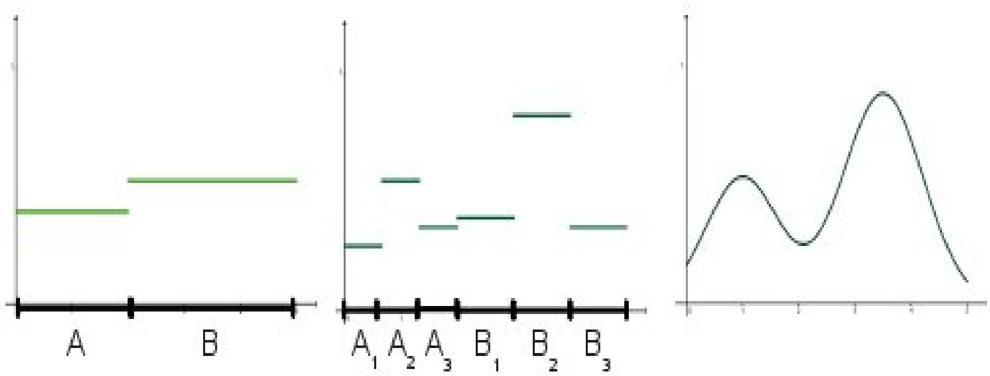


Figure 9.3: Adjusting the graphs of the membership mappings of a user to the user's needs by the availability of more information. (Figure generated with *GeoGebra* and *Libre Office Draw.*)

$\mathfrak{T}_2 = \{\emptyset, X_\Gamma, A, B, A_1, A_2, A_3, B_1, B_2, B_3, A_1 \cup A_2, A_1 \cup A_3, ...\}$ *(*FIGURE *9.3, centred image) and* $\mathfrak{T}_3 = \mathfrak{T}_L$ *(*FIGURE *9.3, right image) the membership mappings are continuous.* $(X_\Gamma, \mathfrak{T}_1), (X_\Gamma, \mathfrak{T}_2)$ *and* $(X_\Gamma, \mathfrak{T}_3)$ *are* T_3*-spaces and the sets* $A, B, A_1, ...$ *are compact, because* $\mathfrak{T}_1$ *and* $\mathfrak{T}_2$ *are finite topologies; thus, the spaces* $(X_\Gamma, \mathfrak{T}_1)$ *and* $(X_\Gamma, \mathfrak{T}_2)$ *are compact (*NOTE 8.1.85*) and, thus, closed subsets of* X_Γ *are compact (*THEOREM 8.1.84*). Further, the sets* $A, B, A_1, ...$ *are open and closed subsets of* X_Γ *with the topologies* $\mathfrak{T}_1$ *and* $\mathfrak{T}_2$*. Thus the mappings can be integrated with the Riemann-integral, because distances in the observed spaces can be understood in the Euclidean sense. When this is not the case, for example in epidemiological cases or within the TCIU (*SECTION 9.4*) or within the space* $(X_{object}, \mathfrak{T}_{object})$ *considered in the SAGU, p.148, the Lebesgue-integral is useful.* $\mathfrak{T}_1 \subset \mathfrak{T}_2 \subset \mathfrak{T}_3$*,* $(\mathfrak{T}_1, \mathfrak{T}_2, \mathfrak{T}_3)$ *is an isotonic set sequence, in other words, the more information we have, the finer the topology. In this way we always get an approximation of the membership mappings of the user's needs, because we only have discrete information and, thus, the topology* $\mathfrak{T}_\Gamma$ *will never be reached. If we had the indiscreet topology on* X_Γ*, that is, if we would not have any information about the user (except the geo-location), each filter (and net) would converge to any point with* THEOREM *8.1.77, in other words, each GUI would be proposed as suitale for the user. Thus, we had a variety of options. In order to get a handle on this, an initial GUI will be provided to users we do not have any information about except the geo-location. The initial GUI is not a global GUI, because it is GIST. We refer to the information about users in similar situations, that is, users located at the same geo-location to assign a GUI to the user that worked out fine for users in similar situations. The GUI adapts then by replacing, adding or deleting GUI element objects. By doing this, the variety of options becomes smaller and the best fitting GUI for the user is isolated. If it was* $X_\Gamma \neq A \cup B$*,* μ*-null sets could be included to measure the membership mappings. Nevertheless, changes to a mapping* f *on a* μ*-null set will not have an impact on* $\mu(f)$*. This is important, because in this way it is possible to separate important measurable user properties from the unimportant via measures.*
With this example, the finiteness of the implementation, which is fundamental for the OOA, can be illustrated. The mathematical description is infinite dimensional.

The GUI consists of a main frame that is accumulated with GUI elements which are available in different designs, i.e. objects.

$$M_{object} = E_1 \cup E_2 \cup ... \cup E_m.$$

Each GUI element $E_i = \{o_{1,E_i}, ..., o_{p,E_i}\}$, $i \in \{1, ..., m\}$, $m \in \mathbb{N}$, $p = |E_i|$ is available in different objects o_{j,E_i}. M_{object} is bounded because the amount of different objects is finite, thus $|M_{object}| < \infty$. To construct the best GUI for a user, several GUI elements are chosen, changed or deleted. Usually, the same GUI element does not appear twice as different objects on the screen. But it is, as well, possible, that a user uses frequently one GUI element as two or more different objects. The SAGU will propose to replace these GUI element objects not until another GUI element object will be needed more necessary and there is no more space on the screen to place the GUI element object. In FIGURE

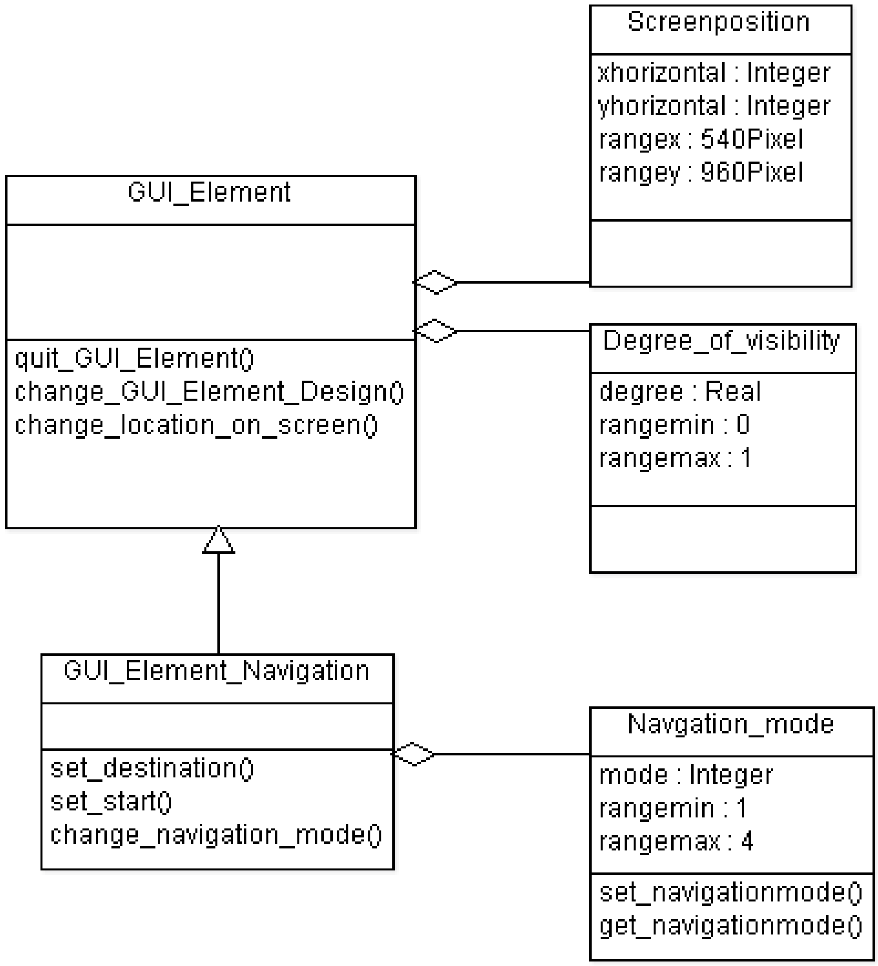


Figure 9.4: Exemplary GUI element class. (Figure generated with *ArgoUML*.)

9.4 an exemplary GUI element class (GUI element navigation) is illustrated as class diagram in UML. Open sets are required to separate GUI properties (e.g. GUI elements or GUI element objects) (separation axiom T_2). This is necessary to enable the assignment of one particular GUI adjusted to the user. In a data space about the objects of the GUI or a GIS, we try to adjust properties of the GUI using the methods of OOA. If there are two points, which are not separable in a topology, it is not possible to assign open sets (cluster) to the two points for the properties of the GUI. If those properties of the GUI are not separable in the topology, they are dependent. The separation axiom T_{3a} plays an important role for the development of an SAGU. T_{3a} enables the separation of a measurable set and a point. This is important for the evaluation of a set of collected data (about a user) and the determination of a membership mapping representing the user's needs. Furthermore, it is important to separate the set of GUI elements from the points contained therein, the GUI element objects.

Relations between topological spaces are mediated by continuous mappings. With homeomorphisms neighbourhood-"information" can be transferred from a topological space $(X, \mathfrak{T}_X)$ to a topological space $(Y, \mathfrak{T}_Y)$. THEOREM 8.1.29 showed how information (about a user) can be transferred from $(X, \mathfrak{T}_X)$ to $(Y, \mathfrak{T}_Y)$. Topologies are assigned to an object in OOA (CHAPTER 6) as information content via Cartesian Product of status-spaces of object-classes. On neighbourhoods in $(\mathbb{R}, \mathfrak{T}_{\mathbb{R}})$, continuous mappings $f : (X, \mathfrak{T}_X) \overset{\mathfrak{T}}{\to} (\mathbb{R}, \mathfrak{T}_{\mathbb{R}})$ behave similarly. Within the topological space $(X, \mathfrak{T}_{dis})$ the one-point sets are open. Each mapping $f : (X, \mathfrak{T}_{dis}) \overset{\mathfrak{T}}{\to} (Y, \mathfrak{T}_Y)$ is continuous and for a one-point set $\{y\} \subset Y$ we cannot imply the behaviour of f in a larger neighbourhood $U_Y \subset Y$ of y, because we can not extrapolate single point information of the codomain of a mapping to the topological neighbourhood. This is not an advantage if we want to transfer information (about a user) from $(X, \mathfrak{T}_X)$ to $(Y, \mathfrak{T}_Y)$.

The GUI for an application for digital devices (e.g. smartphones) to deliver decision support to users in hazardous situations consists of several GUI elements in certain GUI element objects. The degree of the quality of a GUI element object can be represented by fuzzy membership mappings

$$f_{o_{j,E_i}} : (X_\Gamma, \mathfrak{T}_\Gamma) \to ([0,1], \mathfrak{T}_{nat,\mathbb{R}|_{[0,1]}}),$$

with $o_{j,E_i} \in M_{object} = E_1 \cup ... \cup E_m$, M_{object} is the set of the GUI element objects. We have a bidirectional net on the set M_{object}. The nodes of this net are the GUI element objects and the edges are distances between the GUI element objects determined via measures. The symmetry property is, among other things, lacking, when the comprehension of a GUI element object o_{j,E_i} is dependent on a GUI element object o_{k,E_j} (thus o_{k,E_j} can be comprehended without o_{j,E_i} but not the other way round) the distance from o_{j,E_i} to o_{k,E_j} is lower than the distance from o_{k,E_j} to o_{j,E_i}. This way, the quality of the GUI element objects and thus the mapping $f_{o_{j,E_i}}$ is influenced.

$$X_\Gamma := S^2_{R_0}(z) \times T$$

with $S^2_{R_0}(z) = \{\omega \in [0, R_0]^3 \mid \|\omega - z\|_2 = R_0\}$ with

$$\omega = \begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix} = \begin{pmatrix} R_0 \cdot \sin(\theta) \cdot \cos(\varphi) \\ R_0 \cdot \sin(\theta) \cdot \sin(\varphi) \\ R_0 \cdot \cos(\theta) \end{pmatrix}$$

with $\theta \in [-\frac{\pi}{2}, \frac{\pi}{2}]$ which corresponds to the latitude $[-90°, 90°]$ and $\varphi \in (-\pi, \pi]$ which corresponds to the longitude $(-180°, 180°]$, $z = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ is the barycentre of the three-dimensional object describing the earth, $T = \mathbb{R}$ the time and R_0 the distance from the barycentre z to the earth's surface at the timepoint $t \in T$ (averaged $6371km$).

$$\|\omega - z\|_2 = \sqrt{(\omega_1 - 0)^2 + (\omega_2 - 0)^2 + (\omega_3 - 0)^2}.$$

The degree of the quality of a GUI element for a user is dependent on geo-location and time, because certain infectious diseases only occur in areas with suitable environmental conditions, and depending on the weather, for example. $\mathfrak{T}_\Gamma$ is generated by $L_1 \times L_2 \times T_0$ with

$$L_1 := \{(a_1, b_1] \mid -180 < a_1 \leq b_1 \leq 180\} \subset (-180, 180],$$

$$L_2 := \{[a_2, b_2] \mid -90 \leq a_2 \leq b_2 \leq 90\} \subset [-90, 90] \text{ and}$$

$$T_0 := [a_0, b_0] \subset T = \mathbb{R}.$$

Thus,

$$\mathfrak{T}_\Gamma = \mathfrak{T}_{nat,(-180,180]}|_{(a_1,b_1]} \otimes \mathfrak{T}_{nat,[-90,90]}|_{[a_2,b_2]} \otimes \mathfrak{T}_{nat,\mathbb{R}}|_{[a_0,b_0]}$$

with $-180 < a_1 \leq b_1 \leq 180$, $90 \leq a_2 \leq b_2 \leq 90$, $a_0, b_0 \in \mathbb{R}$.

$$f_{o_{j,E_i}} \in \mathscr{L}^1((X_\Gamma, \mathfrak{T}_\Gamma), \mu_{L,X_\Gamma}) =: L_\Gamma,$$

μ_{L,X_Γ} is the Lebesgue-measure restricted to X_Γ. L_Γ is a function space which is infinite dimensional. We use the Lebesgue-measure within the SAGU, because changes of a mapping f on a μ-null set will not have an impact on the Lebesgue-measure $\mu_L(f)$. This is important for the SAGU because this way it is possible to separate important measurable properties of a user from the unimportant via measures.

With

$$f_{entire} : (X_\Gamma, \mathfrak{T}_\Gamma) \xrightarrow{\mathfrak{T}} ([0,1]^{|M_{object}|}, \underbrace{\mathfrak{T}_{nat,\mathbb{R}}|_{[0,1]} \otimes ... \otimes \mathfrak{T}_{nat,\mathbb{R}}|_{[0,1]}}_{|M_{object}| \text{ times (product topology)}})$$

$$(x, y, t) \mapsto (f_{o_{1,E_1}}(x, y, t), ..., f_{o_{k,E_m}}(x, y, t)),$$

$k = |E_m|$,

$$f_{entire} \in X_{object} \subset L_\Gamma^{|M_{object}|},$$

we can determine the quality of each GUI element object $o_{j,E_i} \in M_{object}$ at each geo-location on earth at time points $\omega = (x, y, t) \in X_\Gamma$, with $t \in T_0$ and T_0 a compact subset of T, and the best fitting GUI elements can be chosen for a user. X_{object} is eventually larger, because computer scientists can add more attributes in the future, that have not been taken into account yet. X_{object} is a topological space. The Lebesgue-topology $\mathfrak{T}_{L_0}$ induced by the $\mathscr{L}_1$-norm: $\|F\| = \sum_{i=1}^{m} \|f_{o_{j,E_i}}\|_1$ is not sufficient to describe the space in hand properly. The topology $\mathfrak{T}_{object}$ on X_{object} is induced by

$$d_{object} : X_{object} \times X_{object} \to [0, \infty)$$

$$(f_{entire}, g_{entire}) \mapsto d_{object}(f_{entire}, g_{entire})$$

$d_{object}(f_{entire}, g_{entire})$ is determined by a bidirectional net on X_{object}. The nodes of this net are the membership mappings of X_{object} and the weighted edges are distances between the membership mappings determined via measures, whereby the distance form f_{entire} to g_{entire} can be another than from g_{entire} to f_{entire}; f_{entire} $g_{entire} \in X_{object}$. Thus $(X_{object}, \mathfrak{T}_{object})$ is no metric space. The net can be represented by a weighted matrix $M_t \in (n \times n, \mathbb{C}) \in \mathbb{C}^{n^2}$ with n nodes, i.e. n membership mappings from X_{object}. This net is seized again in the NN2 (SECTION 9.5). $(X_{object}, \mathfrak{T}_{object})$ is a locally compact T_3-space. A membership mapping of a user f_{entire} describing the quality of each GUI element object for the user can be measured via

$$\mu_{entire}(f) :(L_{\Gamma}^{|M_{object}|}, \underbrace{\mathfrak{T}_L \otimes ... \otimes \mathfrak{T}_L}_{|M_{object}| \text{ times (product topology)}}) \xrightarrow{\mathfrak{T}} (\mathbb{C}, \mathfrak{T}_{nat})$$
$$f \mapsto \frac{1}{|M_{object}|} \sum_{k=1}^{|M_{object}|} \mu_{L,X_{\Gamma}}(f_k),$$

$f_k \in L_{\Gamma}$. $\mu_{entire}(f)$ is a complex Radon measure, because it fulfils the properties of DEFINITION 8.2.10: it is a linear form, because

1. $\forall\ f, g \in L_{\Gamma}^{|M_{object}|} : \mu_{entire}(f+g)$
$= \frac{1}{|M_{object}|} \sum_{k=1}^{|M_{object}|} \mu_{L,X_{\Gamma}}(f_k + g_k)$
$= \frac{1}{|M_{object}|} \sum_{k=1}^{|M_{object}|} \mu_{L,X_{\Gamma}}(f_k) + \mu_{L,X_{\Gamma_0}}(g_k)$
$= \frac{1}{|M_{object}|} \sum_{k=1}^{|M_{object}|} \mu_{L,X_{\Gamma}}(f_k) + \frac{1}{|M_{object}|} \sum_{k=1}^{|M_{object}|} \mu_{L,X_{\Gamma}}(g_k)$
$= \mu_{entire}(f) + \mu_{entire}(g)$

2. $\forall\ f \in L_{\Gamma}^{|M_{object}|}, \lambda \in \mathbb{C} : \mu_{entire}(\lambda f)$
$= \frac{1}{|M_{object}|} \sum_{k=1}^{|M_{object}|} \mu_{L,X_{\Gamma}}(\lambda f_k)$
$= \frac{1}{|M_{object}|} \sum_{k=1}^{|M_{object}|} \lambda \mu_{L,X_{\Gamma}}(f_k)$
$= \lambda \frac{1}{|M_{object}|} \sum_{k=1}^{|M_{object}|} \mu_{L,X_{\Gamma}}(f_k)$
$= \lambda \mu_{entire}(f_k)$

And the following property is fulfilled:
For each compact set $K \subset X_{\Gamma}$ there is a $c_K \geq 0$ with

$$f \in L_{\Gamma}^{|M_{object}|}, Tr(f) \subset K \Rightarrow |\mu_{entire}(f)| \leq c_K \|f\|_{\infty}^{entire},$$

with the semi-norm

$$\|f\|_{\infty}^{entire} :L_{\Gamma}^{|M_{object}|} \to \mathbb{R}_0^+$$
$$(f_1, ..., f_{|M_{object}|}) \mapsto \max\{\|f_1\|_{\infty,\mu}, ..., \|f_{|M_{object}|}\|_{\infty,\mu}\},$$

with the semi-norm

$$\|f\|_{\infty,\mu} :\mathscr{L}^1((X_{\Gamma}, \mathfrak{T}_{\Gamma}), \mu_L) \to \mathbb{R}_0^+$$
$$f_k \mapsto \inf\{c \geq 0 \mid M_c := \{x \in X_{\Gamma} \mid |f_k(x)| > c\} \wedge (\mu(M_c) := \mu(1_{M_c}) = 0)\},$$

and with $K := \bigcup_{k \in \{1,...,|M_{object}|\}} K_k$ with K_k compact set for f_k,
because

$$|\mu_{entire}(f)| = |\frac{1}{|M_{object}|}\sum_{k=1}^{|M_{object}|}\mu_{L,X_{\Gamma_0}}(f_k)|$$
$$\leq \frac{1}{|M_{object}|}\sum_{k=1}^{|M_{object}|}|\mu_{L,X_\Gamma}(f_k)|$$
$$\leq \frac{1}{|M_{object}|}\sum_{k=1}^{|M_{object}|}\widetilde{c_k}\cdot\|f_k\|_{\infty,\mu}$$
$$\leq \frac{1}{|M_{object}|}\cdot\sum_{k=1}^{|M_{object}|}\widetilde{c_k}\cdot\|f\|_\infty^{entire}\cdot|M_{object}|$$
$$= \frac{1}{|M_{object}|}\cdot|M_{object}|\cdot\left(\sum_{k=1}^{|M_{object}|}\widetilde{c_k}\right)\cdot\|f\|_\infty^{entire}$$
$$\overset{\sum_{k=1}^{|M_{object}|}\widetilde{c_k}:=c_K}{=} c_K\|f\|_\infty^{entire}$$

We look for membership mappings from X_{object}, which deliver with the measure μ_{entire} the same or a similar value, i.e. the difference is smaller than $\varepsilon > 0$. Therewith, we can approximate the membership mapping of a user. We have collected training data about the user: $D = \{(A_k, m_k) \mid k \in \{1, ..., N\}\}$, whereby A_k is a compactum and m_k is a measured value assigned to this set. Including the level surface, we receive:

$$\mathfrak{L}_\varepsilon = \{f_{entire} \in X_{object} \mid |\mu(f_{entire}\cdot 1_{A_k}) - m_k| < \varepsilon\}$$

for all $k \in k \in \{1, ..., N\}$. If $\mathfrak{L}_\varepsilon = \emptyset$, we can find nearest surfaces with $\inf\{\varepsilon > 0 \mid \mathfrak{L}_\varepsilon \neq \emptyset\}$.
The SAGU works like a Filter $\mathfrak{F}_{object}$ on the set X_{object} of the membership mappings determining the GUI. For the SAGU it is important to collect data (about a user). The more information we have about the user, the precise the result and, thus, the assigned GUI will be. If we have no information about the user (except the geo-location) our situation could be comparable to having the indiscreet topology $\mathfrak{T}_{ind} = \{\emptyset, X_{object}\}$ on X_{object}. With Theorem 8.1.77 we know that in the indiscreet topology $\mathfrak{T}_{ind}$, every filter is convergent to every point $x \in X_{object}$ and, thus, each GUI would be proposed as suitale for the user. Thus, we had a variety of options. In order to get a handle on this, an initial GUI will be provided to users we do not have any information about except the geo-location. The initial GUI is not a global GUI, because it is GIST. We refer to the information about users in similar situations, that is, users located at the same geo-location to assign a GUI to the user that worked out fine for users in similar situations. The GUI adapts then by replacing, adding or deleting GUI element objects. By doing this, the variety of options becomes smaller and the best fitting GUI for the user is isolated.
The filter $\mathfrak{F}_{object}$ possesses at least one point of adherence representing one GUI. In a compact space, each filter has a point of adherence, as Theorem 8.1.86 signifies. If the space the SAGU operates in was a compact space it could be guaranteed that a GUI is proposed as suitale for the user. Thus, if the filter is not convergent, a refinement of the filter is convergent to the point of adherence. If the refinement of this filter is convergent to more than one point, that is, more than one GUI would be proposed as suitale for the user, one proposed GUI is tested by the user for some time, after which a question for securing a decision is asked with the possibility to obtaining another GUI, or the user

could choose the GUI manually. As X_{object} is not Hausdorff, we know with THEOREM 8.1.78: a filter $\mathfrak{F}$ on X_{object} is convergent to one point at most, if and only if $(X_{object}, \mathfrak{T}_{object})$ is a T_2-space, that it can happen that more than one GUI are proposed as suitale for the user. The involvement of the user in the decision process or random experiments can resolve such a situation as one GUI can be assigned to the user. But as we can only collect discreet and thus incomplete information about the user's needs and the user's needs can change in each time-step we only can try to approximate the membership mapping of a user, we do not have a Banach Space. Several adjustment steps cannot necessarily be represented by a Cauchy sequence, because the user needs can change and thus another GUI is required.
In the following, the SAGU and its single components are described in more detail. First, we will have a look on the initial state determiner (SECTION 9.1), who determines inter alia membership mappings belonging to X_{object} representing different user groups. Those mappings are the cluster-centres used within the NN2 (SECTION 9.5).

9.1 The Initial State Determiner

An initial state determiner, e.g. a person or a group of persons, feeds once the TCIU with self-determined evaluations of the sensors of the TCIU and self-determined membership mappings for the NN1 (membership mappings determining the time to edit a sensor and the quality of the information gained by using a sensor), the NN2 (membership mappings describing the users and the user groups) and the membership mappings for the FLC, see FIGURE 9.5. This happens with the use of a digital device with an appropriate access assistance, e.g a GUI. The comparison of membership mappings is important for the initial state determiner. Assume that one membership mapping for the initial state has to be determined and we have two initial state determiners, who generate membership mappings for the same situation independently of each other. If the mappings of the two initial state determiners are strictly order-perceiving, the initial state determiners decide similarly and one of the two generated membership mappings can be chosen. If the mappings are not strictly order-perceiving, we have to calculate the error. Consequently, if this error is bigger than a certain limit, we have to figure out why the membership mappings are different. Accordingly, the initial state determiners have to communicate to find a common solution. Therefore, the measure μ_{SOP} can be used to compare certain membership mappings. For the definition of μ_{SOP} the definition of an ordinal homomorphism is important:

Definition 9.1.1 *Ordinal Homomorphism (cf [Mes72], p.189)*
$(M_1, \prec_1)$ and $(M_2, \prec_2)$ are ordered sets. A mapping

$$m_{OH} : M_1 \to M_2$$
$$x \mapsto m_{OH}(x)$$

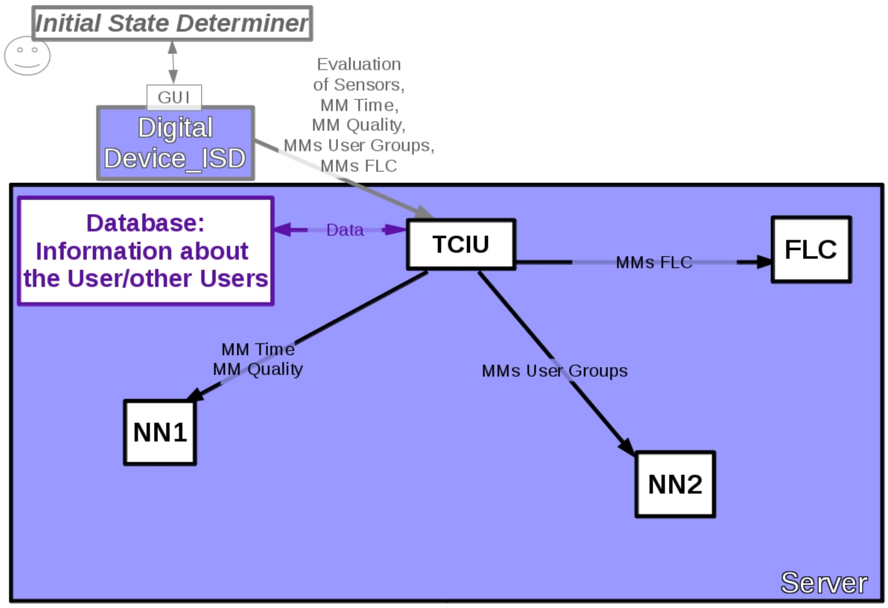


Figure 9.5: The initial state determiner within the system. (Figure generated with *Libre Office Draw.*)

with the property

$$x \prec_1 y \Leftrightarrow m_{OH}(x) \prec_2 m_{OH}(y)$$

is called ordinal Homomorphism.

The mappings m_j, $j \in \{1,2\}$ are membership mappings with $m_j : (X, \mathfrak{T}_X) \overset{\mathfrak{T}}{\to} ([0,1], \mathfrak{T}_{nat,\mathbb{R}|_{[0,1]}})$, order-relations are defined on X and $[0,1]$, fulfilling the properties

$$\forall\ x, y \in X : m_1(x) < m_1(y) \Leftrightarrow m_2(x) < m_2(y).$$

The mappings are strictly order-preserving. Assume that we have two membership mappings m_1 and m_2 which are not strictly order-preserving. Then we can determine the range M_{range}, where m_1 and m_2 are not strictly order-perserving with

$$M_{range} := \{(x,y) \in X \times X \mid m_1(x) \geq m_1(y) \Leftrightarrow m_2(x) < m_2(y)\}.$$

$$h_{m_1,m_2} : (X \times X, \mathfrak{T}_X \otimes \mathfrak{T}_X) \overset{\mathfrak{T}}{\to} (\mathbb{C}, \mathfrak{T}_{nat})$$
$$(x,y) \mapsto \begin{cases} 0, \text{ if } (x,y) \notin M_{range} \\ \mu_L(m_1(x,y), m_2(x,y)), \text{ if } (x,y) \in M_{range} \end{cases}$$

$h_{m_1,m_2} \in \mathscr{L}^1((X \times X, \mathfrak{T}_{X, \times X}), \mu_{SOP})$ and μ_{SOP} is a measure with

$$\mu_{SOP} : (\mathscr{L}^1((X \times X, \mathfrak{T}_X \otimes \mathfrak{T}_X), \mu_{SOP}), \mathfrak{T}_L) \overset{\mathfrak{T}}{\to} (\mathbb{C}, \mathfrak{T}_{nat})$$
$$h_{m_1,m_2} \mapsto \int_{M_{range}} h_{m_1,m_2} d\mu_{SOP}.$$

The error can be determined with μ_{SOP}. Additionally to the initial state determiner, μ_{SOP} is important when generating risk, resource or decision support maps.

9.2 The Database

In addition to the initial state determiner's membership mappings, the user can decide to deliver information with her/his digital device (e.g. in a crowd-sourcing environment) about her/his previous settings, her/his geo-location and information about previous experience of users in similar situations, e.g. users located at the same geo-location, is collected in a database, see FIGURE 9.6, in the form of an encrypted user profile with the user's permission, for data protection. Via the TCIU, the following data is saved in the database: the membership mappings required by the NN1, the NN2 and the FLC, the evaluation of the sensors and the sensor clusters for the TCIU, the error value based on the (re)action of the user and the user group cluster of the user, which are further processed in the FLC and the GUI control.

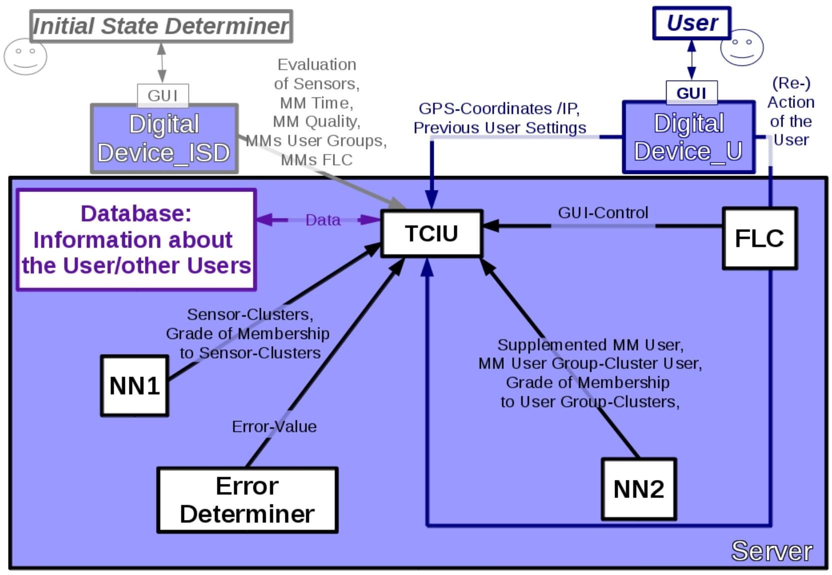


Figure 9.6: Database: information about the user/other users within the system. (Figure generated with *Libre Office Draw*.)

9.3 The Neural Network *NN1*

The NN1 clusters the sensors (see p.156) of the TCIU for the internal logic of the TCIU, see FIGURE 9.7. NN1 is a recurrent fuzzy ART ANN (see DEFINITION 7.2.14), because we need an ANN that can go back to earlier collected information and, subsequently, fuzzy information can be processed.
In our case, the fuzzy ART ANN is used to cluster the sensors of the TCIU within SECTION 9.4.

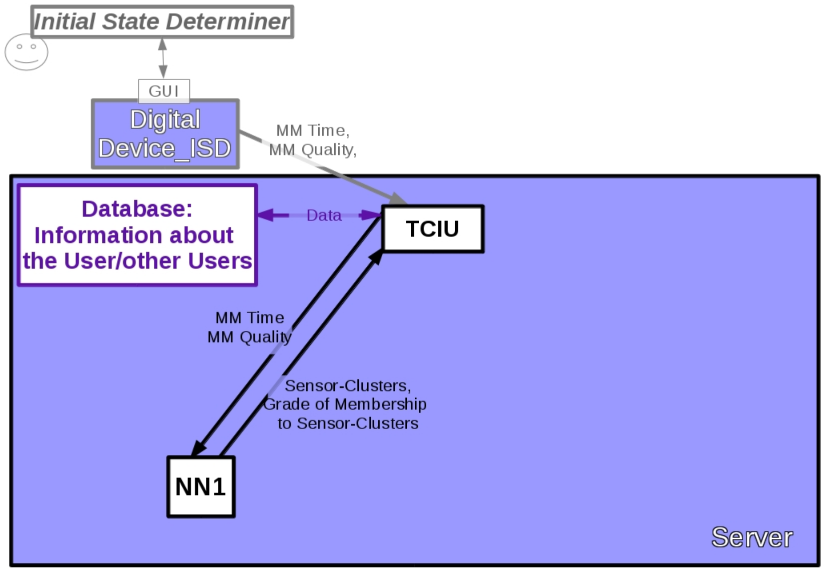


Figure 9.7: The NN1 within the system. (Figure generated with *Libre Office Draw.*)

To each sensor, a membership mapping determining the time to edit a sensor (see p.156) and a membership mapping determining the quality of the information gained by using a sensor is assigned. Therefore a cluster analysis is done with the fuzzy ART ANN and the grade of membership of the sensor to the sensor-clusters is determined. The cluster-centers are changed dependent to the grade of membership of the sensor to the clusters. The one cluster with the largest grade of membership, that is the weight-vector describing the best fitting membership mappings for the respective sensor is chosen for the TCIU. w_k with $k \in \{1, ..., n\}$ and v are vectors consisting of membership mappings: $w_k = \begin{pmatrix} \tau_k \\ \varpi_k \end{pmatrix} \in \mathfrak{K}_+(([0, 100], \mathfrak{T}_{nat,\mathbb{R}|_{[0,100]}}))^2$ and

$$i_k : ([0, 100], \mathfrak{T}_{nat,\mathbb{R}|_{[0,100]}}) \xrightarrow{\mathfrak{T}} ([0, 1], \mathfrak{T}_{nat,\mathbb{R}|_{[0,1]}}),$$

$i \in \{\varpi, \tau\}$. Thus, the distances of $v = \begin{pmatrix} \tau_v \\ \varpi_v \end{pmatrix}$ to the weight vectors are determined with

$$\begin{aligned} f_{error_k} :& \mathfrak{K}_+(([0, 100], \mathfrak{T}_{nat,\mathbb{R}|_{[0,100]}}))^2 \times \mathfrak{K}_+(([0, 100], \mathfrak{T}_{nat,\mathbb{R}|_{[0,100]}}))^2 \to \mathbb{C} \\ &(w_k, v) \mapsto \lambda_S + |\mu_{NN1}(\tau_k) - \mu_{NN1}(\tau_v)| + |\mu_{NN1}(\varpi_k) - \mu_{NN1}(\varpi_v)| \end{aligned}$$

with $k \in \{1, ..., n\}$, with $\mu_{NN_1}(f) = \int_{[0,100]} |f| d\mu_{NN1}$ for $f \in \mathfrak{K}_+(([0, 100], \mathfrak{T}_{nat,\mathbb{R}|_{[0,100]}})$ and λ_S is dependent on the number of sensors within a cluster described by a weight-vector, that is, λ_S is low, if many sensors are assigned to a weight-vector and λ_S is large if few sensors are assigned to a weight vector. The vectors consisting of membership mappings w_k are initially determined by an initial state determiner. With the user's permission, the weight vectors can be saved in a database and can be used again by the same user or by other users. Open sets are used for the learning task of the ANN to identify the similarity of outputs to similar inputs. Convergence is applied to the learning task of the ANN in order to investigate if similar inputs produce "more similar" outputs in a certain time-period, that is, whether the learning task is solved better if the system is improved.

9.4 Tool for Collecting Information about the User

One task of the TCIU is the distribution of data. The TCIU uses information delivered by the user via the digital device of the user. For example, the GPS or IP information of the user is used to determine

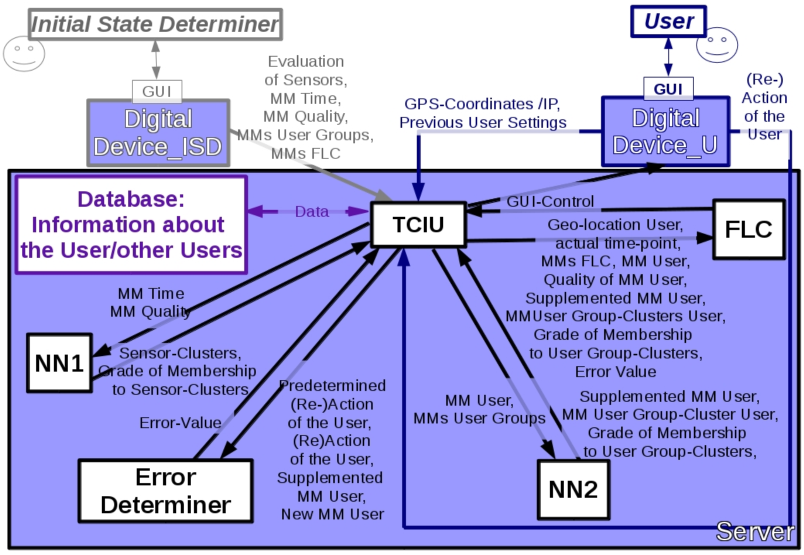


Figure 9.8: The TCIU within the system. (Figure generated with *Libre Office Draw*.)

a GPS block. The GPS blocks are linked to local areas (GPS blocks are described in more detail on p.156). The other information referred to (a membership mapping determining the time to edit a sensor, a membership mapping determining the quality of the information gained by using a sensor and the evaluation of the sensors) is used to determine an output, that is, identify the next question or policy proposal or policy for the user. If we have information about the previous user settings, the questionnaire can work more effectively, because we know for example which questions have already been asked and we do not have to ask them again. Output is the geo-location of the user, the current time-point, the approximated membership mapping of the user and the quality of the approximated membership mapping of the user, which is delivered to the FLC. The approximated membership mapping of the user is delivered to the NN2 (Section 9.5), see Figure 9.8, as well. The information about the evaluation of the sensors is important for the approximation of the membership mapping of the user. Within the TCIU there is a dynamical network which represents the internal logic for generating an output submitted to the digital device based on the action or reaction of the user. This dynamical network works similar to a questionnaire. Based on the definition of a questionnaire, a definition for the TCIU can be derived, whereby information is defined like on p.5.

Definition 9.4.1 *Questionnaire ([AR96])*
The term "questionnaire" is a collective term for a variety of instruments. The indicator of this instrument is that the testee has to relate a predetermined written set of questions, statements, adjectives to herself/himself and she/he has to make a judgement about herself/himself using a prescribed format for the answer. Questionnaires can be interpreted as verbal reports on language stimuli given by test persons. The questions or statements are subjected to the principles of the test theory following selection.

The network used in this thesis is more comprehensive than a questionnaire.

Definition 9.4.2 *Tool for Collecting Information about the User (TCIU)*
The TCIU is a questionnaire with a supplement, that it is GIST (p. 87) and includes not only language stimuli, but visualised stimuli as well. Moreover, not only verbal reports, but information about the action of a user is processed as well. The information is processed even if there are no stimuli. Another task of the TCIU is the distribution of data.

$$X_S = \{s_1, ..., s_n\}$$

is the set of all sensors within the TCIU. A sensor can be for example a question of the GIST questionnaire, hardware of the digital device of the user e.g. a camera, or the GPS for user location or to track the motion of a user.
For the implementation, we use a finite number of sensors (limitation of performance of digital devices), but an infinite dimensional not necessarily countable sensor space.
The TCIU has to face mainly three tasks:

- Determine the use of sensors based on the user (re)action.
- Determine the quality of the approximated membership mapping of the user based on the amount of collected information.
- The distribution of information within the SAGU.

To be able to use appropriate sensors for the user, GPS blocks have to be determined, because the quality of a sensor is geo-location-dependent, for example, it does not make sense to ask the user about malaria if the user is located at the North Pole.

$$X_{GPS} = \{GPS_1, GPS_2, ..., GPS_m\}$$

is the set of all GPS blocks. We use a finite number of GPS blocks to enable an implementation for digital devices but an infinite not necessarily countable space of GPS blocks. $(X_{GPS}, \mathfrak{T}_{GPS})$ is a topological space. The GPS blocks can be represented by fuzzy sets. Thus, the degree of membership of a user to a GPS block can be determined. The GUI assigned to a user is based on the user's geo-location and thus on the GPS block the user is located in. When a user moves towards the boundary of the GPS block she/he is located in, the GUI proposes a change to another GUI adjusted for the GPS block towards which the user is moving, if the user is near to the boundary of that GPS block. The GPS blocks could thus be numbered, as shown in FIGURE 9.9. The GPS blocks are assigned to

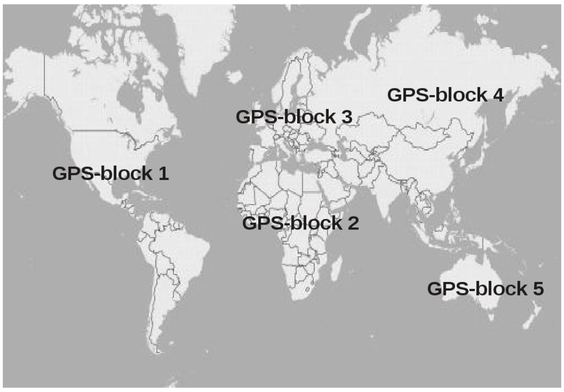


Figure 9.9: Exemplary GPS blocks numbered according to continents. (Figure generated with the use of *OSM* and *Libre Office Draw.*)

sensor blocks containing sensors fitting to the geo-location of the user $X_{SB} = \{SB_1, ..., SB_m\}$ with a mapping:

$$(X_{GPS}, \mathfrak{T}_{GPS}) \stackrel{\mathfrak{T}}{\to} (X_{SB}, \mathfrak{T}_{SB}).$$

We do not have a metric on SB_i, $i \in \{1, ..., m\}$, because the symmetry-property (property 3. of DEFINITION 8.1.1.1), is not fulfilled. The reason for this is if a sensor x was used, for example, a general question would be answered with "no" like: "Are you male?", y, then, would be a question that goes into more detail on the basis of the answer to question x like: "Are you pregnant?", which is answered with "yes". However, with those answers the questions would usually not be asked the other way round, that is, $d(y, x) > d(x, y)$, the distance from y to x is largen than from x to y.

Example 9.4.3: *A dynamic network could work as illustrated in* FIGURE *9.10.*

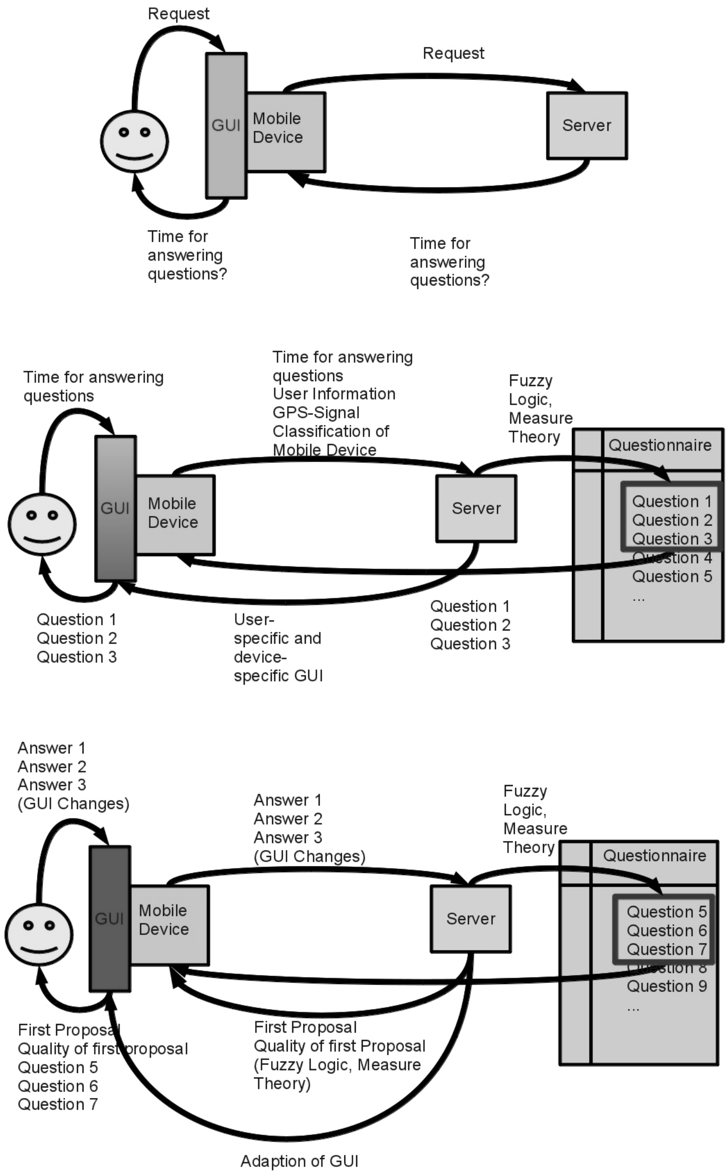


Figure 9.10: Adaption of a GUI and delivering help with the assistance of the TCIU. (Figure generated with *Libre Office Draw.*)

The structure of the TCIU is visualised in FIGURE *9.11.*

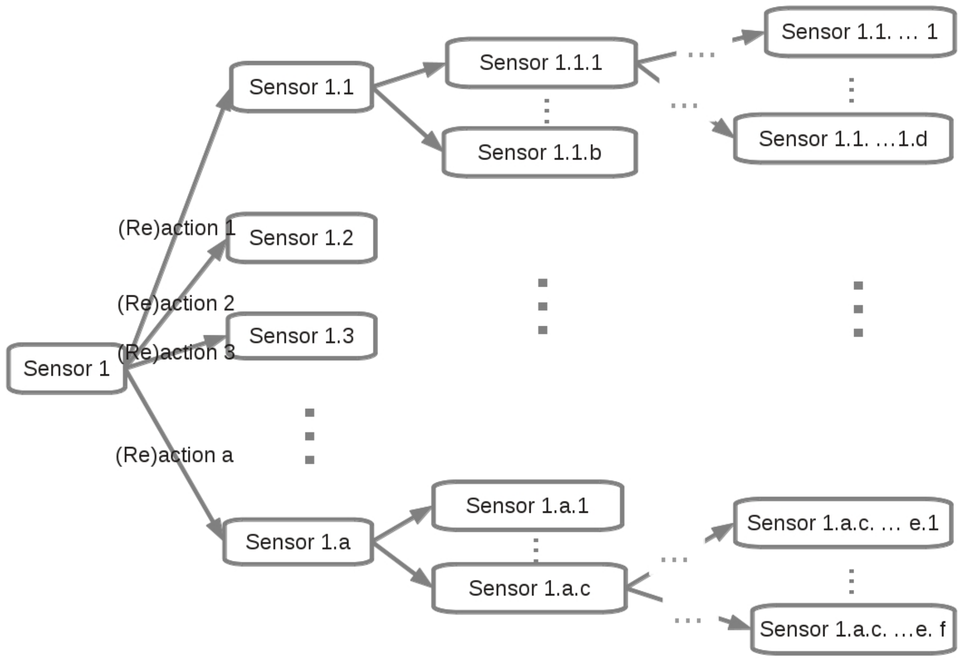


Figure 9.11: Structure of the TCIU with $a, b, c, d, e, f \in \mathbb{N}$. (Figure generated with *Libre Office Draw.*)

The amount of collected information about the user can be determined as following:

$$S = \{1, 2, ..., n\}$$

is the set of the numbers of the sensor columns, and

$$R = \{0, 1, ..., n-1\}$$

is the set of the numbers of (re)actions of a user, e.g. the answers of the user on questions, i.e. the set of numbers of sensors that were used to collect information about a user. The mapping

$$\begin{aligned} i_{user} :& S \times S \mapsto R \\ & (s, \tilde{s}) \mapsto |\tilde{s} - s| \end{aligned}$$

describes the amount of collected information about the user.
i_{user} describes a metric on S.
The sensors should be clearly labelled to ensure that one sensor is used twice only if absolutely necessary. Classes must be built to categorise the sensors (FIGURE 9.12).

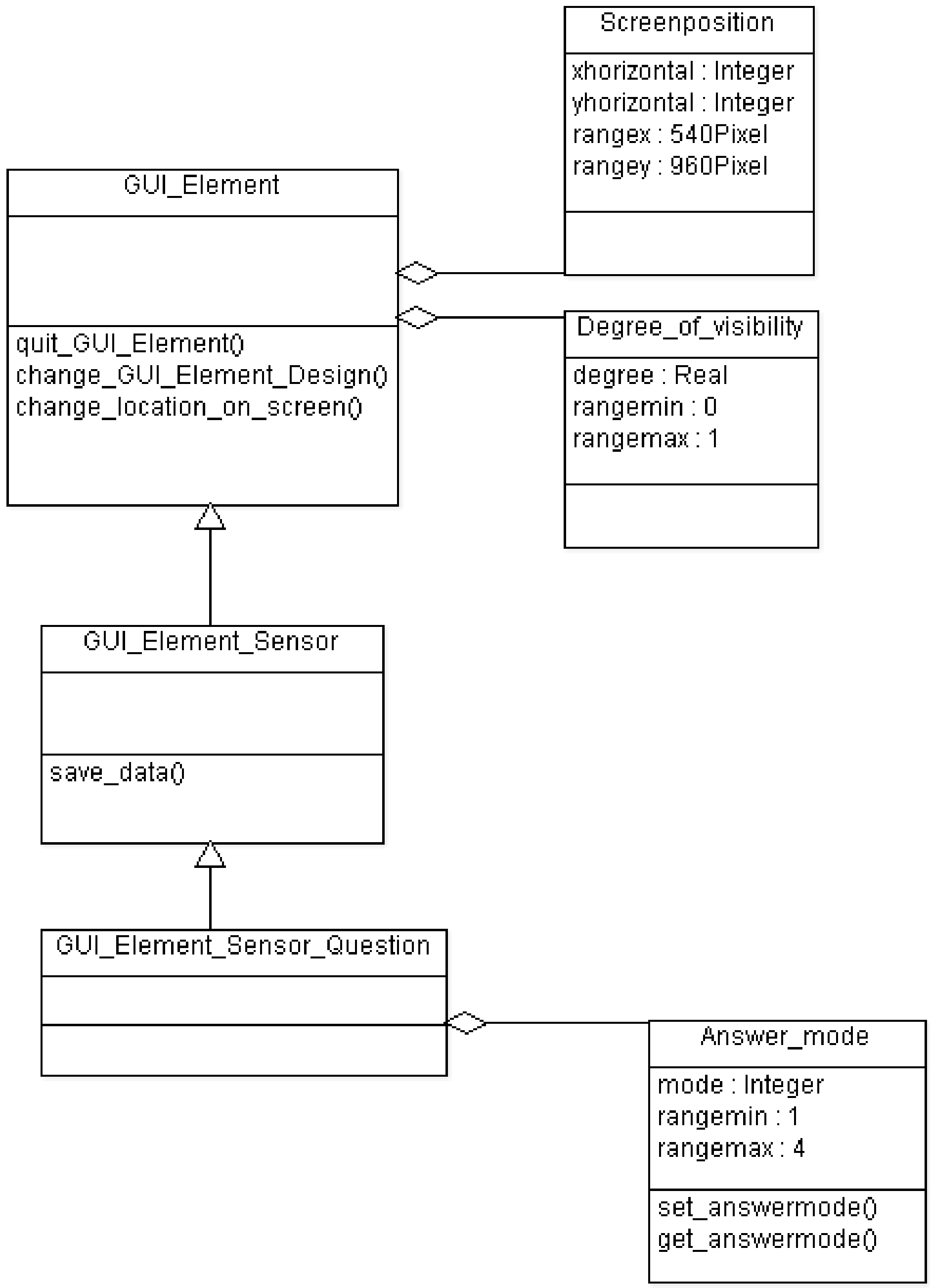


Figure 9.12: Exemplary sensor. (Figure generated with *ArgoUML*.)

Within these classes, the sensors must be evaluated according to, for example, the time that is needed to execute a sensor to, for example, answer a question. This evaluation can be done using a fuzzy mapping, which has the following properties:

- Domain is the interval $[0, 100] \subset \mathbb{R}$ and codomain is the unit interval $[0, 1]$.
- The mapping is continuous and monotonically increasing.
- The mapping is an element of $\mathfrak{K}_+([0, 100], \mathfrak{T}_{nat,\mathbb{R}|_{[0,100]}})$.

Example 9.4.4: *For example, the mapping visualised in* FIGURE *9.13 would be in line for our issues.*

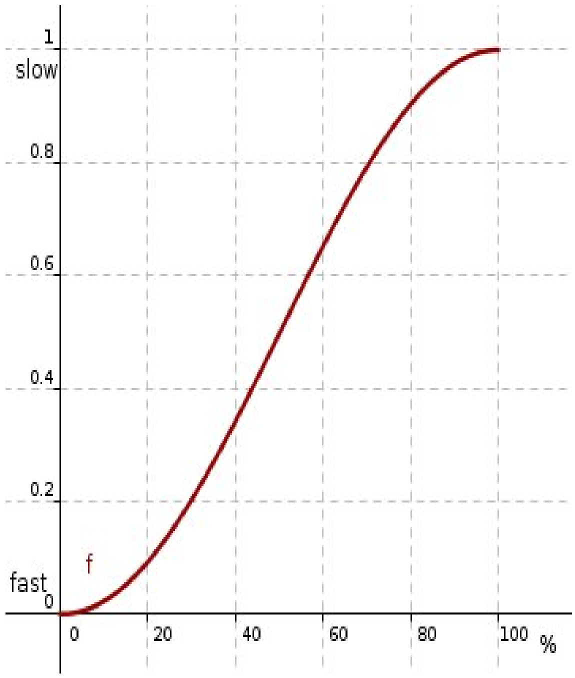


Figure 9.13: Mapping (f) assigning a value between 0 and 1 to the approximated time used for executing a sensor according to the available time to execute sensors in % with $f : ([0,100], \mathfrak{T}_{nat,\mathbb{R}|_{[0,100]}}) \overset{\mathfrak{T}}{\to} ([0,1], \mathfrak{T}_{nat,\mathbb{R}|_{[0,1]}}); x \mapsto -\cos(\frac{x-0.4}{100-0.4} \cdot \frac{\pi}{2})^2 + 1$. (Figure generated with *GeoGebra*.)

We have a network-structure on X_S to describe the distances between the different sensors. The nodes are the sensors. The net changes at each time-point. One method to determine the weights on the edges at the current time-point is described in the following example, EXAMPLE 9.4.5. The ACO is used to find the optimal sensors for a user based on this network. Therewith, a GIST questionnaire can be generated.

Example 9.4.5: *The following algorithm could be used for creating a GIST questionnaire adapted to the user, whereby a sensor can be clearly described via the values $\mathscr{T}$ and $\mathscr{Q}$, a value representing the senor-block SB_i, $i \in \{1, ..., m\}$, a time-value T and the geo-location ω_0 (see* FIGURE *9.14) is assigned to a sensor. The mappings ϖ and τ have been used to cluster the sensors within the NN1. The following mathematical description represents a sequence-diagram (*SUBSECTION *6.1).*

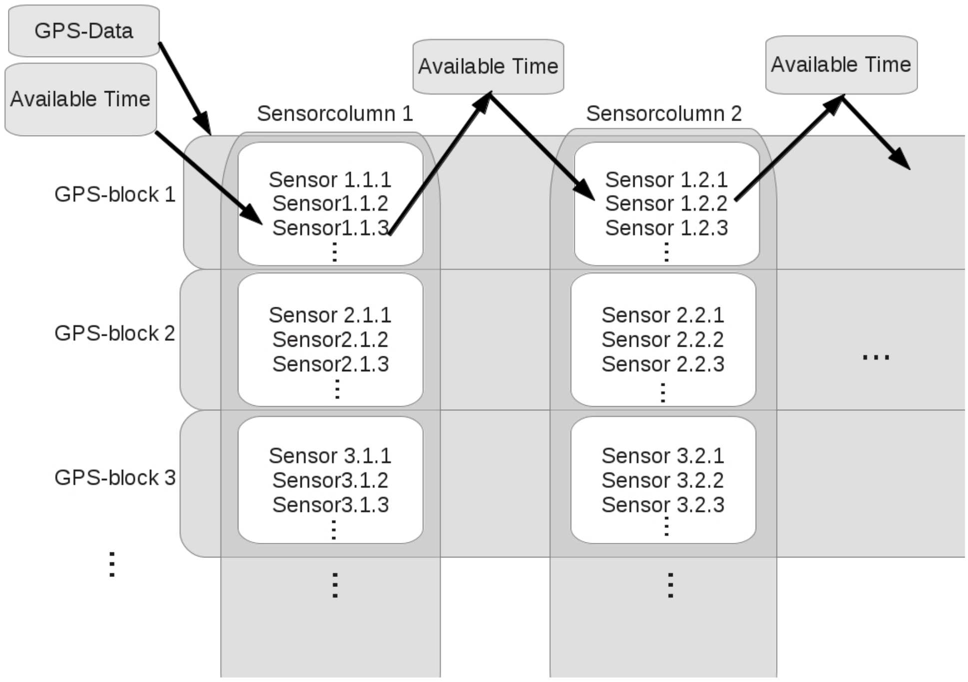


Figure 9.14: Structure of the TCIU. (Figure generated with *Libre Office Draw.*)

1. *First, the GPS block is chosen by using GPS,IP or mobile cell information and, thus, the sensor block can be determined.*

2. *When the user has (re)acted, the mapping f_{entire} is determined. The user is located at a certain geo-location at the current time-point: $\omega_0 = (x_0, y_0, t_0) \in X_\Gamma$.*

$$c : ([0,1], \mathfrak{T}_{nat,\mathbb{R}|_{[0,1]}}) \xrightarrow{\mathfrak{T}} (\{0,1\}, \mathfrak{T}_{\{0,1\}})$$

$$f_{o_j,E_i}(\omega_0) \mapsto \begin{cases} 1, \text{ if } f_{o_j,E_i}(\omega_0) \neq \beta \\ 0, \text{ if } f_{o_j,E_i}(\omega_0) = \beta \end{cases}$$

With $c^{-1}(1)$, the set of the GUI element objects that we do not have information about can be determined. Such a sensor, which delivers information about one of the GUI element objects we do not have information about, should be chosen as the next sensor via, for example, random experimentation.

3. *Afterwards the system takes a look at the available time (AvT) a user has entered. Only sensors with lower time-value T than the available-time-value can be chosen, $AvT - T \geq 0$.*

4. *The system takes a look at the information-quality of a sensor represented by $\mu_R(\varpi) = \int_{[0,100]} \varpi d\mu_R(\varpi) = \mathscr{Q}$ with $\varpi \in \mathfrak{K}_+([0,100], \mathfrak{T}_{nat,\mathbb{R}|_{[0,100]}})$ and the time needed to react on a sensor represented by $\mu_R(\tau) = \int_{[0,100]} \tau\mu_R(\tau) = \mathscr{T}$ with $\tau \in \mathfrak{K}_+([0,100], \mathfrak{T}_{nat,\mathbb{R}|_{[0,100]}})$. With*

a product measure μ_P, the value α is determined:

$$\alpha = \mu_P(\tau, \varpi) = \int_{[0,100]^2} \tau\varpi d\mu_P(\tau, \varpi)$$

Alternatively, the value α can be determined using a convex combination:

$$\begin{aligned} f :&[0,1]^3 \to [0,1] \\ &(\mathscr{T}, \mathscr{Q}) \mapsto \lambda_1 \mathscr{T} + \lambda_2 \mathscr{Q} = \alpha \end{aligned}$$

with $\lambda_1, \lambda_2 \in [0,1]$ and $\lambda_1 + \lambda_2 = 1$, e.g. with $\lambda_1 = \lambda_2 = \frac{1}{2}$.

5. *Now the results will be compared and the sensor with the lowest result is chosen as the next sensor to be used. During the entire process the available time a user entered is counted down.*

$$\begin{aligned} g :&([0,1]^3)^n \to [0,1] \\ &((\mathscr{T}_1, \mathscr{Q}_1), (\mathscr{T}_2, \mathscr{Q}_2), ..., (\mathscr{T}_n, \mathscr{Q}_n)) \mapsto \min\{\alpha_1, \alpha_2, ..., \alpha_n\} \end{aligned}$$

The next sensor can be found by considering the inverse image M_I of $\{\min\{\alpha_1, \alpha_2, ..., \alpha_n\}\}$ regarding to the mapping g. If $|M_I| > 1$, one sensor in M_I is chosen via random experiment.

6. *Afterwards the system takes another look at the available time and the process starts all over again from step 2. until the available time is up.*

With a BP ANN the membership mapping of the user is approximated as described on p.143. The evaluation of the reaction of the user to the sensors is important for the approximation of the membership mapping of the user. The evaluation is a value $\beta \in [0,1] \subset \mathbb{R}$. If, at a certain ω_0, a GUI element object is appropriate for a user, a value larger than β is assigned. If, however, a GUI element object is not appropriate for a user, a value smaller than β is assigned and if no information about a GUI element object is available, the value β is assigned. An evaluation influences the quality of other GUI element objects, this property is implemented via the measure μ_{entire}.

The quality of the approximated membership mapping is based on the amount of information collected about the user, $i_{user}(1, s)$, s is the number of the sensor-column of the last sensor the user edited. With an increasing membership mapping with the domain S, the quality of the approximated membership mapping of the user can be determined.

To approximate an unknown membership mapping, we proceed in the following way: We collect information about the unknown mapping f on the interval $[a, b] \in \mathfrak{T}_t \subset \mathfrak{T}_L$, $t \in \mathbb{N}$. $(X_\Gamma, \mathfrak{T}_{X_\Gamma})$, is a topological space. f is an unknown mapping. We want to approximate $f \in X_{object}$ with the mapping $g_t \in X_{object}$. g_t is a dynamical mapping which changes during time. At each time $t \in \mathbb{N}$, we have a topology $\mathfrak{T}_t$ with $\forall\ t \in \mathbb{N} : \mathfrak{T}_t \subset \mathfrak{T}_L$ and it is $\forall\ t \in \mathbb{N} : \mathfrak{T} \subset \mathfrak{T}_{t+1} \subset \mathfrak{T}_L$ or $\forall\ t_1, t_2 \in \mathbb{N}, t_1 \leq t_2 : \mathfrak{T}_{t_1} \subset \mathfrak{T}_{t_2} \subset \mathfrak{T}_L$. The topology $\mathfrak{T}_t \subset \mathfrak{T}_L$ is not necessarily metrisable for all

$t \in \mathbb{N}$. The information content about the unknown mapping f increases during time. For $O_t \in \mathfrak{T}_t$ the measurement $\mu_L(f) = \int\limits_{O_t} f(x) d\mu_L(x)$ and thus the deviation $|\mu_L(f - g_t) \cdot 1_{O_t}| := e(t, O_t)$ can be calculated, which is the error for O_t. This error-value is delivered to the TCIU and the quality of the membership mapping of the user is reduced to optimise the assignment-process of a GUI.
This error measurement is done within the error determiner, Section 9.7.
One idea to improve the system for collecting information about the user is the implementation of the Choquet integral for fastening processes: this integral can be used to measure the usability of an action of the system. Decision theory is concerned with identifying the values, uncertainties and other issues relevant in a given decision, their rationality, and the resulting optimal decision. Within decision theory, the Choquet integral is a way of measuring the expected utility of an uncertain event. Firstly, here, we have to mention some definitions.

Definition 9.4.6 *Fuzzy Measure ([GR00])*
A fuzzy (or non-additive) measure $\mathcal{F}_\mu$ on $(X, \mathfrak{T}_X)$ is a function

$$\mathcal{F}_\mu : (\mathcal{P}(X), \mathfrak{T}_{\mathcal{P}(X)}) \xrightarrow{\mathfrak{T}} ([0,1], \mathfrak{T}_{nat, \mathbb{R}|_{[0,1]}})$$

satisfying the following axioms.

1. $\mathcal{F}_\mu(\emptyset) = 0$
2. *if $A, B \in \mathcal{P}(X)$, $A \subset B \subset X$, then $\mathcal{F}_\mu(A) \leq \mathcal{F}_\mu(B)$, that is, $\mathcal{F}_\mu$ is a non decreasing set function.*

$\mathcal{P}(X)$ power set.

Note 9.4.7: *Probability measures are fuzzy measures ([Asa95], p.132).*

Definition 9.4.8 *Choquet Integral (cf[Asa95], p.132 f.)*
Let $\mathcal{F}_\mu$ be a fuzzy measure on $(X, \mathfrak{T}_X)$, $x \in X$. The Choquet Integral of a function

$$f : (X, \mathfrak{T}_X) \xrightarrow{\mathfrak{T}} (\mathbb{R}, \mathfrak{T}_{nat})$$

with respect to $\mathcal{F}_\mu$ is defined by

$$\mathcal{C}_{\mathcal{F}_\mu}(f) := \int f(x) d\mathcal{F}_\mu = \int\limits_0^\infty \mathcal{F}_\mu(\{x \mid f(x) \geq \alpha\}) d\alpha + \int\limits_{-\infty}^0 (\mathcal{F}_\mu(\{x \mid f(x) \geq \alpha\}) - \mathcal{F}_\mu(X)) d\alpha.$$

Definition 9.4.9 *Discrete Choquet Integral ([GR00])*
Let $\mathcal{F}_\mu$ be a fuzzy measure on $(X, \mathfrak{T}_X)$, whose elements are denoted $x_1, ..., x_n$ here. The discrete Choquet Integral of a function

$$f : (X, \mathfrak{T}_X) \overset{\mathfrak{T}}{\rightarrow} (\mathbb{R}^+, \mathfrak{T}_{nat, \mathbb{R}|_{\mathbb{R}^+}})$$

with respect to $\mathcal{F}_\mu$ is defined by

$$\mathcal{C}_{\mathcal{F}_\mu}(f) := \int f d\mathcal{F}_\mu = \sum_{i=1}^{n} (f(x_{(i)}) - f(x_{(i-1)}))\mathcal{F}_\mu(A_{(i)})$$

where the subscript $\cdot_{(i)}$ indicates that the indices have been permuted in order to have
$0 \leq f(x_{(1)}) \leq ... \leq f(x_{(n)}), A_{(i)} := \{x_{(i)}, ..., x_{(n)}\}$ *and* $f(x_{(0)}) = 0$ *by convention.*

The probability that an action of the system A is important for the process of assigning a suitable GUI to the user can be expressed by the value $P(A)$. P is the probability measure and, thus, a fuzzy measure. We are interested in how useful it is for executing an action. Using the Choquet measure we can determine the utility of the execution of an action within the decision-making process of the algorithm. f is a membership mapping that shows how well the GUI fits to the user at a certain point x_i. As the system converges, the value $f(x_i)$ should become bigger, the bigger i. Accordingly, we are able to evaluate information and how important an action of the system is in a certain situation. The Choquet measure has not been implemented until now, because we did not know whether it was needed. Empirical studies will be used to investigate whether the system works fast enough to be used in hazardous situations or whether the Choquet measure has to be implemented.

9.5 The Neural Network *NN2*

With the NN2, the user is assigned to a user group represented by membership mappings in X_{object}, see Figure 9.15. NN2 is, as NN1, a recurrent fuzzy ART ANN (see Definition 7.2.14), because we need an ANN that can go back to information that has been collected earlier and, consequently, fuzzy information can be processed.
In our case, the fuzzy ART ANN is used to determine the user group cluster best fitting the user, as well as the degree of membership to this cluster. Those issues are used in the FLC, Section 9.6: Therefore, a cluster analysis is done with the fuzzy ART ANN and the weight-vector describing the best fitting user group cluster for the user is chosen for the FLC. c_k with $k \in \{1, ..., n\}$ and v are one-dimensional vectors consisting of a membership mapping: $c_k \in X_{object}$. Thus, the deviations of $v \in X_{object}$, to the weight vectors are determined with

$$\begin{aligned} f_{error_k} :& X_{object} \times X_{object} \rightarrow \mathbb{C} \\ & (c_k, v) \mapsto \lambda_U + \lambda_{OU} + d(c_k, v), \end{aligned}$$

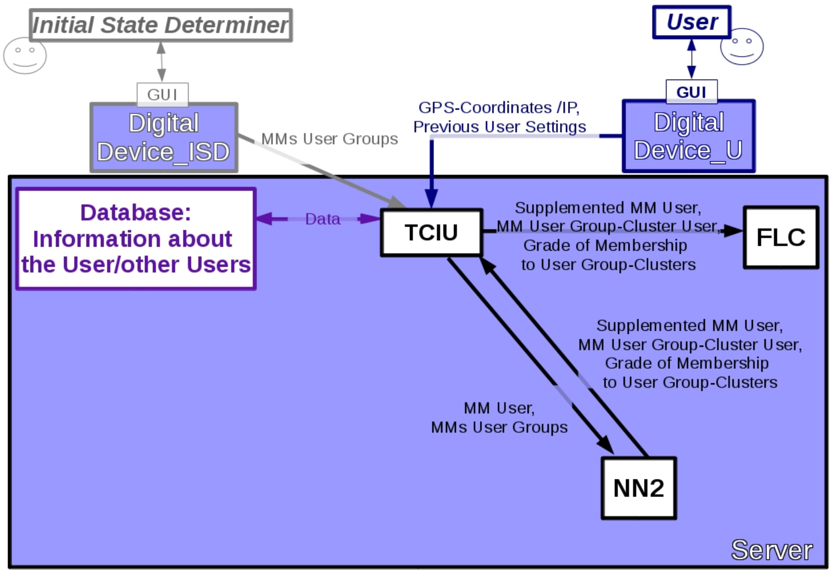


Figure 9.15: The NN2 within the system. (Figure generated with *Libre Office Draw*.)

with $k \in \{1, ..., n\}$, $d(c_k, v)$ influences the bidirectional net on X_{object} mentioned on p.148.
$d(c_k, v) := \gamma \cdot \mu_L(c_k, v) = \gamma \cdot \int_{X_\Gamma} |c_k - v| d\mu_L$, $\gamma \in [0, \infty)$. Furthermore, as in NN1, λ_{OU} is dependent on the number of users using a weight-vector, that is, λ_{OU} is low, if many users use a weight-vector and λ_{OU} is large if few users use a weight vector. Further, λ_U is dependent on the previous user settings, that is, λ_U has lower values for weight-vectors assigned many times to the user than for weight-vectors that are rarely assigned, $\lambda_U, \lambda_{OU} \in [0, 1]$. The vectors consisting of a membership mapping c_k are initially determined by an initial state determiner.

With the user's permission, the weight vectors are saved in a database and can be used again by the same user or by other users.

Open sets are used for the learning task of the ANN to identify the similarity of outputs to similar inputs. Convergence is applied to the learning task of the ANN to investigate in a certain time-period, if similar inputs produce "more similar" outputs, i.e. if the learning task is solved better, when improving the system.

The cluster centers are adjusted based on the grade of membership of the membership mapping of the user to the clusters. The membership mapping representing the cluster-centre c_k to which the user was assigned is determined. The membership mapping of the user f_{entire} is supplemented with information from the membership mapping representing the adjusted cluster-centre at points, where no information about the user is available. The output is the adjusted cluster-centre c_k, to which the user was assigned, the membership mapping of the user f_{entire} and the supplemented mapping of the user.

9.6 The Fuzzy Logic Controller

The FLC determines an output, that is, a GUI control by evaluating the supplemented mapping of the user at the geo-location of the user and the current time-point. The instruction is delivered to

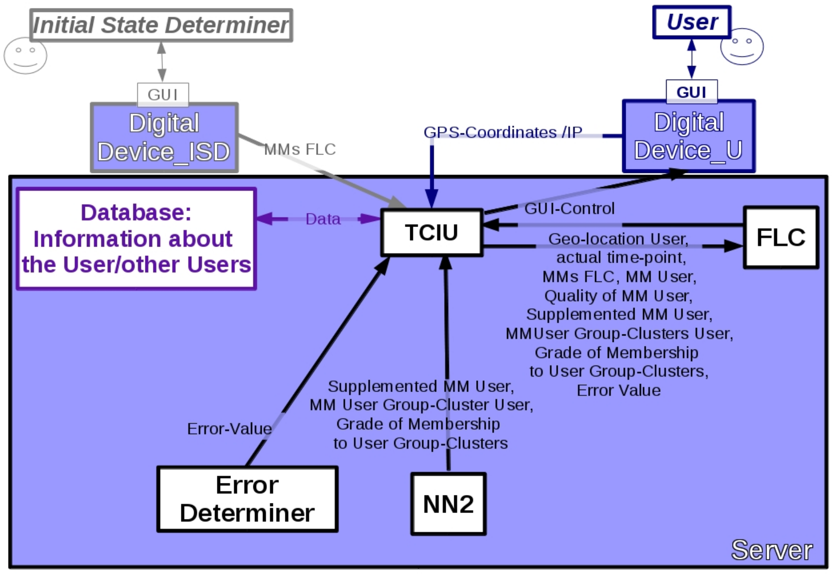


Figure 9.16: The FLC within the system. (Figure generated with *Libre Office Draw.*)

the digital device, see FIGURE 9.17 and, thus, the GUI is influenced. An error can be determined by observing the reaction of the user, that is, if the user is following an instruction, for example moving to a location by checking the user's GPS data.

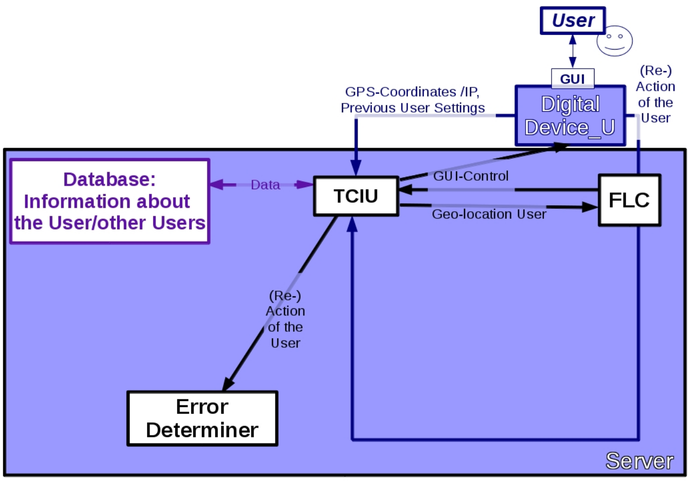


Figure 9.17: The digital device of the user with the GUI. (Figure generated with *Libre Office Draw.*)

With $f_{entire}(x, y, t)$, a value can be assigned to each GUI element object at certain time points and each geo-location on earth. Within the FLC, the geo-location of the user at the actual time-point are processed.
The following fuzzy-rules are used for fuzzy logic control:

- **IF** the quality of the GUI element object is good

  AND the GUI element object has the best quality compared to the other GUI element objects containing to the same GUI element,

  THEN make GUI element object available.

- **IF** the quality of the GUI element object is good

  AND the GUI element object has the best quality compared to other GUI element objects containing to the same GUI element, but there are others with the same quality,

  THEN do a probability experiment to choose the GUI element object that is made available.

- **IF** the quality of the GUI element object is good

  AND the GUI element object does not have the best quality compared to the other GUI element objects containing to the same GUI element,

  THEN do not make GUI element object available.

- **IF** the quality of the GUI element object is not good

  AND the GUI element object has the best quality compared to the other GUI element objects containing to the same GUI element,

  THEN do not make the GUI element object available.

- **IF** the quality of the GUI element object is not good

  AND the GUI element object has not the best quality compared to the other other GUI element objects containing to the same GUI element,

  THEN do not make the GUI element object available.

FIGURE 9.18 depicts a graph of an exemplary membership mapping for describing the degree of "good GUI element object quality".
Further, in FIGURE 9.19 the graph of an exemplary membership mapping for describing the degree of "good GUI element object quality" is illustrated.

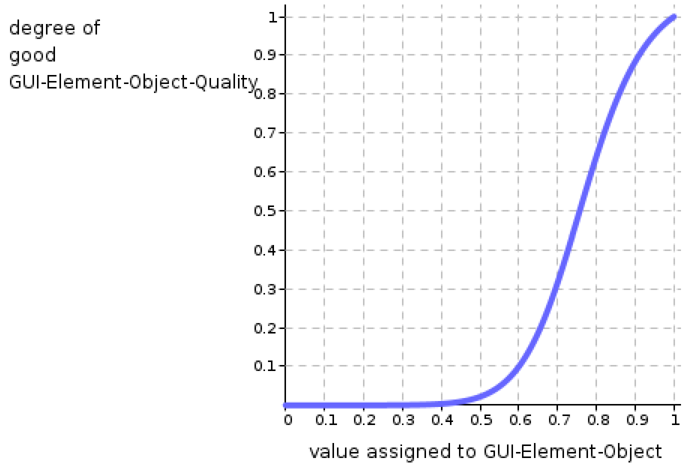


Figure 9.18: Graph of an exemplary membership mapping for describing the degree of "good GUI element object quality". (Figure generated with *GeoGebra*.)

Figure 9.19: Graph of an exemplary membership mapping for describing the degree of "best GUI element object quality compared to each other GUI element object belonging to the same GUI element". MAX is the maximum value of all values assigned to the GUI element objects belonging to a GUI element. (Figure generated with *GeoGebra*.)

The two membership mappings that describe the degree of "good GUI element object quality" and the degree of "best GUI element object quality compared to each other GUI element objects containing to the same GUI element" can be combined with the MIN operator.
In Figure 9.20 the graph of an exemplary membership mapping for describing the values "make GUI element object available" or "do not make GUI element object available" when combining "good GUI element object" and "best GUI element object compared to each other GUI element objects belonging to the same GUI element" is illustrated.

Example 9.6.1: *If the value assigned to a GUI element object is* 0.8 *and thus the GUI element object is "good" with a degree of* 0.6*, and this GUI element object is the best compared to each other GUI element object belonging to the same GUI element, that is, the degree of best GUI element object*

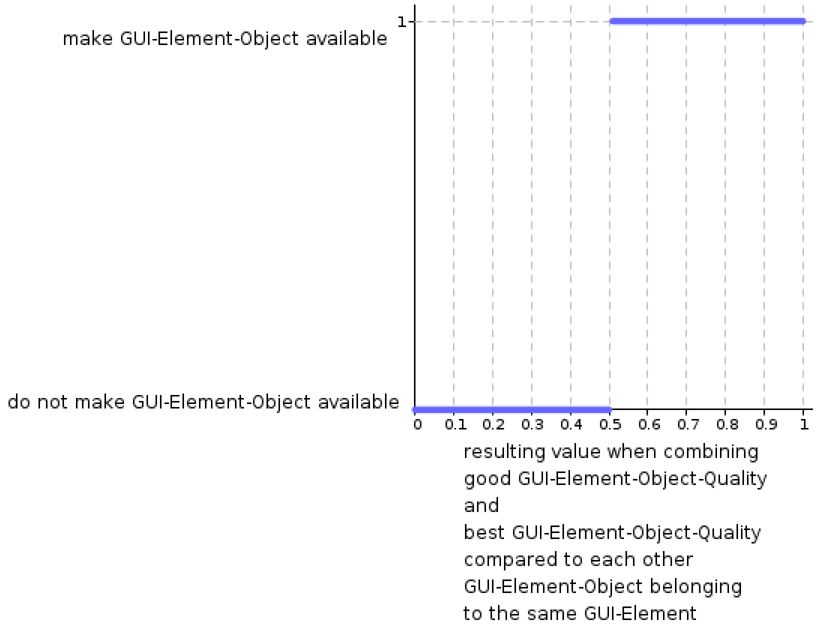


Figure 9.20: Graph of an exemplary membership mapping for describing the values "make GUI element object available" or "do not make GUI element object available" when combining "good GUI element object" and "best GUI element object compared to each other GUI element object belonging to the same GUI element". (Figure generated with *GeoGebra*.)

quality compared to each other GUI element object belonging to the same GUI element is 1*. Therefore, the resulting value is* $\min\{0.6, 1\} = 0.6$*. Thus, the GUI element object is made available for the user.*

If the error value, that is, the deviation between user reaction and predetermined reaction is large, the following fuzzy rule can be implemented in the FLC:

IF the grade of membership of a user to a GUI cluster is not very good

AND the quality of the membership mapping of the user is not very good

AND the error is large

THEN switch to the GUI cluster with the second-best grade of membership and supplement membership mapping of the user.

From the findings gained in Chapter 5 we know that no more than seven choices (i.e. GUI elements) should be offered on a display at the same time. This information can be included by means of an FLC as well:

IF the situation the user is located in is not particularly hazardous

AND the amount of GUI element objects is larger than seven,

THEN reduce the amount of GUI element objects to the seven that have been best evaluated.

Output is a vector with the value "0" for "GUI element-object not available" and "1" for "GUI element object available". The vector has as many dimensions as the amount of GUI element-objects: $|M_{object}|$. Thus, the user's digital device can process the GUI control.
In FIGURE 9.21, the FLC is illustrated as class diagram in UML.

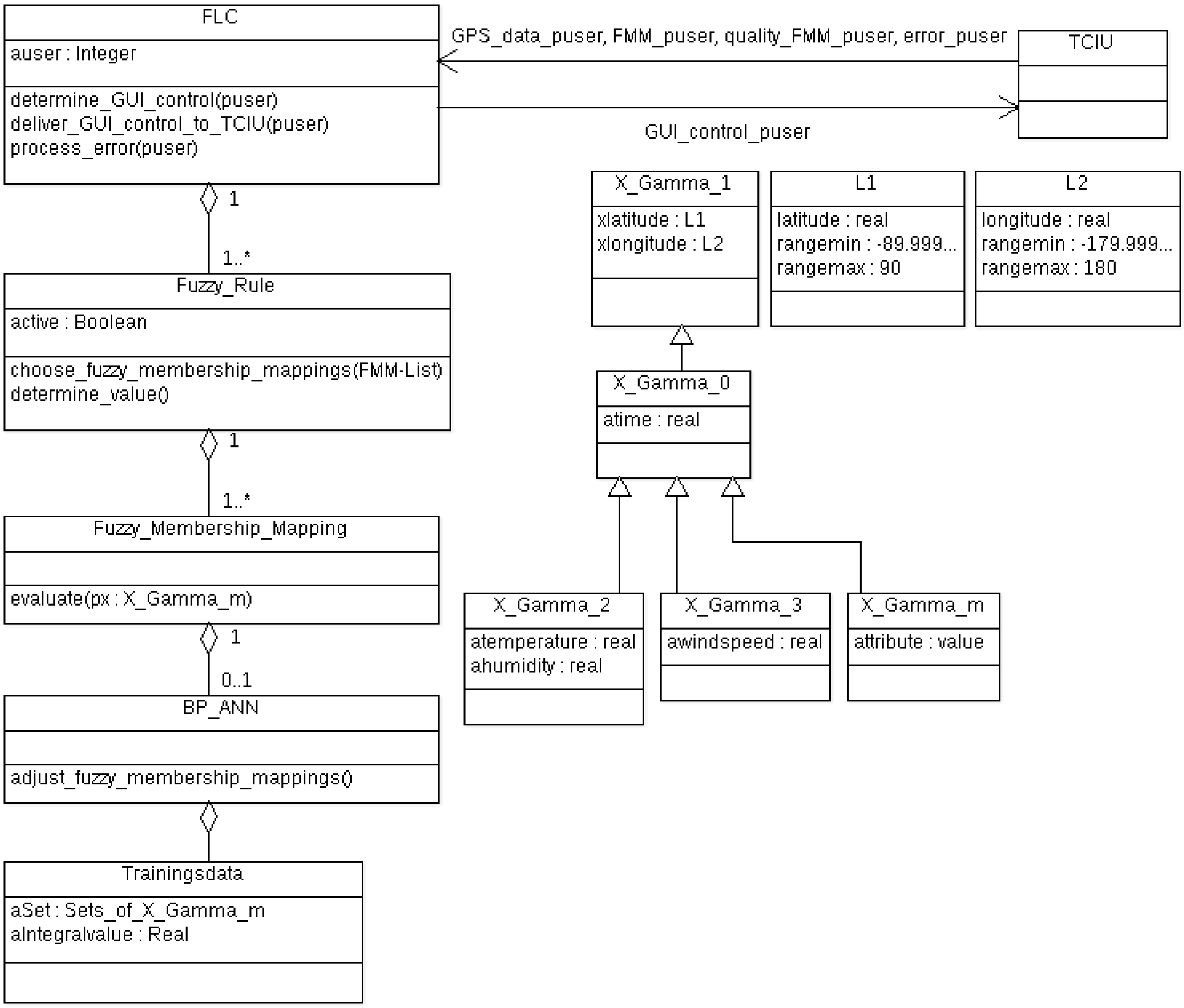


Figure 9.21: The FLC. (Figure generated with *ArgoUML*.)

9.7 Error Determiner

An error value is delivered by the error determiner to the TCIU and to the FLC in order to adjust the SAGU, see FIGURE 9.22. In the system the quality of the action or reaction of a user is measured with

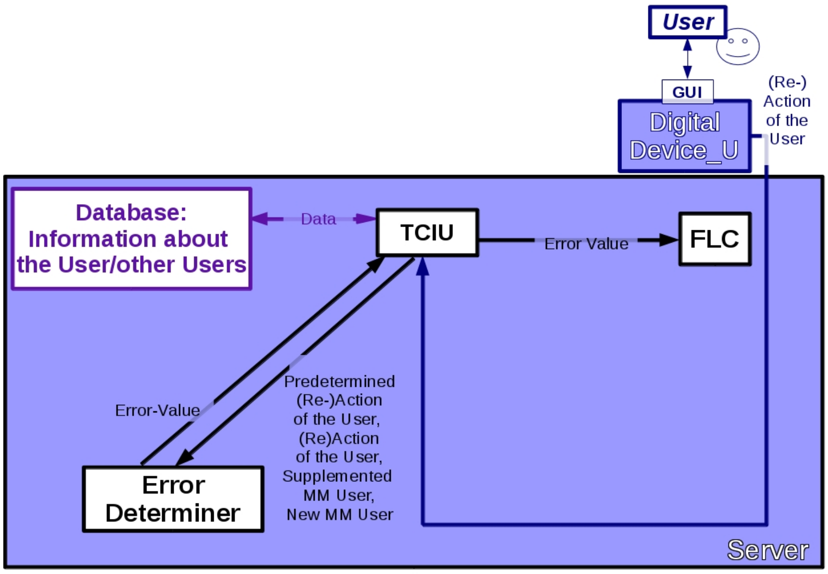


Figure 9.22: The error determiner within the system. (Figure generated with *Libre Office Draw.*)

respect to the proposed reaction. Thus, an expected (re)action of the user has to be predetermined and each predetermined (re)action has to be valued, as well as the actual (re)action of the user. Hence, the expected value can be compared to the actual value and the error can be measured. If the distances in the observed space can be understood in the Euclidean sense, the Riemann-Integral can be used as measure. When this is not the case, for example in epidemiological cases or within the TCIU (SECTION 9.4) or within the space $(X_{object}, \mathfrak{T}_{object})$ considered in the SAGU, p.148, the Lebesgue-integral is useful. If it is detected that the user often, that is, more than β times, (re)acts with large deviation, in other words, larger than a value α to the predetermined (re)action, i.e.

$$\forall\ t \in T \subset \mathbb{N} : (|\mu(reaction_{predet}) - \mu(reaction_{user})| > \alpha) \wedge (|T| > \beta),$$

the FLC reacts by proposing another GUI to the user. This process is repeated until the deviation of user (re)action to the predetermined (re)action is smaller than a value α for several times, that is, more than γ times. In other words:

$$\forall\ t \in \tilde{T} \subset \mathbb{N} : (|\mu(reaction_{predet}) - \mu(reaction_{user})| \leq \alpha) \wedge (|\tilde{T}| > \gamma).$$

Therefore, the predetermined reaction of a user has to be determined for each sensor and the actual user reaction has to be evaluated to measure the deviation.

As it would be confusing for the user, if the GUI changed after each time-step, a proposal of the

system for changing the GUI should only be supported, if it is really necessary, to assist the user in an optimal way. The threshold-property of the ANNs contributes to this, as well as measures like $F_A(x) := \frac{1}{\mu(A)} \int\limits_{A+x} f(x)d\mu(x)$ to smooth a mapping.
A method to approximate an unknown membership mapping is the following: We are collecting information about the unknown mapping f on the interval $[a,b] \in \mathfrak{T}_t \subset \mathfrak{T}_L$, $t \in \mathbb{N}$. $(X_\Gamma, \mathfrak{T}_{X_\Gamma})$, is a topological space. f is an unknown mapping. We want to approximate $f \in X_{object}$ with the mapping $g_t \in X_{object}$. g_t is a dynamical mapping which changes over time. At each time $t \in \mathbb{N}$, we have a topology $\mathfrak{T}_t$ with $\forall\ t \in \mathbb{N} : \mathfrak{T}_t \subset \mathfrak{T}_L$ and it is $\forall\ t \in \mathbb{N} : \mathfrak{T} \subset \mathfrak{T}_{t+1} \subset \mathfrak{T}_L$ or $\forall\ t_1, t_2 \in \mathbb{N}, t_1 \leq t_2 : \mathfrak{T}_{t_1} \subset \mathfrak{T}_{t_2} \subset \mathfrak{T}_L$. A topological space equipped with the topology $\mathfrak{T}_t \subset \mathfrak{T}_L$ is not necessarily metrisable for all $t \in \mathbb{N}$. The information content about the unknown mapping f increases during time. For $O_t \in \mathfrak{T}_t$ the measurement $\mu_L(f) = \int\limits_{O_t} f(x)d\mu_L(x)$ and, thus, the deviation $|\mu_L(f - g_t) \cdot 1_{O_t}| := e(t, O_t)$ can be calculated, which is the error for O_t. This error value is delivered to the TCIU, where it is distributed, to optimise the assignment process of a GUI by adjusting the quality value of the approximated membership mapping of the user.
In FIGURE 9.23 the error determiner is illustrated as class diagram in UML.

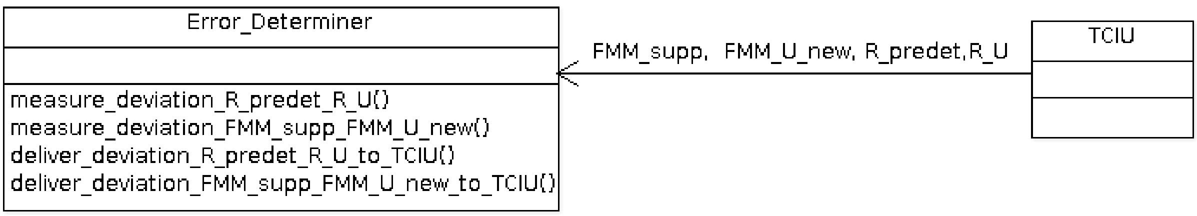


Figure 9.23: The error determiner. (Figure generated with *ArgoUML*.)

In the sections above, the mathematical structure behind an SAGU was presented. The system assigns exactly one GUI to the user after each time-step and it changes according to user needs. Because of the generic structure of the algorithm, it can be transferred to other spatial decision support that does not operate on Euclidean spaces.

In the next chapter, the object oriented illustration of the SAGU is presented, among others. The OOA models in this chapter and in the next chapter are generated with *ArgoUML*. The added value of the use of *ArgoUML* is, that program hulls are written in scripting language, e.g. PHP5, when creating a diagram.

10 Object Oriented Illustration of the SAGU, Maps for Visualisation and exemplary GUIs tailored to different User Groups

One goal of the *ReGLaN-Health and Logistics* project is the development of an executable prototype of an OS-application for a digital device using an adaptive GUI tailored to different user groups. Therefore, a System for Adapting a GUI to a User (SAGU) was developed in this thesis.
In Figure 10.1, a state diagram, illustrating the functioning of the SAGU, is presented. Firstly, an

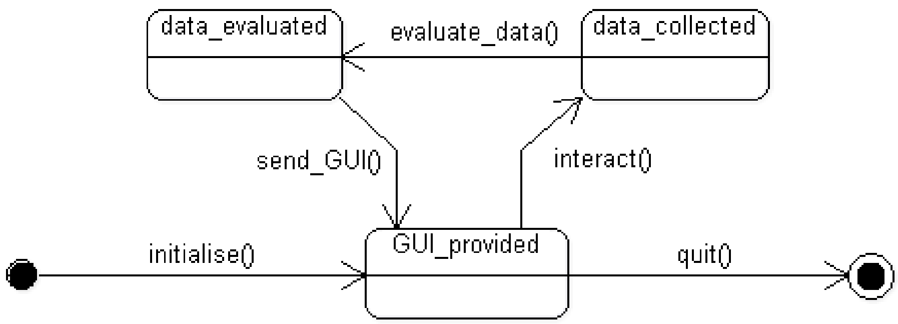


Figure 10.1: State Diagram illustrating the functioning of the SAGU in UML. (Figure generated with *ArgoUML*.)

initialisation takes place, thus providing an initial GUI. The user is then given the option to interact with the application via the GUI or to quit the application. If the user interacts, data is collected and evaluated in order to identify a GUI that fits closely to the user's needs. If it is found that a GUI other than the GUI actually provided to the user fits the user better, the user is asked whether the GUI should be changed (in the GUI element for questions using a certain GUI element object). The user's answer comprises the next interaction; this answer will be evaluated and the GUI will be changed if the user answers "yes". Conversely, if the user answers "no", the current GUI will be kept. With each interaction between the user and the system, the system determines the best fitting GUI for the user and, thus, the process is repeated until the user quits the application.
On pages 196 and 200, specifications for certain user groups and use cases are illustrated. The digital devices the application is supposed to be generated for were elucidated in Section 2.2 In this chapter the generation of risk, resource and decision support maps is described, among others.

These are intended for use in the developed GUIs for risk visualisation and decision support. The application is intended to provide decision support for the user in a hazardous situation, for example the epidemiological outbreak of a disease, ecotoxicological or radioactive exposure or an accident situation. Therefore, we have to generate risk and resource maps and combine them in such a way that maps for decision support can be derived. GUIs tailored to different user groups, for example for users who find themselves in hazardous situations and governmental facilities that support the understanding of the spatial sense for optimal decision support, have to be generated.

In FIGURE 10.2, the contents considered in this thesis incorporated into the early warning and

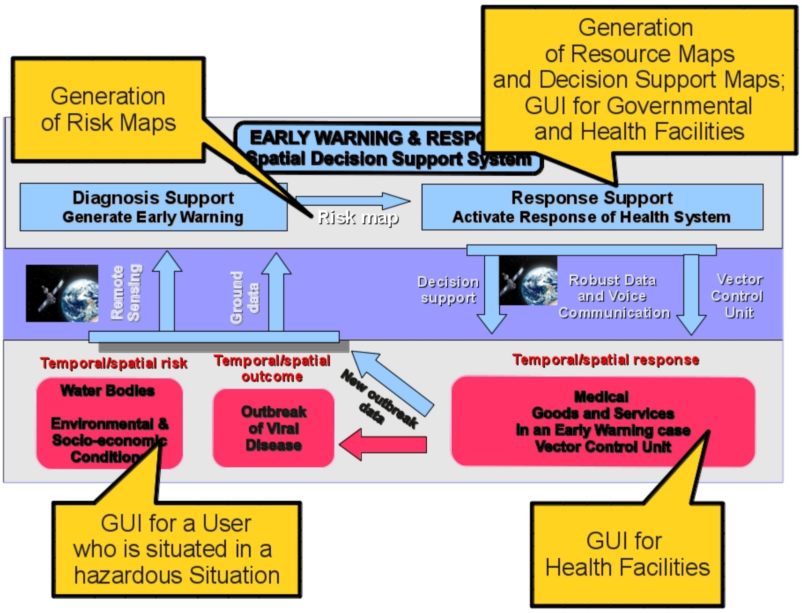


Figure 10.2: Contents of this thesis incorporated into the Early Warning and Response Cycle. (Figure generated with *Libre Office Draw*.)

response cycle are visualised. For the generation of the risk maps, we have to predict the spatial risks of, for example, diseases like malaria. We do this using mathematical modelling. In order to produce a good model, we have to find out which environmental parameters for generating a malaria risk map are important, for example temperature, rainfall, wind speed, wind direction, human population, animal population, pathogen (protozoa), the mosquito life cycle and the S-I-R model of the disease in the infected person and the mosquito. A first attempt at generating a risk map is done in SUBSECTION 10.1.1.

Furthermore, we also have to model the distribution of resources in order to combine the risk maps with maps of available resources so as to create optimal decision support and early warning and response (see SUBSECTION 10.1.2). The combination of risk and resource maps delivers a decision support map (see SUBSECTION 10.1.3). As "all models are wrong, but some are useful" ([Box76]), we have to determine the quality of the maps by, inter alia, observing the SQI (see p.185). Such maps can be visualised with using a GIS. Different GUIs are needed because different user groups have different demands. Risk is the product of probability and impact (see Definition 1.0.1). The impact of such risk differs for different user groups. Such user groups include, for example, governmental facilities, vector control units, medical doctors, nurses and users who find themselves in hazardous situations. If we focus on the above groups, we have the following demands:

- Governmental facilities:
  - Where and how to intervene?
    - Who must be warned?
    - Where to allocate resources?
    - What advice and decision support can be delivered to the population?
- Vector control units:
  - Where should resources be located?
    - Who must be warned?
    - Where should pesticides be applied?
    - Where is medication needed?
    - Where are medical doctors needed?
    - Where are mosquito nets needed?
    - Where are insect repellents needed?
- Users in hazardous situations:
  - How can I protect myself?:
    - Am I located in a hazardous area?
    - How can I get out of/cross this hazardous area possibly unscathed?
    - What preventive measures can be taken?

The user may belong to various user groups, for example someone working for vector control unit who has to travel into a hazardous area to apply pesticides. While travelling, she/he may use the GUI of a user who is situated in a hazardous situation.

One requirement for this research is the exclusive use of OS software for the development of the application. The benefits of OS were mentioned in Chapter 3.

The application should be available offline, as internet access is not available everywhere. For this application, the processes have to be automated with the use of fuzzy logic, that is, membership functions are used to provide automated decision support.

In the following section, the generation of risk, resource and decision support maps is addressed and example GUIs for the user groups, users in hazardous situations and governmental facilities, are presented.

10.1 Maps for Visualisation

Maps are generated in order to visualise an overview of a situation. Some examples showing the procedure for generating risk and resource maps and maps for decision support will follow.
Infectious diseases are considered in this thesis because they influence the spatial mathematical modelling of public health issues. Accordingly, epidemiological risk maps can be generated and early warning and decision support can be delivered for the population located in a hazardous area; for example, for health facilities in order to optimise health service delivery. To develop a risk map for a disease, accurate information about the disease is essential, for example the transmission of the disease. Furthermore, treatment, prevention and possible mitigation strategies for the disease are considered, because if this information is communicated to the population via an appropriate GUI on a digital device it may help to reduce the spreading of the disease.
In this section we will concern ourselves with malaria as an infection caused by protozoa. This disease is transmitted by mosquitoes and occurs in South Africa among many other countries. Protozoa enter the human body as parasites. Accordingly, a parasitic disease is an infectious disease caused or transmitted by a parasite. Parasites are animals or plants that survive at the expense of another organism (host). External parasites sit on the host, for example lice, mosquitoes and mites, while internal parasites live in the body of the host, for example tapeworms and disease causing bacteria. They feed on the host's body parts (cf [Pau00], p.802).
Parasitic diseases caused by helminths and protozoa are major causes of human disease and misery in most countries in the tropics. They plague billions of people and kill millions annually, as well as having debilitating consequences such as blindness and disfiguration for additional millions of people (`http://grants.nih.gov/grants/guide/rfa-files/RFA-AI-93-015.html` (retrieved 16/02/2014).
"Most deaths occur among children living in Africa where a child dies every minute from malaria." (`http://www.who.int/mediacentre/factsheets/fs094/en/` (retrieved 16/02/2014)).
Malaria is a febrile, life threatening, infectious disease, caused by single cell protozoa, plasmodium parasites, that enter the bloodstream of humans through the bite of an infected malaria mosquito, called the "malaria vector". These parasites multiply, invade the red blood cells and multiply there again. The malaria mosquito (anopheles) is a particular mosquito species, some of them which transmit types of malaria, but only the females suck blood. These mosquitoes bite mainly between dusk and dawn. There are about 420 different species of anopheles mosquitoes of which 40 species are capable of transmitting the tropical disease malaria. (`http://www.malaria.info/` (retrieved 16/02/2014)).
The anopheles species is described in more detail on p.179f. In malaria, there are three types of pathogens with different varying durations of development:

- 24 to 48 hours, almost daily attack of fever. This is referred to as malaria tropia, and is caused by plasmodium falciparum.

- 48 hours, attack of fever occurs every third day. This is called malaria tertiana, and is caused by plasmodium vivax or plasmodium ovale.

- 72 hours, an attack of fever occurs every fourth day. Referred to as malaria quartana, and is caused by plasmodium malariae.

Plasmodium falciparum and plasmodium vivax are the most common and plasmodium falciparum is the most deadly. The intensity of transmission depends on factors related to the parasite, the vector, the human host, and the environment. In 2012, malaria caused an estimated 627 000 deaths, mostly among African children. Non-immune travellers from malaria-free areas are also very vulnerable to the disease when they become infected. Malaria is preventable and curable, and it is treated with anti-malarial drugs. ([Pau00], p.575 and `http://www.who.int/mediacentre/factsheets/fs094/en/` (retrieved 16/02/2014)).

Approximately half of the world's population is at risk of malaria, although most malaria cases and deaths occur in sub-Saharan Africa. FIGURE 10.3 depicts the development of the number of cases of malaria and the number of deaths due to malaria in 2000, 2005 and 2009 is visualised. The largest

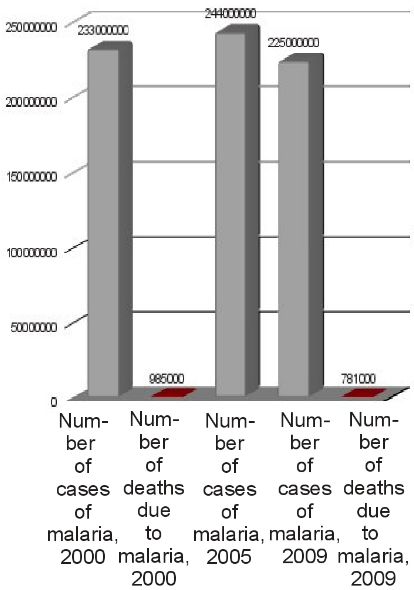


Figure 10.3: Development of the number of cases of malaria and the number of deaths due to malaria. (Figure generated with *Libre Office Calc*. Compare: [Wor10], p.xi.)

absolute decline in malaria deaths has been observed in Africa, although an increase in malaria cases in 2009 was registered in Rwanda, Sao Tome and Principe and Zambia. This increase in malaria cases underscores the fragility of malaria control programmes and the need to control those inter alia by monitoring the disease data, which is difficult because the available data are often insufficient. (cf [Wor10], p.xi).

This symptom of the lack of data can also be seen in [Wor10], p.119. If there is no adequate data, major epidemiological events could be occurring in additional countries without being detected and investigated ([Wor10], p.xi).

On risk maps, South Africa is often not marked as area where malaria transmission occurs. This finding could also be justified by a lack of data, see [Wor10], p.119. When generating a risk map for South Africa the first step is to visualise the risk for anopheles mosquito survival depending on rainfall and humidity (see SUBSECTION 10.1.1). If certain parameters for malaria risk were added, this approach could result in a malaria risk map.

To improve the reporting of malaria cases, crowd-sourcing could be used. In other words, if a person suffers from malaria, this fact is reported via a mobile device to a spatial database and the GPS-coordinates of the person is noted. Subsequently, a map for the distribution of malaria cases could be generated and new risk maps could be developed. In addition, health service delivery could be improved by sending medication and/or medical doctors to the areas, where the malaria cases were reported.

10.1.1 Generating a Risk Map that presents the Risk for Anopheles Mosquito Survival depending on Rainfall and Humidity

A first step in the provision of decision support is the generation of risk maps. The generation of reliable risk maps for a disease transmitted by mosquitoes is based on the generation of maps visualising the plotting of the distribution of infected mosquitoes transmitting the disease. Therefore, it is important to know, where, when and how the mosquitoes live, where, when and why they bite humans and which environmental parameters influence their life.

The family culicidae contains approximately 2900 different species of mosquitoes, whose common feature is a long proboscis ([BH07], p.169f).

Culicidae belongs to the kingdom animalia, the phylum arthropoda, the class hexapoda and the order diptera. Pictorial keys exist for culicidae, which could be used e.g. to identify anopheles mosquitoes, tag malaria risk zones (e.g. via crowd-sourcing) and deliver early warnings by enhancing the approach described on p.49, where AR is used for the identification of species. We will also take a closer look at the genera anopheles as the carrier of malaria.

The life cycle of an anopheles mosquito comprises four stages (see FIGURE 10.4). The first three stages of egg, larva and pupa develop in water, with only the adult mosquitoes living on land. The anopheles mosquito breeds in permanent collections of fresh water. Some species prefer a dense plant cover, as this provides protection to the larvae against fish and other predators. Others can breed in correspondingly high humidity in hoof prints, discarded containers or other small water clusters. To reduce anopheles occurrence, predators of the anopheles could be released into suitable waters to reduce the anopheles-occurrence. This could be done by the GUI user group vector control unit. Eggs are laid singly on the water surface, and each egg has its own floating device in the form of side-mounted air-filled flotation chambers. The larvae hatch after one to three days. They usually feed on small plant particles that they filter from the water. If they are disturbed, they dive down, but

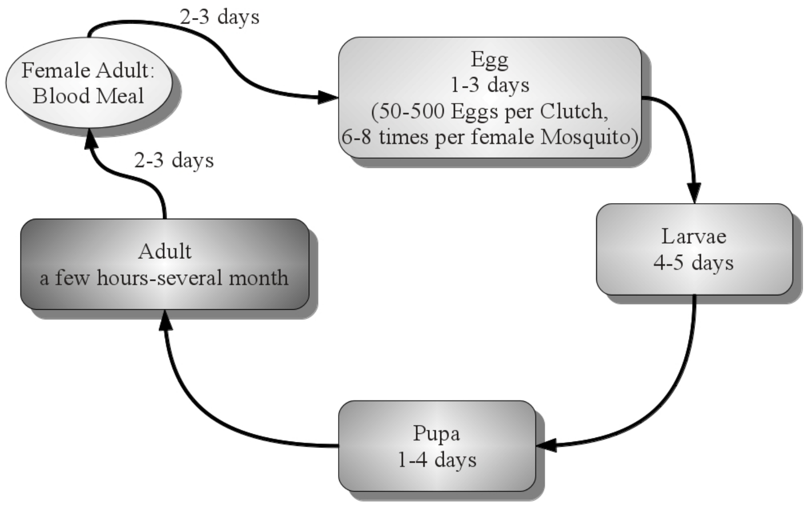


Figure 10.4: Life cycle of anopheles mosquitoes. (Figure generated with *Libre Office Draw.*)

must have to return to the surface in order to meet their oxygen demands. During their development in the water, the larvae moult four times. Depending on the type of mosquito and the prevailing temperatures, the larvae transform into pupa after four to five days, or even after several weeks. During the larva-like pupa stadium stage, the anopheles mosquitoes do not feed, but they are extremely agile and come to the surface occasionally to absorb oxygen. After one to four days, the adult anopheles mosquitoes hatch. Once hatched, the anopheles mosquito sits briefly on the water surface in order to spread its wings for drying them. Thus, the development time varies by one to three weeks depending on the climatic situation. The generations follow one another consecutively and adult anopheles mosquitoes can move up to two kilometres away from their breeding grounds. Depending on the type of anopheles, the life span is between a few hours to several months, but the male has a shorter lifespan than the female. Both the males and the females feed on plant nectar and plant juices. The female takes her blood meal in order to obtain a protein donor for the formation of eggs. After dark or in the early morning hours the mosquito looks for a victim. During the day, however, the females prefer to stay in dark and damp places, since the thin exoskeleton provides only a poor protection against drying out. The female anopheles takes its first blood meal two to three days after hatching. After another two to three days, it lays its eggs. Depending on the species, the number of eggs per clutch varies between 50 and 500. Throughout its life, a female produces six to eight clutches, each of these is preceded by a blood meal. In order to track down their victims, female anopheles have the capability of detecting carbon dioxide, lactic acid and infrared light. Worldwide, approximately 400 anopheles species are known of which 60 are considered to be carriers of malaria. In most regions of the world a few species are the main vectors, while other species occur as secondary vectors. (cf [Peu50], p.38ff and `http://www.gigers.com/matthias/malaria/anophele.htm` (retrieved 16/02/2014)).

The life cycle of the anopheles mosquito (FIGURE 10.4) may be important for the GUI user group vector control unit, because this way a suitable intervention that matches the state of mosquito development could be delivered. For example the releasing of predators during the larvae-stage of the mosquito or the use of insecticides and behavioural-information for the population, such as "use Insecticide-Treated Mosquito Net (ITN)" or "use insect repellent", could be delivered in the adult stage of the mosquito. These early-warning and decision support measurements can be communicated to the vector control unit via the adaptive GUI. In order to generate risk maps visualising the occurrence of mosquitoes transmitting the disease, inter alia the following information could be important as parameters for mathematical modelling: weather conditions, that is, temperature, wind speed and humidity (precipitation, stagnant water and flowing water) in combination with the life cycle of the mosquitoes and the S-I-R model of the disease in the infected person and the mosquito. Here, the parameters "temperature" and "precipitation" are combined with certain information about the life cycle of the anopheles mosquitoes to generate a first attempt at a malaria risk map. A risk map

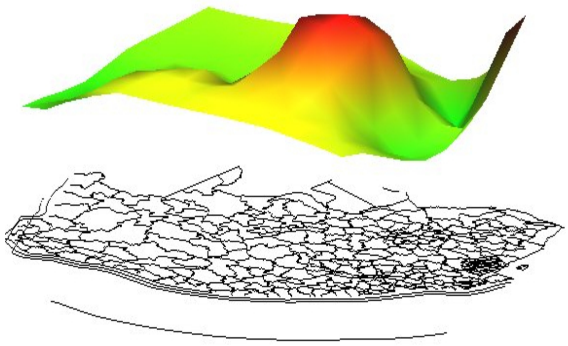


Figure 10.5: Exemplary risk map. (Figure generated with *GRASS GIS*.)

is a three-dimensional map that presents the risk assigned to a two-dimensional geo-coordinate, (see FIGURE 10.5). Thus, risk can be visualised by means of graphs of the space $\mathbb{R}^3$. An attempt at generating a risk map is described here. The chosen parameters for the model are temperature and humidity. "There is good reason for using rainfall to indicate the probable presence of vectors, their survival and the potential for malaria transmission. Although it is known that flooding often causes destruction of breeding sites and a temporary reduction of vectors, it never eliminates a vector, so that very high rainfall was still considered optimal for transmission." ([CSS99]).

First membership mappings describing the mosquito survival that are dependent on rainfall and temperature (FIGURE 10.6) are generated, (see FIGURES 10.7 and 10.8). These are combined by intersecting fuzzy sets (see FIGURE 10.9).

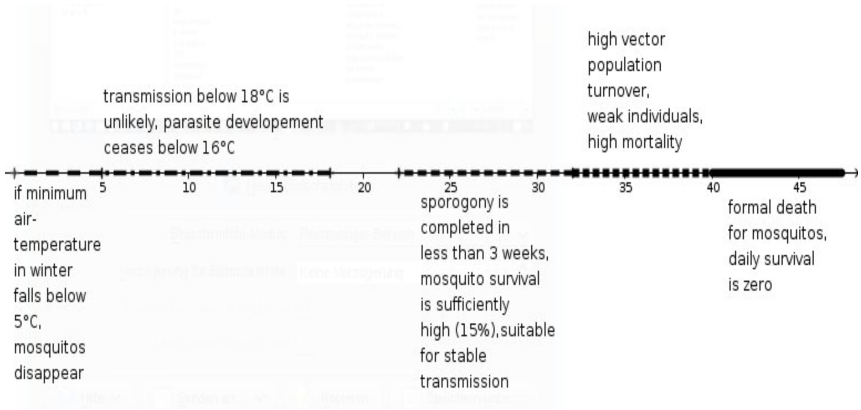


Figure 10.6: Mosquito survival dependent on temperature. (Figure generated with *Libre Office Draw*. Compare: [CSS99].)

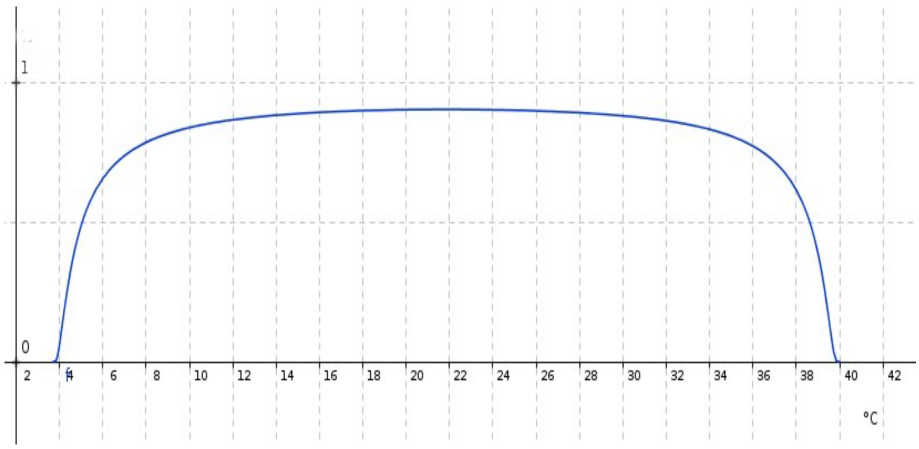


Figure 10.7: Daily mosquito survival f defined by [Mar97] as $f : [3.7, 40] \to [0, 1], x \mapsto e^{\frac{-1}{-4.4+1.31x-0.03x^2}}$ assuming constant humidity. (Figure generated with *GeoGebra*.)

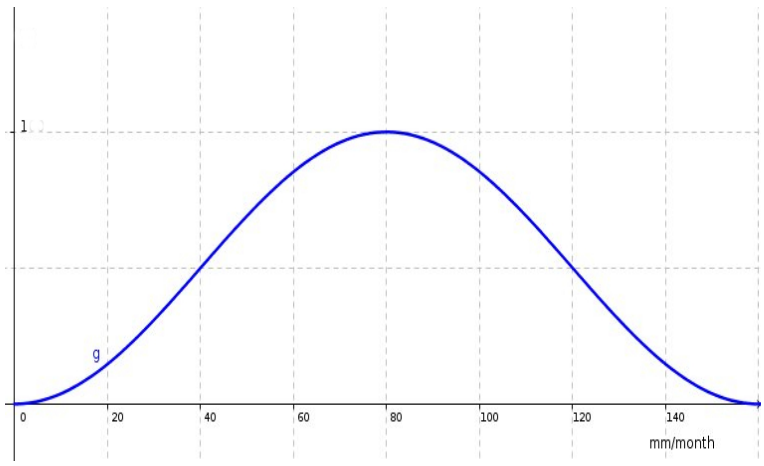


Figure 10.8: Daily mosquito survival depending on rainfall g is defined by [CSS99] as $g : [0, 160] \to [0, 1], x \mapsto -\cos^2(\frac{x}{80} \cdot \frac{\pi}{2}) + 1$. (Figure generated with *GeoGebra*.)

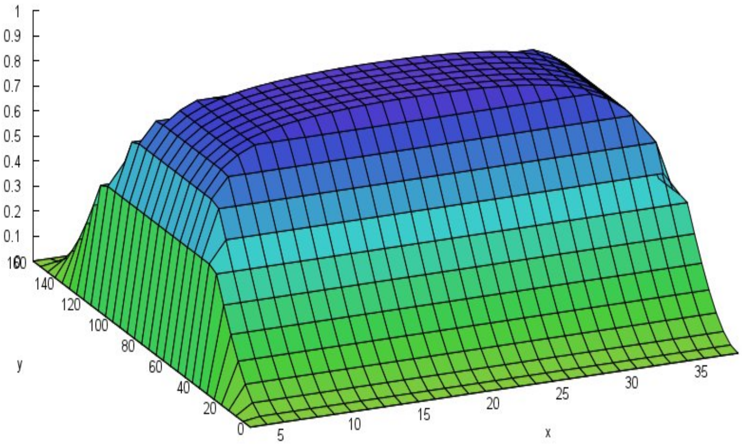


Figure 10.9: Daily mosquito survival h dependent on temperature (f) and rainfall (g) defined as $h : [3.7, 40] \times [0, 160] \rightarrow [0, 1], x \mapsto \min \{f(x), g(y)\}$. (Figure generated with *Maxima*.)

Subsequently, the resulting mapping is applied to data of for South Africa on

- humidity and rainfall, source: `http://www.weathersa.co.za/web/index.php/sclimate` (retrieved 16/02/2014), monthly averages of the 30-year period, 1961-1990,
- temperature, source: `http://data.giss.nasa.gov/csci/stations/` (retrieved 16/02/2014), monthly averages of the 130-year period, 1880-2010 (partly incomplete).

The data points are visualised in Figure 10.10.

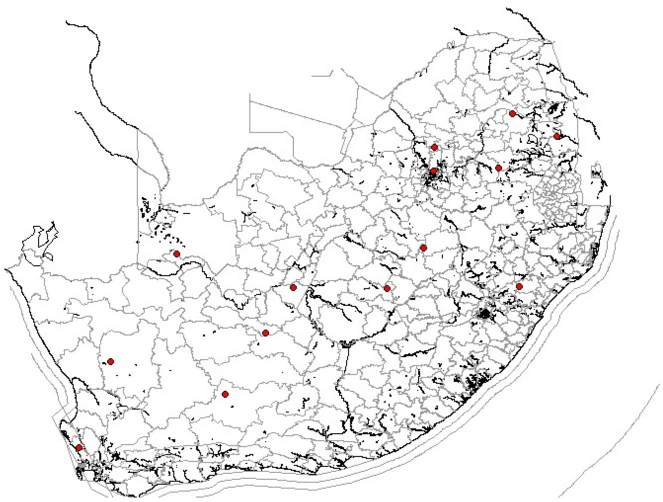


Figure 10.10: Map of South Africa with data points. (Figure generated with *GRASS GIS*.)

The surface is interpolated and a risk map is visualised using *GRASS GIS* (see FIGURE 10.11). Accordingly, red denotes high risk and green denotes low risk. The risk map can be described by a mapping:

$$M_{risk} : (\mathrm{I\!R}^2, \mathfrak{T}_{nat}) \overset{\mathfrak{T}}{\to} ([0,1], \mathfrak{T}_{rel}).$$

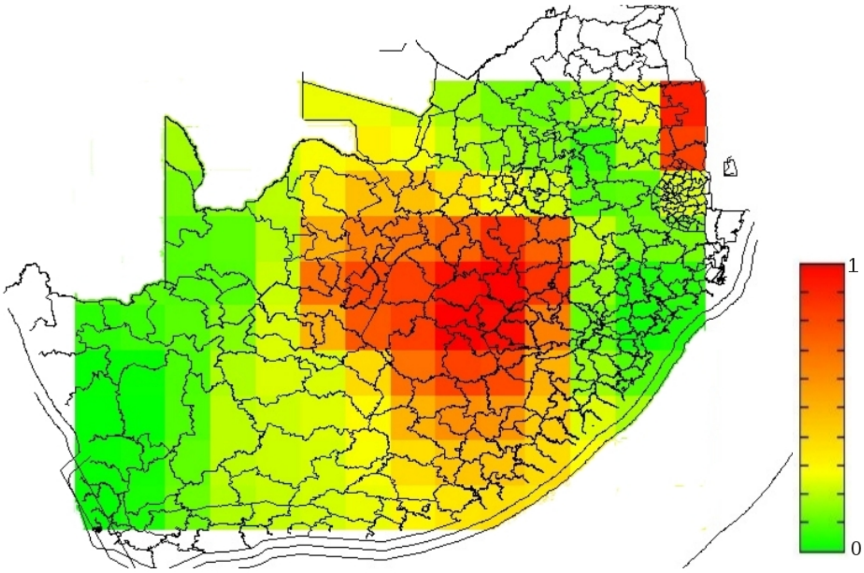


Figure 10.11: Map of South Africa with interpolated surface showing the risk for anopheles mosquito survival depending on rainfall and humidity in January. (Figure generated with *GRASS GIS*.)

However, the resulting map is not a risk map for malaria, but a map showing the risk for anopheles mosquito survival, depending on rainfall and humidity. This becomes obvious, as well, when comparing these resulting maps with the results of the *World Health Organisation* (WHO): Malaria is present in the three northern provinces of South Africa bordering Mozambique and Swaziland. Seasonal transmission takes place between October and April, with 4% of the population being at high risk, and 6% at low risk. (cf [Wor10], p.119). Our risk map shows too many risk zones (red areas), as well as a risk zone in central South Africa, where there is actually no malaria risk zone. In generating a risk map for malaria, some additional parameters have to be involved such as wind speed, wind direction, human population, animal population, pathogen (protozoa), the mosquito life cycle and the S-I-R model. Nevertheless, the developed map is not useless, as it could be used for warning people infected with malaria or other diseases transmitted by anopheles mosquitoes against entering the risk areas with high risk for anopheles mosquito survival depending on rainfall and humidity.
The SQI describes the quality of a map.

"The notion of usefulness of quality depends first on the use that is made of the data it quantifies, particularly during and after a decision. Decision and quality are part of a dialectic that is similar to that of risk and hazard. Decision may be regarded as the conclusion of an informed and logical process, in which the treatment of uncertainty must be present. The purpose of the usefulness of quality is then measured by its ability to reduce the uncertainty of a decision." ([DJ06], p.18).

Definition 10.1.1 *Spatial Quality of Information (SQI)*
SQI is the degree to which data accurately reflects a condition at a particular location in the real-world, this location is element of $\mathbb{R}^2$*, that the data represents.*
The SQI can be represented by a mapping:

$$f_{SQI} : (\mathbb{R}^2, \mathfrak{T}_{nat}) \xrightarrow{\mathfrak{T}} ([0,1], \mathfrak{T}_{rel})$$

Example 10.1.2: *SQI is important with respect to the ageing of data and data per area.*

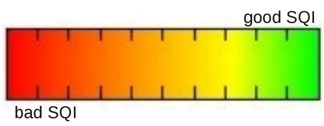
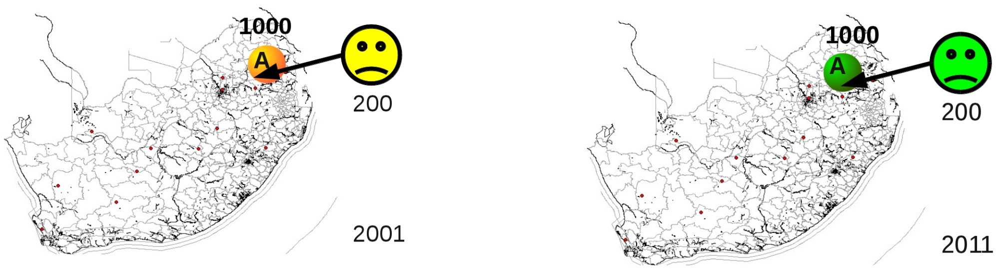


Figure 10.12: Colourscale for determination of SQI. (Figure generated with *GRASS GIS*.)

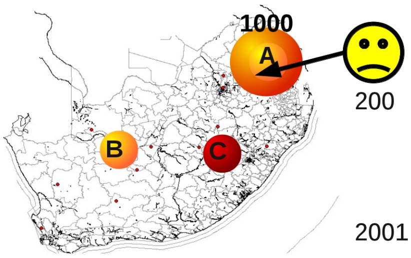


(a) Area A in the year 2001 with 200 infected of 1000 inhabitants.

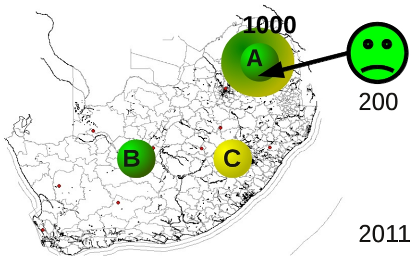
(b) Area A in the year 2011 with 200 infected of 1000 inhabitants.

Figure 10.13: Ageing of data. (Figure generated with *GRASS GIS* and *Libre Office Draw*.)

Imagine that data was collected in area A in the year 2001. One thousand inhabitants of area A *were asked if they were infected with malaria for example. Two hundred inhabitants answered "yes". We do the same in the year 2011 with a similar result. In* FIGURE 10.13 *the SQI is determined by colours that rely on the scale in* FIGURE 10.12 *with green representing good SQI and red representing bad SQI. In* FIGURE 10.13 *(a) the SQI is worse than in 10.13 (b), because the data is older. We have to be careful when we want to combine the data from 2001 with 2011, because we do not know anything about the 1000 interviewed inhabitants in area A. It is possible that we have asked the same inhabitants and still 200 inhabitants are infected. However, maybe we have asked other inhabitants and a subset of the inhabitants of 2001 could have moved away from area A or they even could have died (of malaria). If we have measured data and we want to determine data in areas where we do not*



(a) SQI in area A and area B and impact on SQI of area C and surrounding of area A in the year 2001.

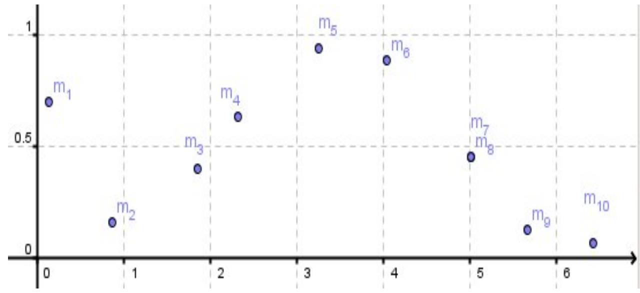


(b) SQI in area A and area B and impact on SQI of area C and surrounding of area A in the year 2011.

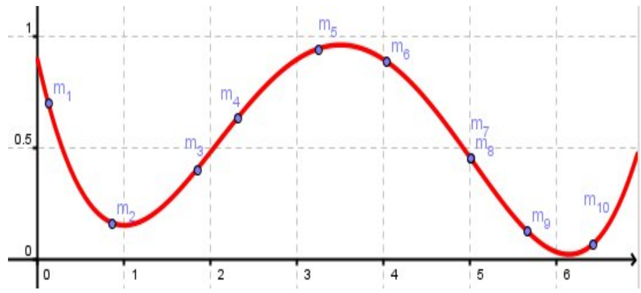
Figure 10.14: Data per area. (Figure generated with *GRASS GIS* and *Libre Office Draw.*)

have measured data, the SQI becomes worse, the bigger the distance to measured data becomes. This is depicted in FIGURE 10.14*. Again, the SQI is determined by colours relying on the scale in* FIGURE 10.12.

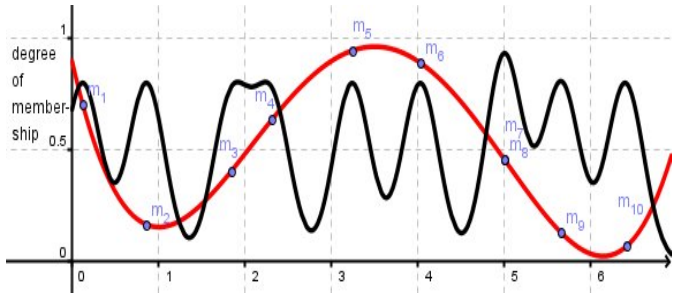
Example 10.1.3: FIGURE 10.15 *shows how the quality of information can be visualised two-dimensionally. In (a) collected data is visualised; the missing data is interpolated in (b); and the graph of the quality of information is determined in (c). The quality is good at the places, where data was collected and the farther the distance to a place where data was collected, the worse the quality becomes. In (d) the graph of the quality of information is determined with the precondition that the data in m2 and m5 is older than the other data and thus the quality at these points is worse than at newer datapoints.*



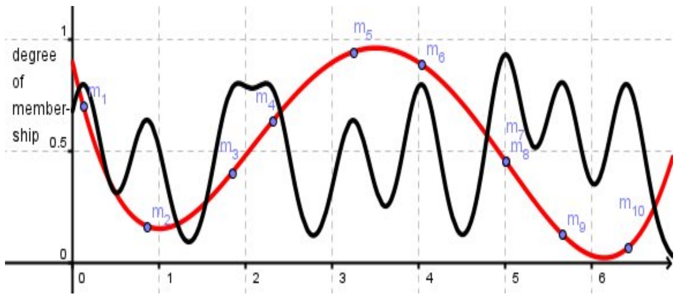
(a) Graph of the collected data.



(b) Graph of the interpolated membership mapping.



(c) Graph of the quality of information.



(d) Graph of the quality of information, if the data m2 and m5 is older than at the other data.

Figure 10.15: Graph of the quality of information. (Figure generated with *GeoGebra*.)

Figure 10.16 shows the SQI for the risk map illustrated in Figure 10.11. The map for the SQI was generated by determining the relative number of measurements at a data point. White in this case denotes good quality of information, while black denotes poor information quality.

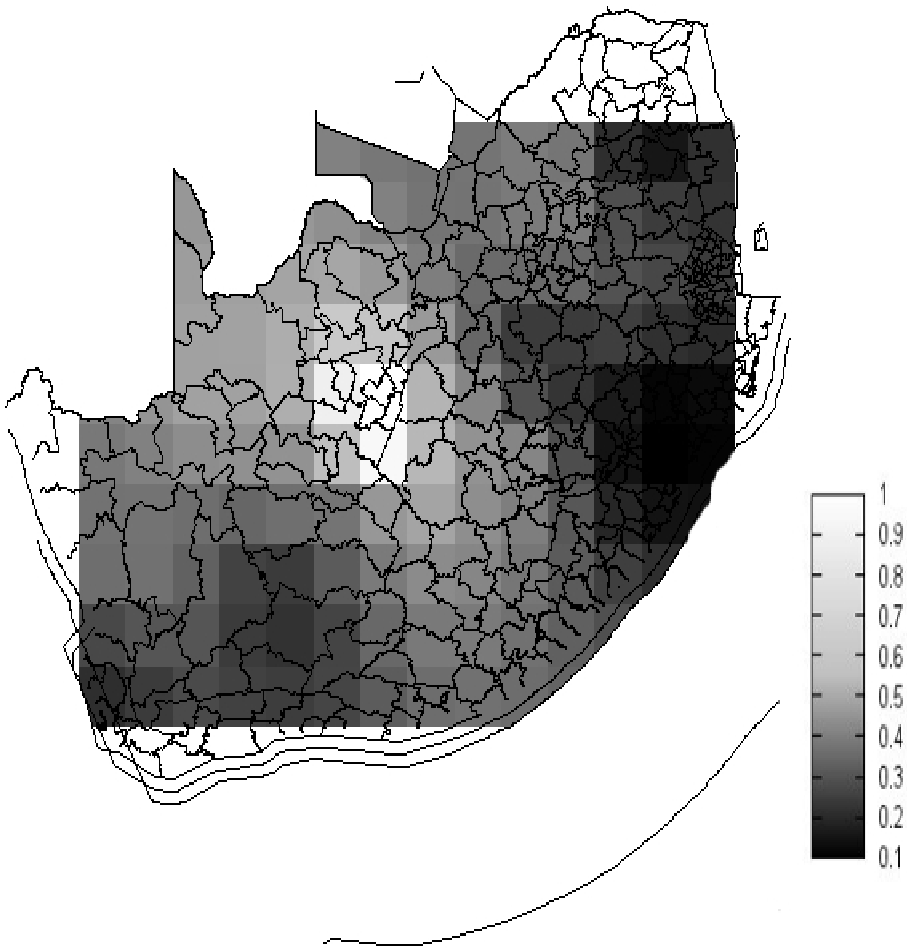


Figure 10.16: SQI for the datapoints used for generating the risk map in FIGURE 10.11. (Figure generated with *GRASS GIS*.)

Animations about the SQI for the datapoints used for generating the risk map for anopheles mosquito survival depending on rainfall and humidity and about the risk map for anopheles mosquito survival depending on rainfall and humidity from January to December generated with *GNUPLOT* can be found at the link below:

`http://mathematik.uni-landau.de/download/Platz/Animations.zip` (retrieved 16/02/2014)

10.1.2 Generating a Resource Map visualising Locations in South Africa, where Mosquito Nets are sold

For developing a resource map, prevention options for malaria have to be investigated.
Prevention is done individually by taking prophylactic drugs, and generally through water restoration and mosquito control. One type of prevention is the provision of an ITN. The lifespan of a long-lasting ITN is currently estimated to be three years. Since 2007, additional funding has resulted in huge progress in increasing access to ITNs. [Wor10] on p.xi estimates, that in mid-2010, 42% house-

holds in Africa owned at least one ITN and 35% of African children slept under an ITN. Presumably, because ITN ownership remained low in some of the largest countries in Africa until the end of 2009, the goal of the *World Health Assembly* (WHA) that at least 80% of children should use an ITN, could still not be reached. From some surveys, it is apparent that low use is mainly due to a lack of adequate nets for all members of the household. Results of the household survey suggest that the majority (80%) of the available ITNs are used. (cf [Wor10], p.xi)

Thus the communication of the importance of ITNs and the availability of sufficient nets accompanied by an optimised distribution of mosquito nets plays an important role. In Africa, current methods

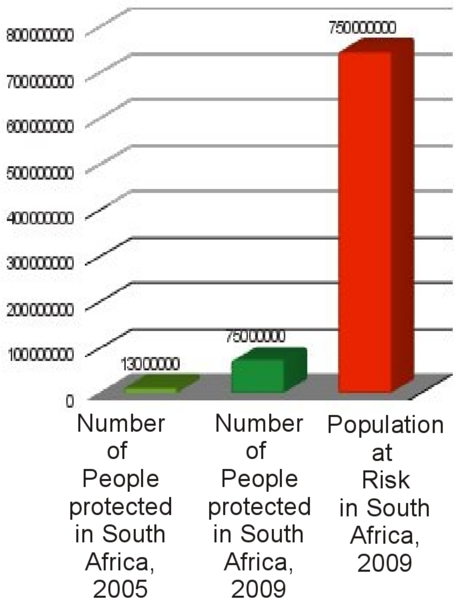
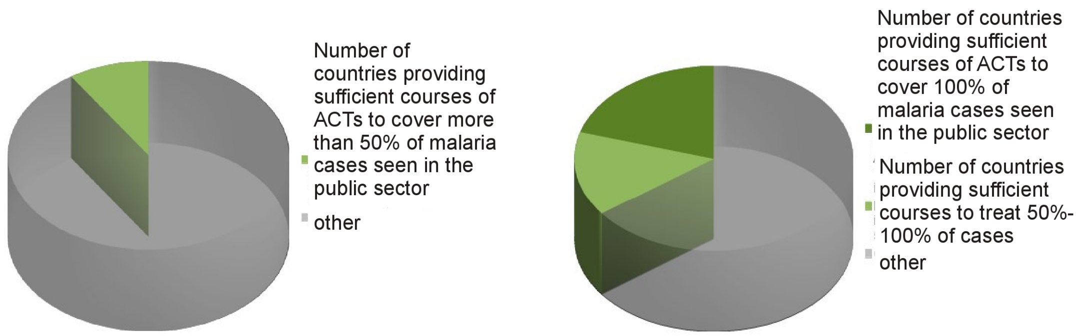


Figure 10.17: Number of people protected by IRS programmes in sub-Saharan Africa. (Figure generated with *Libre Office Calc.* Compare: [Wor10], p.xi.)

of malaria vector control are based on a single class of insecticides, the pyrethroids, which are the most widely used compounds for Indoors Residual Spraying (IRS) and may be used as the only class of insecticides for ITNs. In FIGURE 10.17, the number of people protected by IRS programmes in 2005 and 2009 in sub-Saharan Africa is depicted. However, the widespread use of a single class of insecticides increases the risk that the mosquitoes will become resistant to the insecticide, which could quickly become a major public health problem. (cf [Wor10], p.xi).

In the regard, Dichloro-Diphenyl-Trichloroethane (DDT) resistance, permethrin resistance, deltamethrin resistance and lambdacyhalothrin resistance in anopheles gambiae s.s. and s.l. become a problem. DDT is a synthetic pesticide, which acts as a contact poison and stomach poison to insects. However, because of side effects, including the suspicion that DDT could cause cancer in humans, DDT is banned in Germany and other countries. Permethrin is a synthetic chemical, widely used as an insecticide, acaricide, and insect repellent. Deltamethrin is a pyrethroid ester insecticide, while cyhalothrin is a pyrethroid insecticide and lambdacyhalothrin is a mixture of highly active isomers of cyhalothrin. Permethrin, deltamethrin and lambdacyhalothrin are all types of pyrethroids and used for IRS. ([Pau00], p.178).

The best available treatment for malaria, particularly for plasmodium falciparum malaria, is Artemisinin-Based Combination Therapy (ACT). In FIGURE 10.18, the availability of ACTs in 2005 and 2009, based on information from manufacturers, is illustrated. Information on access to treatment is generally incomplete, especially for the significant proportion of patients who are treated in the private sector. However, the effect of oral artemisinin-based monotherapies is threatened by the spread of resistance to artemisinin. Nevertheless, some African countries still allow the marketing of monotherapies. (cf [Wor10], p.xi).



(a) Availability of ACTs in 2005. **(b)** Availability of ACTs in 2009.

Figure 10.18: Availability of ACTs based on information from manufacturers. (Figure generated with *Libre Office Calc*. Compare [Wor10], p.xi.)

Thus, communication and information with respect to the effect and, eventually, resistance to ACTs is very important. Cases of resistance could be reported via mobile device, that is, crowd-sourcing. Based on this data, resistance maps could be developed and the ACT manufacturers could develop new medication, if the areas of resistance grow too big. The distribution of ACTs could be optimised using the combination of risk and resource maps to develop a decision support map, as done below in the development of a resource map. In this case, the resource is the mosquito nets, see FIGURE 10.19, which provide protection against mosquito bites (see p.188). The map was developed by identifying locations in South Africa, where mosquito nets are sold, like

- *Campworld* (`http://campworld.campworld.co.za/` (retrieved 16/02/2014)),
- *Kiwinet* (`http://www.kiwinet.co.za/` (retrieved 16/02/2014)),
- *Sleep Eezzi* (`http://www.sleepeezzimosquitonets.co.za/` (retrieved 16/02/2014)) and
- *Cape Union Mart* (`http://www.capeunionmart.co.za/` (retrieved 16/02/2014)).

Figure 10.19: Bed with mosquito net in Africa.

We consider the locations where mosquito nets are sold, because data relating to the distribution of free mosquito nets is not easy to find. Accordingly, the distribution of mosquito net stores may be related to the distribution of mosquito nets among the population. Accordingly, the resource map can be described by a mapping:

$$M_{resource} : (\mathbb{R}^2, \mathfrak{T}_{nat}) \stackrel{\mathfrak{T}}{\rightarrow} ([0,1], \mathfrak{T}_{rel}).$$

The resource map is visualised in FIGURE 10.20.

This map could be improved by, among other things, including more data points for higher resolution and quality, while being careful not to weaken the result by including too many data points, or by including the demand for mosquito nets by determining the exact amount of available mosquito nets per location (the identification of tourist locations is important, because for the protection of tourists more mosquito nets are made available free for the population) and by including the prices paid for mosquito nets and the income of the population to determine, which products are affordable for the population.

The *World Malaria Report 2010* ([Wor10], p.121) delivers only little information on the use of mosquito nets in South Africa, thus a comparison of the resource map with reality is difficult.

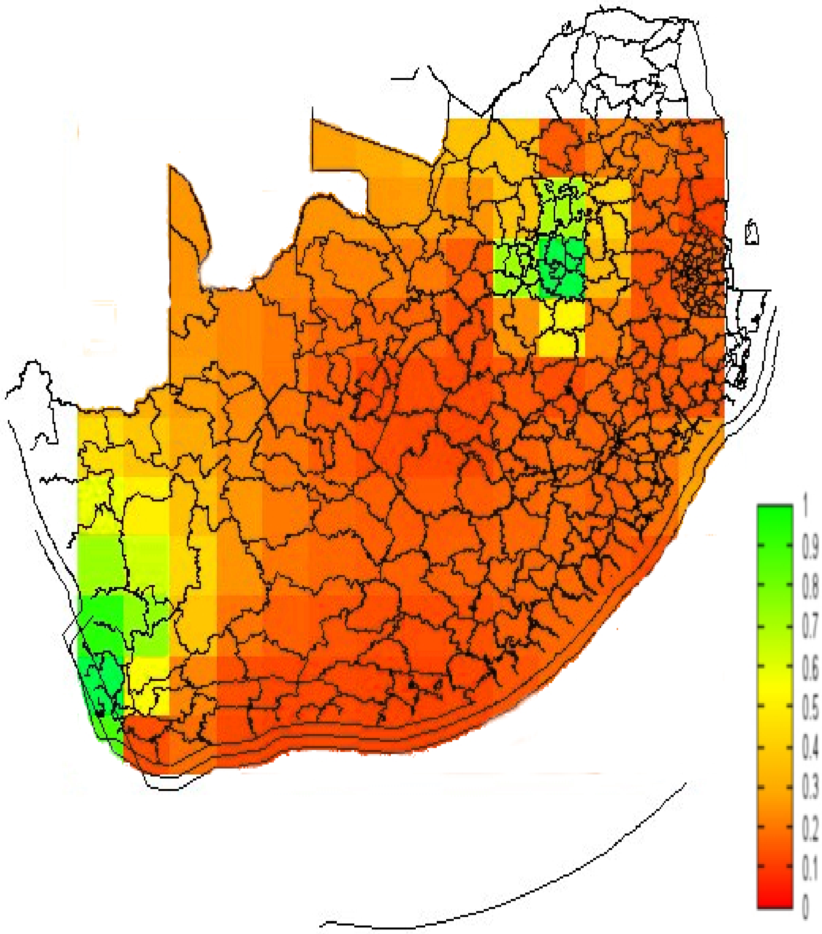


Figure 10.20: Map of South Africa with interpolated surface showing the availability of mosquito nets depending on the amount of shops and manufacturers selling mosquito nets in South Africa. 0 denotes low mosquito net availability and 1 denotes high mosquito net availability. (Figure generated with *GRASS GIS*.)

10.1.3 Generating a Map for Decision Support visualising the Locations where Mosquito Nets are sold in Relation to Anopheles Mosquito Survival based on Rainfall and Humidity in South Africa

A map for decision support could be generated by combining risk and resource maps:

$$\begin{aligned} M_{decision} :& (\mathbb{R}^2, \mathfrak{T}_{nat}) \overset{\mathfrak{T}}{\to} ([0,1], \mathfrak{T}_{rel}) \\ & (x,y) \mapsto \frac{M_{risk}(x,y) - M_{resource}(x,y) + 1}{2} \end{aligned}$$

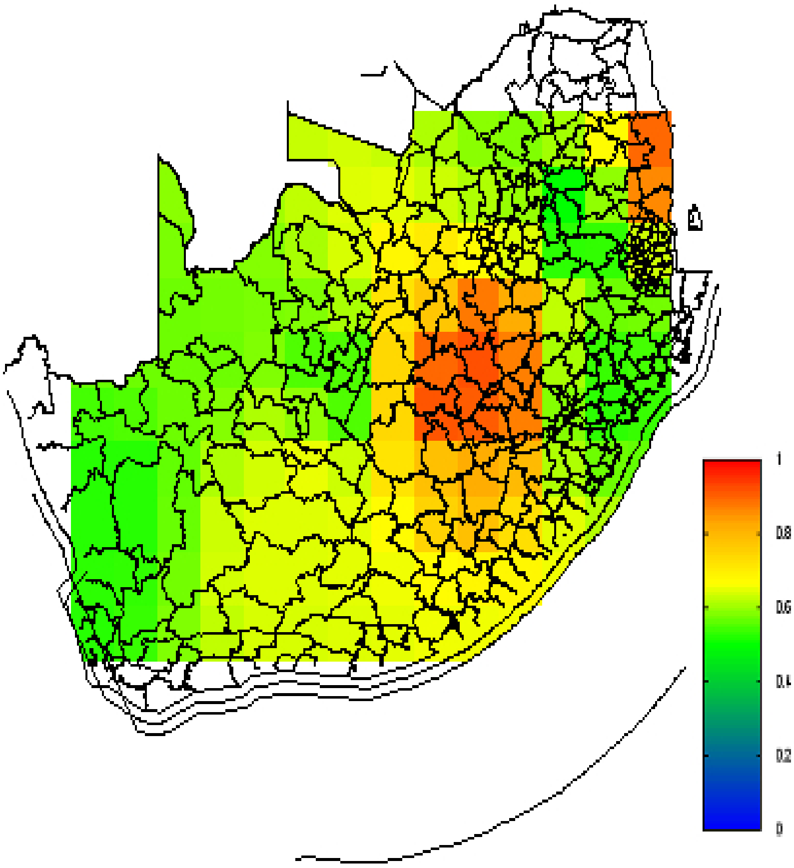


Figure 10.21: Map of South Africa with interpolated surface showing the availability of mosquito nets in relation to mosquito survival. 0 denotes many more mosquito nets than mosquitoes, 0.5 denotes equal distribution and 1 denotes many more mosquitoes than mosquito nets. (Figure generated with *GRASS GIS*.)

When combining the map in FIGURE 10.21 with the population density (FIGURE 10.22), decision support could be delivered for a governmental facility by, for example, illustrating optimal locations for the distribution of free mosquito nets. Two proposals for favourable locations for distributing free mosquito nets are highlighted with white circles in FIGURE 10.23. Those locations are “good”, because there is higher mosquito survival expected than the mosquito nets are available and the population density is high. The map for decision support could be improved by improving the risk and resource maps and by improving the way of combining the risk and resource maps.

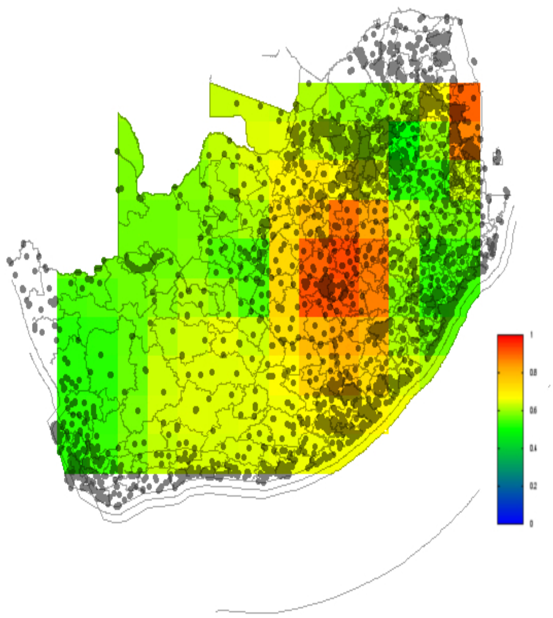


Figure 10.22: Map of South Africa with interpolated surface showing the availability of mosquito nets in relation to mosquito survival combined with the population density. 0 denotes many more mosquito nets than mosquitoes, 0.5 denotes equal distribution and 1 denotes many more mosquitoes than mosquito nets. (Figure generated with *GRASS GIS*.

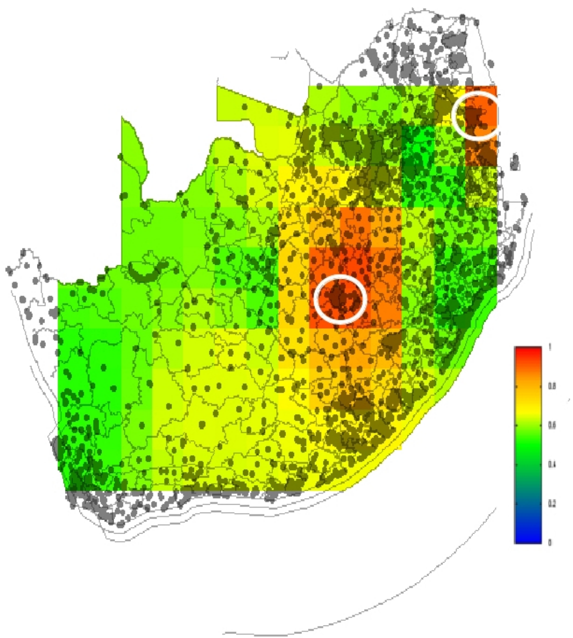


Figure 10.23: Map of South Africa with interpolated surface showing the availability of mosquito nets in relation to mosquito survival combined with the population density with proposals for "good" locations for distributing free mosquito nets. 0 denotes many more mosquito nets than mosquitoes, 0.5 denotes equal distribution and 1 denotes many more mosquitoes than mosquito nets. (Figure generated with *GRASS GIS* and *Libre Office Draw*.

10.2 Exemplary GUIs tailored to different User Groups

In the following section, exemplary GUIs for the user groups "user who is situated in a hazardous situation" and "governmental facilities" are presented.

10.2.1 Exemplary Design for a User who is situated in a hazardous Situation

The demands of a user who is situated in a hazardous situation are the following:

- How can I protect myself?:
  - Am I located in a hazardous area?
  - How can I get out of/cross this hazardous area possibly unscathed?
  - What preventive measures can be taken?

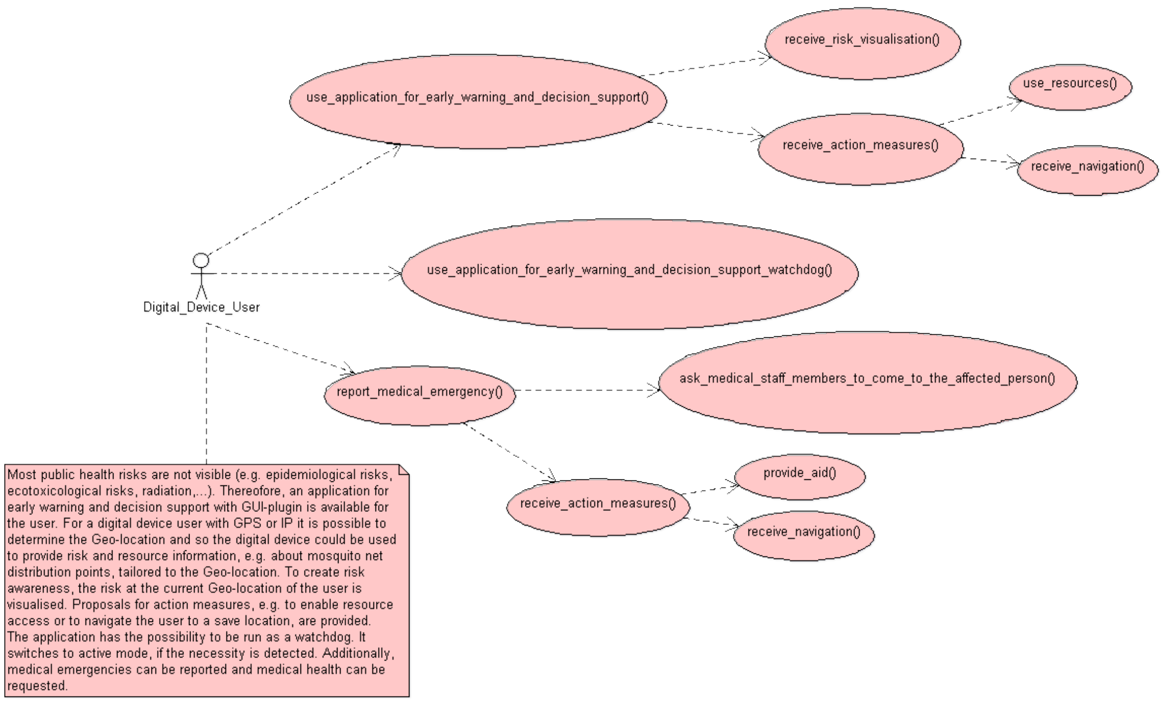


Figure 10.24: Use case diagram for a user who is situated in a hazardous situation. (Figure generated with *ArgoUML*.)

In FIGURE 10.24, a use case diagram for a user who is situated in a hazardous situation is illustrated. In the following section, a sequence diagram for the use case

use_application_for_early_warning_and_decision_support()

is illustrated.

The first step is the initialisation (Figure 10.25). During the initialisation, the user is provided with an initial GUI. The next step, the adaption process (Figure 10.26), is executed until the user quits the system (Figure 10.27). During the adaption process the user is provided with several GUIs (one GUI at the same time) wherein two or more GUIs can be the same, if the GUI provided is the most appropriate for the user. However, the system constantly controls whether the provided GUI still fits to the user and provides another GUI if necessary.

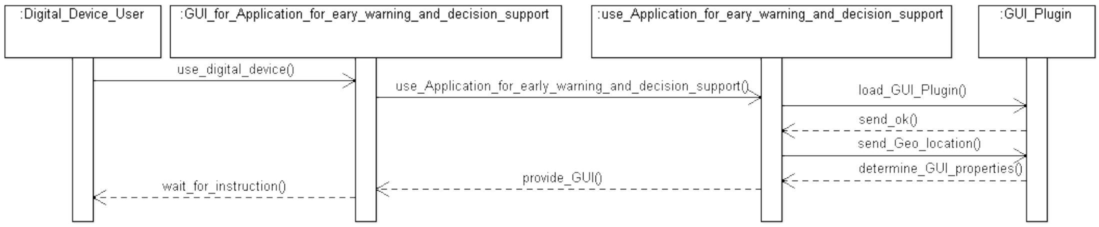


Figure 10.25: Sequence diagram illustrating the functioning of the initiation of the SAGU in UML. (Figure generated with *ArgoUML*.)

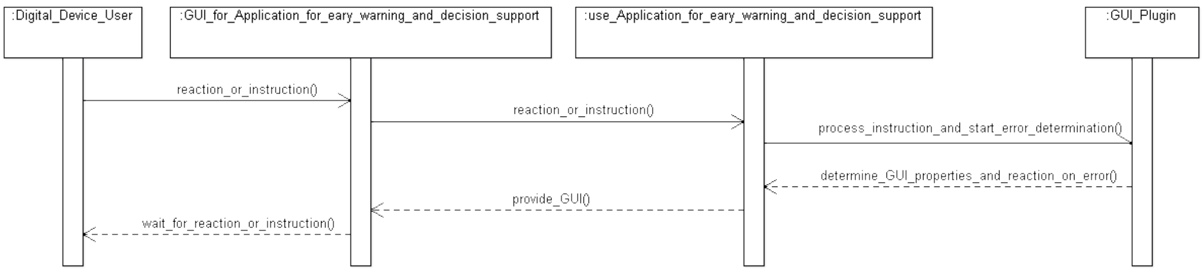


Figure 10.26: Sequence diagram illustrating the functioning of the application for adapting a GUI to the user in UML. (Figure generated with *ArgoUML*.)

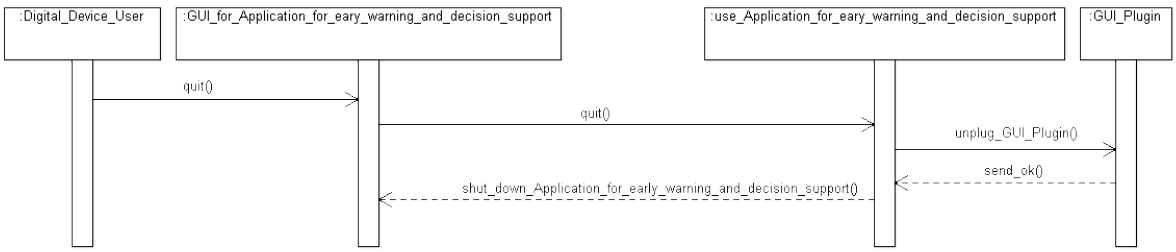
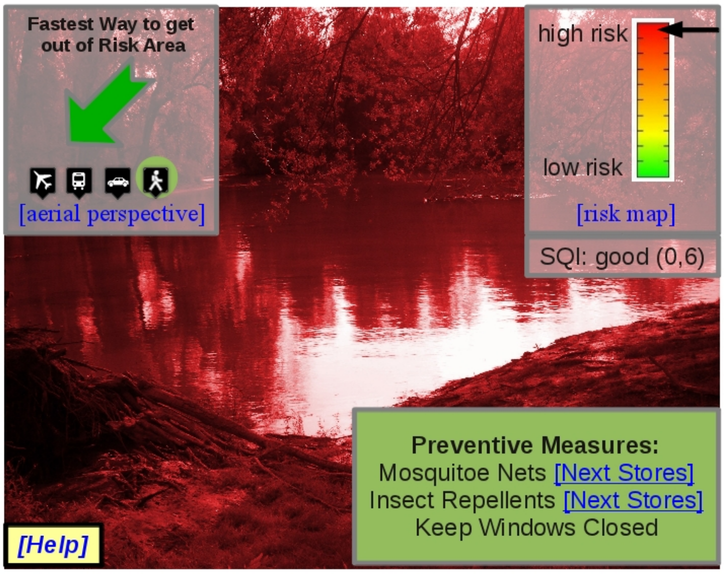


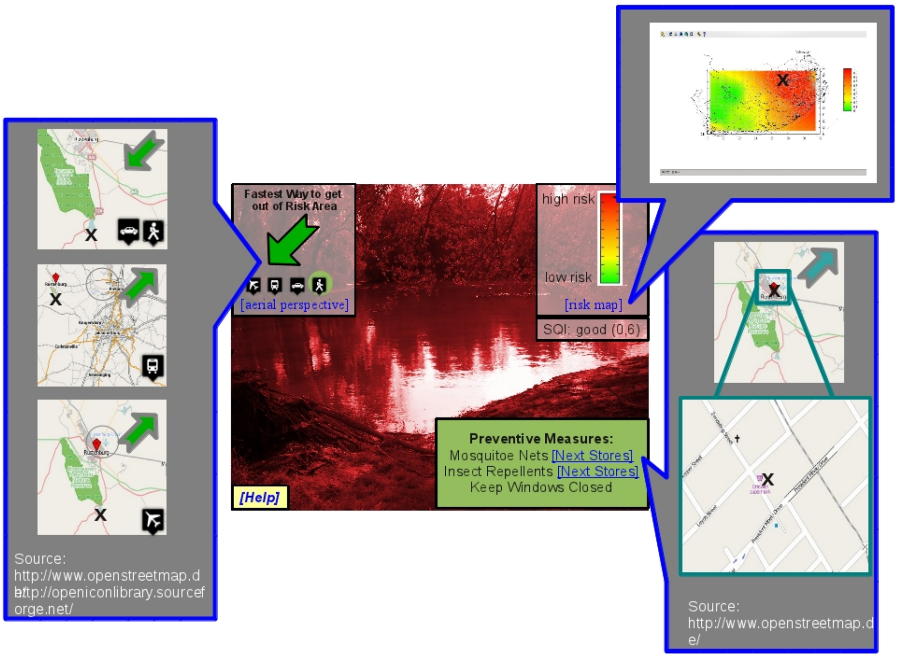
Figure 10.27: Sequence diagram illustrating the functioning of the quitting of the SAGU in UML. (Figure generated with *ArgoUML*.)

The GUI for a user who is situated in a hazardous situation is developed for a mobile device. Such a GUI design should consider the demands of the user. For an optimal decision on questions of navigation, we first require a net of possible ways and times used to pass a travel path. The return path may be faster or more passable (path problem). We have to identify the possible options for going a certain route: by foot, with a vehicle (car, public transport) and adapt the net accordingly. The time used could be rated using fuzzy mappings (slow/ fast). One idea is to measure the motion speed of a user for better calculation and to adapt the decision support to the user conditions. Risk maps for decision support and an algorithm, for example ACO (see p.140f), to solve the path problem mentioned above should be included. Different versions of the application should be developed, for disabled people, wheelchair users, colour-blind individuals, illiterates, and so forth. As image-heavy applications use a great deal of energy and, in case of danger, access to electricity can not be assured, an energy-saving version should be developed. The same applies to internet access; thus an offline version should be developed. It should be possible for the user to identify priorities: Whether it is important to her/him to move as fast as possible or to be exposed to as little risk as possible or whether she/he even wants to enter a danger zone (e.g. to save a person). In FIGURE 10.28, an exemplary GUI for the user group "a user who is situated in a hazardous situation" is presented. The background is coloured red, which means, that the user is located in a hazardous area. This is underlined by the scale in the upper right-hand side of the figure. Thus we have two different GUI element objects of the GUI element for visualising the grade of risk. Some further GUI elements are used within the GUI: The SQI is quoted below. In the lower right-hand side of the figure, the preventive measures which can be taken are listed. On the lower left-hand side, a help-button is placed. On the upper left-hand side, an arrow appears, pointing into the direction for moving out of the hazardous area as fast and unscathed as possible. At the moment, the device is set in pedestrian mode. On the upper right-hand side a scale for supporting the comprehension of the grade of risk and the SQI at the current geo-location is displayed.
First prototypes concerning a GUI for the user group "user who is situated in a hazardous situation" can be found at the links below:

- `http://mathematik.uni-landau.de/download/Platz/Prototype/Riskmap.html` (retrieved 16/02/2014)
- `http://mathematik.uni-landau.de/download/Platz/Prototype/FrameGUI.html` (retrieved 16/02/2014)



(a) Example for a GUI for the user group a user who is situated in a hazardous situation.



(b) Description of the GUI.

Figure 10.28: Example for a GUI. (Figure generated with *Libre Office Draw*.)

10.2.2 Exemplary Design for Governmental Facilities

The demands of a governmental facility, for example the *PHA*, are the following:

- Where and how to intervene?
  - Who must be warned?
  - Where to allocate resources?
  - What advice and decision support can be delivered to the population?

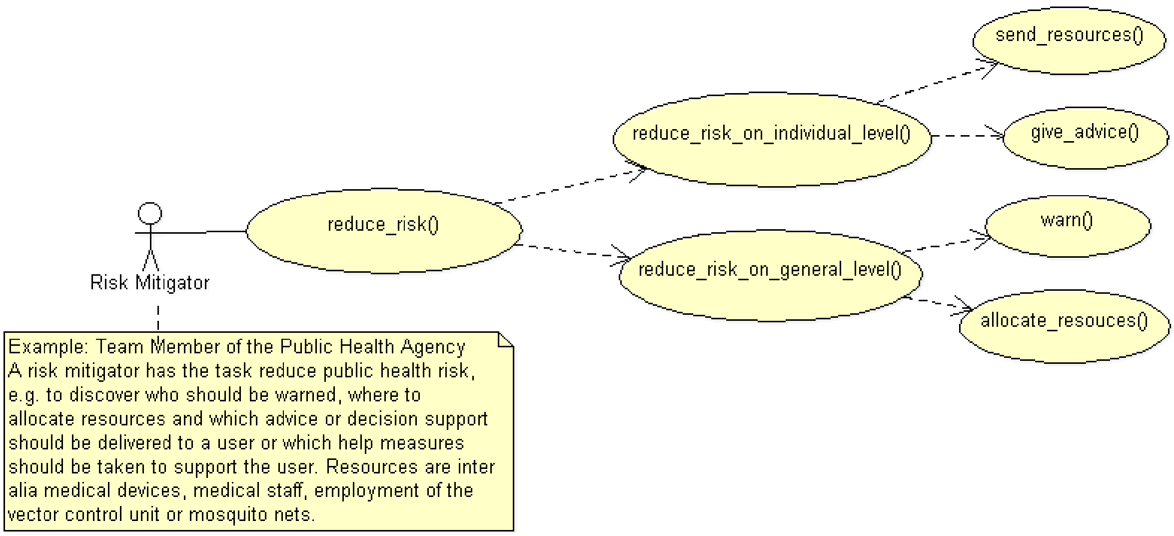


Figure 10.29: Use case diagram for a decision maker. (Figure generated with *ArgoUML*.)

In Figure 10.29, a use case diagram for a decision maker is presented. Accordingly, a sequence diagram for the use case

use_ application_for_ Risk_ Reduction()

is illustrated.

The first step is the initialisation (Figure 10.30). During the initialisation, the user is provided with an initial GUI.

The next step, the adaption process (Figure 10.31), is executed, until the user quits the system (Figure 10.32). During the adaption process the user is provided with several GUIs (one GUI at the same time). In this case, two or more GUIs can be the same, if the GUI provided is the most appropriate for the user. However, the system constantly controls whether the GUI provided still fits to the user and provides another GUI if necessary.

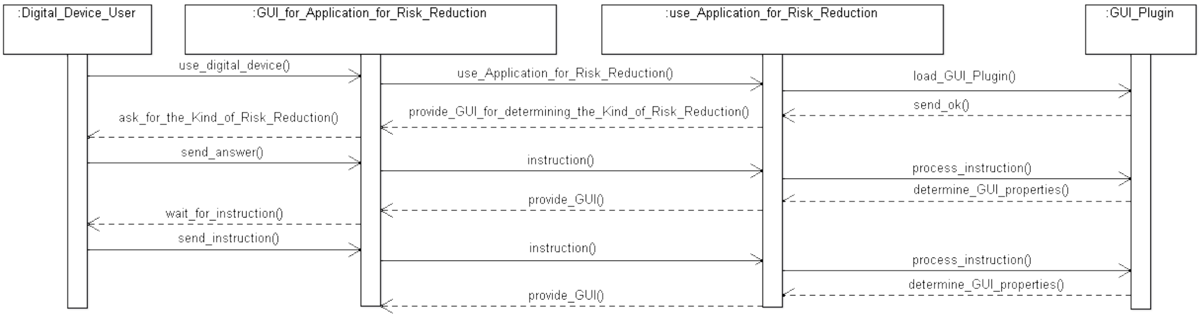


Figure 10.30: Sequence diagram illustrating the functioning of the initiation of the System for adapting a GUI to the user in UML. (Figure generated with *ArgoUML*.)

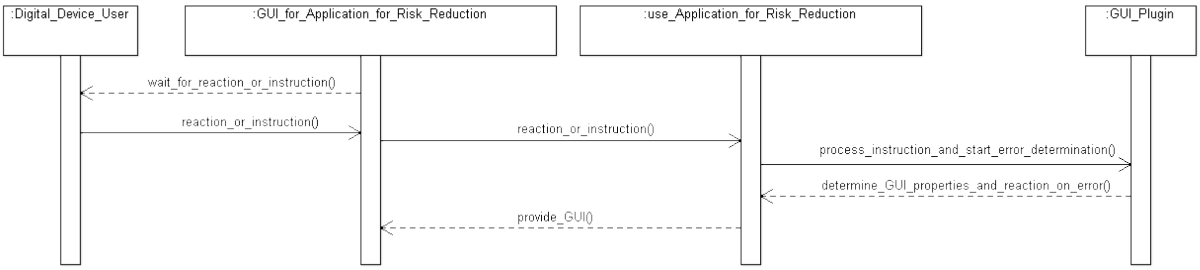


Figure 10.31: Sequence diagram illustrating the functioning of the application for adapting a GUI to the user in UML. (Figure generated with *ArgoUML*.)

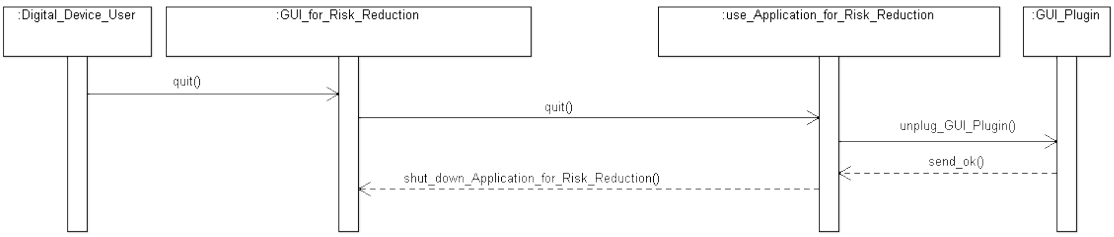


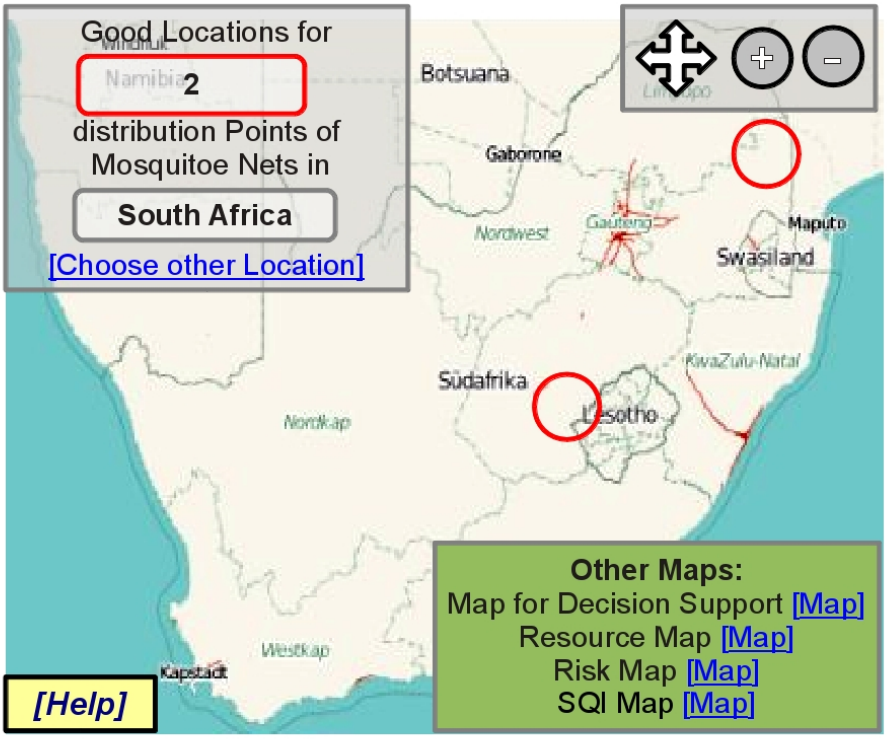
Figure 10.32: Sequence diagram illustrating the functioning of the quitting of the System for adapting a GUI to the user in UML. (Figure generated with *ArgoUML*.)

In the example of a GUI for the *PHA*, Figure 10.33, we confine ourselves to the demand "Where should resources be located?". In the background, an *OSM* map is displayed with proposals for locations for new resource distribution points, depicted by red circles. On the upper left-hand side, the number of locations for new resource (e.g. mosquito net) distribution points can be entered and the intervention area can be chosen. On the upper right-hand side, navigation tools are supplied. On the lower right-hand side, several maps, which were used for the generation of the decision support map can be displayed. A help button appears on the lower left-hand side.

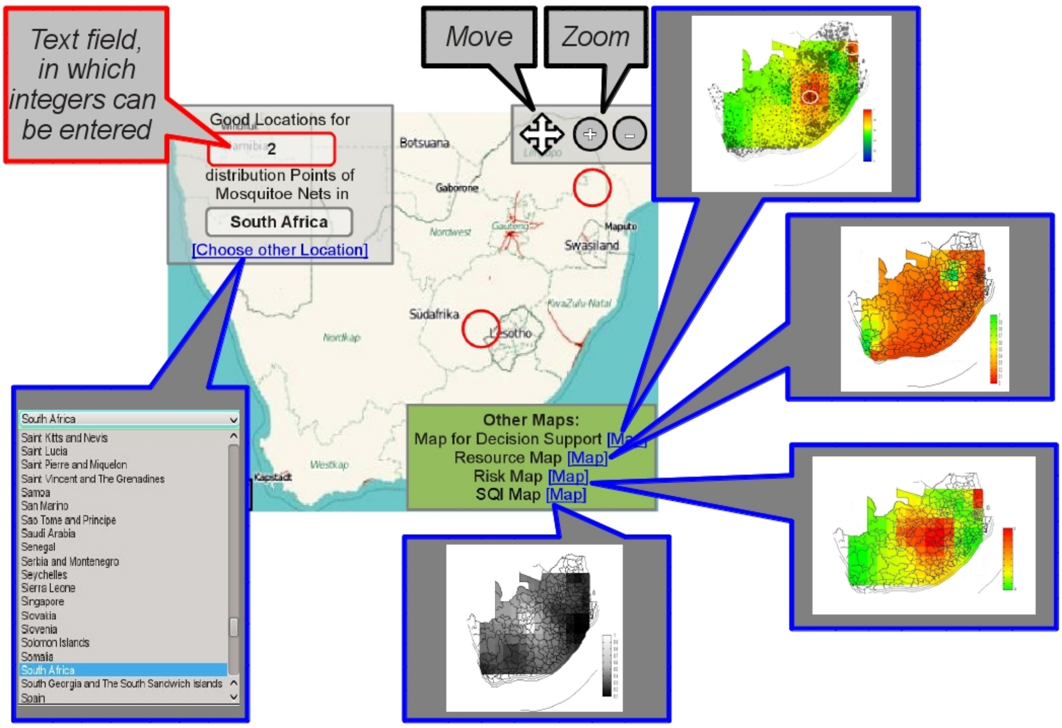
When developing the various GUI designs in the future, the GUI guidelines described in CHAPTER 5 will be implemented. Additionally, telemedicine (SUBSECTION 5.5.1), risk perception (SUBSECTION 4), AR (SUBSECTION 5.5.2) and the use of AR for identification of species (p.49) will be included. Telemedicine will be used to support a person e.g. in an accident-situation, where this person has to intervene to help an injured person if no medical doctor is available physically. Risk perception can be used to replace scales for better comprehension of risk, while AR can be used e.g. for displaying an invisible risk (e.g. an epidemiological risk) or better visualisation of complex tasks (e.g. a medical treatment done by a lay-person. Therefore, AR can be implemented into telemedicine). The approach in terms of which species are identified could be adapted for the identification of anopheles mosquitoes. Accordingly, crowd sourcing could be used for the collection of data on the occurrence of anopheles mosquitoes in certain areas. In other words, when somebody identifies an anopheles mosquito using our application, the location of the user will be delivered via GPS and stored in a database. Based on this data, very up-to-date risk maps could be generated.
The *Project Reality View*, a *Navit* extension based on AR, will be developed in cooperation with the *Geo-information Group of Potsdam University* based on the *Android* platform. (see `http://wiki.navit-project.org/index.php/Augmented_Reality` (retrieved 16/02/2014)). The *Project Reality View* could easily be modified for the application for early warning and decision support to improve the comprehensibility of the navigation. Furthermore, *Navit* could be equipped with an adaptive GUI design based on the approach developed in this thesis.
Several GUIs for different digital devices, for example smartphones, cell phones with smaller displays or cell phones displaying only text information, PCs and digital doorways will be developed. Some other requirements for the digital devices and the GUI are described on p.197. Certain GUI elements will be evaluated within further empirical studies with regard to adaptivity, comprehensibility, benefit for the user and conceiving the spatial sense.

As a result of the requirement to include measure theory on topological spaces to address our concerns, a generic mathematical structure for a system adapting a GUI to a user was developed (CHAPTER 9). In order to implement the developed system, an Object-Oriented Analysis (OOA) was performed in this chapter and in CHAPTER 9 and, thereby a first method for bridging the fields measure theory on topological spaces and Object Orientation (OO) was developed. The scope of this thesis was, inter alia, to conduct an OOA as a precondition for the realisation of Object-Oriented Programming (OOP).

In order to adapt the GUI to South Africa conditions it is important to determine an initial state for the GUI. Therefore, an empirical pilot study was carried out, which is described in the next chapter.



(a) Example for a GUI for the user group *PHA*.



(b) Description of the GUI.

Figure 10.33: Example for a GUI. (Figure generated with *Libre Office Draw*.)

11 The Empirical Pilot Study *Geo-GUI*

"Given their increased importance, testing GUIs for correctness can enhance the entire system's safety, robustness, and usability." ([Mem02]).

In order to investigate complex dynamical systems and the use of adaptive GUIs for decision support, an initial state is required. In a preliminary empirical pilot study the way in which an initial GUI design can be chosen and what effect this has on the learning success should be demonstrated. After a prototype has been developed, the GUI can be tested using empirical research and subsequently improved according to the results. Certain GUI elements will be evaluated within further empirical studies with regard to adaptivity, comprehensibility, benefit for the user and conceiving the spatial sense.

A popular approach of GUI testing is to release beta copies of the software and let the users do part of the testing, ([Mem02]), in other words including User Experience (UX) (see DEFINITION 5.0.8).

A GUI test case execution can be defined as follows:

Definition 11.0.1 *GUI Test Case Execution ([Mem02])*

In a GUI test case execution, a tester gives the inputs one step at a time and compares the expected output with the GUI's output after each step.[1] *Although GUIs differ from conventional software in ways that require unique testing techniques, the overall process for implementing this methodology should follow the steps for conventional software:*

- *Determine what to test by defining coverage criteria.*
- *Generate test case inputs from software specifications and structure.*
- *Generate expected output to compare with actual output.*
- *Execute test cases and verify output.*
- *Determine whether the GUI has been adequately tested.*

The experimental design to evaluate our GUIs is described below. The main questions to answer, that is, the coverage criteria, are as follows:

[1] This interplay complicates GUI testing, however.

- What happens when dealing with the pictorial representation?
- How are images processed?
- How are the instructions interpreted spatially?

Certain elements used in our GUIs will be examined and evaluated. Therefore a test-environment in the form of an empirical pilot study will be created, in which the items mentioned above about testing techniques of GUIs will be considered.

First, the concept of empirical research will be discussed in more detail.

Each applied science uses the experiment as a way of gaining knowledge, for example in research to explore new issues or in the monitoring and control of processes. Every experimental investigation must be based on a precise problem. The formulation of this problem is not the task of mathematics; however, efforts to make use of the language of mathematics for this formulation, leads to an exact re-thinking of the problem and usually also to a subsequent clarification. By clarifying the problem, the variables under investigation and the main conditions under which they should be considered, are determined. However, it is impossible to define all the conditions for a test. Furthermore, of course, the statements expected from the test results usually apply only for the specified test conditions. A meaningful description of such issues and practical problems allows the application of the methods of stochastic, for example probability theory and mathematical statistics. Here, the influences acting on the test result that we do not know or are unable to keep constant, can be considered as random effects. The results of the tests are, thus, random events. (cf [BB79], p.7).

In this thesis, an empirical experiment is realised. Firstly, some important terms are defined.

Definition 11.0.2 *Test (cf [BB79], p.7)*
A test can be defined as the observation of a corresponding random element. These random elements depend on specified test-conditions, which can be systematically changed from test to test.

Definition 11.0.3 *Experiment (cf [BB79], p.8)*
A well-defined, finite set of tests is called experiment.

The experiment carried out in order to show how an initial GUI design can be chosen and what effect this has on the learning success is described in SECTION 11.1.

Definition 11.0.4 *Empirically (cf [Sti96], p.4 f)*
The word "empirically" is generally understood as being "based on experience" or "belonging to experience".

Definition 11.0.5 *Experience (cf [Sti96], p.5)*
Experience means information gathering with the help of the senses.

"Senses" does not have to be limited to the human senses, but also includes technical devices, such as microscopes, cameras, and suchlike. (cf [Sti96], p.5).
In the empirical pilot study described in Section 11.1, a screen-cast software, inter alia, was used to observe the proband's activities.

Definition 11.0.6 *Empirical Research Methods (cf [Sti96], p.5)*
Empirical research methods are understood to be those that can be used for information gathering or data acquisition about partial aspects of reality.

Empirical research methods are constructed, inter alia, as certain instruments for data acquisition such as content analysis, observation, survey, experiment and scaling methods. Besides instruments for data acquisition and instruments for data analysis, such as statistical methods, play an important role in empirical research. The main principle of empirical research methodology can be formulated as follows: All statements of an empirical science have to be verifiable by experience and they have to be able to founder on experience. Thus, the following three conclusions can be made with regard to practical empirical work (cf [Sti96], p.5 ff):

- All terms occurring in a statement that shall be verified empirically, have to refer to the experiential reality.
- Statements that shall be verified empirically have to refer to issues, that are basically experiential.
- Statements that shall be verified empirically have to be formulated in such a way, that they can be basically disproved.

The following issues are important for the empirical research process (cf [Sti96], p.17 ff):

- The research problems have to be specified.
- The used terms have to be defined.
- The operationalisation of the terms enables them to be measured.
- Data has to be collected at statistical units.
- It must be determined whether a primary analysis is necessary, or whether it may be limited to a secondary analysis.
- Finally the collected data has to be analysed.

11.1 Design of the Study

11.1.1 Research Questions (Coverage Criteria)

The following questions would be of interest to us:

- What impact do the developed GUIs have on the user?
- How intuitive are the developed GUIs?
  - What impact does the visual design of a GUI have or what impact do certain GUI elements have on the user?
  - What impact does the adaptivity of a GUI have?

During this research process, only the last question is intended to be partly answered:

- What is the impact of the initial GUI design tailored to a user based on the user profile?

In this study, the user profile is determined by means of a questionnaire, see Subsubsection 11.2.1. The research question will be approximately answered by answering the following question:

- What influence does an adjusted GUI have within the competence area of spatial orientation on learning success?

11.1.2 Quality Criteria of the Test Instruments

The research questions are intended to be answered with the use of a survey and by observing the action of the user, that is, the pupil, when solving certain tasks. The data collection tools, the survey and observation are elucidated as follows:

Data collection tool: Survey. A survey is the most widely used data collection tool. In this thesis, a written, partially standardised survey is carried out. Questions can be classified according to the type of information that can be gained with them:

- attitude/ opinion questions,
- belief questions,
- behavioural questions,
- property questions.

Another classification that is not oriented to the desired information is the one that considers the formal structure of questions, which also includes the answers:

- Closed questions: multiple choice answers are provided.
- Open questions: a response in self-chosen words is required.

The wording and sentence structure of a question plays an important role. Some rules for formulating questions include:

- Questions should include simple words.
- Questions should be short.
- Questions should relate to specific and not abstract issues.
- Questions should be adjusted to the language level of the respondent.
- Suggestive questions should be avoided.
- Questions should be formulated in neutral terms.
- Questions should not be formulated hypothetically.
- Questions should only relate to one issue.
- Questions should not contain double negations.
- Questions should not overtax the respondent.
- Questions should be formally balanced, that is, all positive and negative response options should be included in a question.
- Questions should contain only clear words.

The term “questionnaire” is defined in Definition 9.4.1. The main purpose of a questionnaire is a “correct” translation of the research questions into the items of a questionnaire, but it should also be designed motivating to be encouraging for the respondents and increase their willingness to cooperate. A few guidelines for questionnaire design are as follows:

- Introduction questions play an important role as they should create a “favourable climate” for the survey. Therefore, these questions should be easy to answer and should introduce the topic of the survey in an interesting way.
- Questions that refer to the same topic have to be grouped and should not be spread over the entire questionnaire.
- Belief questions serve to distinguish different topics.
- One should pay attention to possible distortion effects. They can occur, when each question is not seen in isolation by the respondent, but the questions are related to each other.

- Filter questions have the function that some (subsequent) questions can be skipped depending on the given answer of "not applicable".
- The layout of a questionnaire plays an important role. It should have a clearly recognisable structure and it should make a "professional" overall impression. (cf [Sti96], p.173 ff).

Data collection tool: Observation. In this thesis an open, non-participative, partly systematic observation in an artificial situation is carried out by screen-casting the actions of the pupils.

Definition 11.1.1 *Scientific Observation (cf [Sti96], p.169 ff)*
An observation is called scientific, if it is carried out

- *focused, i.e. if it serves a specific research purpose.*
- *systematic, i.e. the observations are made and recorded according to predetermined categories of observation.*
- *methodically controlled, i.e. possible confounding factors that distort an observation are considered.*

11.2 Method of the Study

The empirical pilot study is embedded into the *Spatial Orientation and Mobile Devices* project.

Definition 11.2.1 *Spatial Orientation (cf [Dam07], p.6)*
Spatial orientation features the ability to classify the own person correctly in a local situation, that is, finding the mental or real route in a room.

With respect to spatial orientation, the ability to use maps for orientation is mentioned in the *Geography curriculum of the fifth grade in Germany* (compare `http://erdkunde.bildung-rp.de/sek1/lehrplan/5klasse.html` (retrieved 16/02/2014)).
The following conditions are given for the empirical pilot study: 30 pupils (one class, *class d*) of fifth grade students in a comprehensive school in Landau, Rhineland Palatinate, Germany were observed.
Prior to the study, the following was performed:

- Preparatory geography-mathematic lessons were given within the *Spatial Orientation and Mobile Devices* project:

- Two double lessons on subjective mapping - *My Route to School*:
    - first double lesson: introduction to the handling of mobile devices
    - second double lesson: preparation of orientation aids (landmarks)
- The class was separated into two groups of equal size, both groups working with mobile devices.

Following this, two double lessons à 90 minutes were held, thus a total of 180 minutes, followed by a Posttest à 45 minutes and a Follow-up-test à 45 minutes.
A data collection was done as follows:

- Questionnaires determining the current initial status of the pupils with respect to
    - subject-specific level (domain knowledge),
    - experience with digital devices and GUIs (domain knowledge),
    - expectations of the pupils about the following lessons.
- The students were observed while completing the tasks using the screencast software *vokoscreen*.
- Questionnaires aimed at psychological aspects of pupils that occur while completing the tasks using/ not using a GUI/ an adjusted GUI.
- Questionnaires aimed at assessing the clarity, comprehensibility and pleasure (of visual elements) provided by a GUI.
- Questionnaires aimed at the learning success of the pupils.

Evaluation was done using multivariate data analysis, that is, cluster analysis (see p.212) and Analysis of Variance (ANOVA), (see p.227).

11.2.1 First Double Lesson (Pretest)

(90 min) (20/09/2012, 10:30am-01:00pm)
During the first double lesson, the state of the pupils' knowledge was tested within a Pretest with tasks relating to:

- resource distribution
- path problems
- spatial and distributional justice

both in the plane, and on uneven surfaces.
Justice can be defined as follows:

Definition 11.2.2 *(Social) Justice ([SS05], p.136)*
A society is sufficiently fair, if it offers the same fundamental rights to each citizen, if there exist fair equal opportunities in access to privileged positions and if it is organised, that the most disadvantaged fare better, than it would happen to them if society was organised differently.

Models of justice do not provide a constructplan for a better world, but articulate the hidden ideals, that drive the resistance against injustice. ([SS05], p.125).
Thus many parameters have to be included to determine a sufficiently justice solution for a certain situation. In this study the way in which the pupils determine justice can be observed, as well as the parameters they include within the determination process. During the Pretest questions about prior experience with GUIs and digital devices were answered by the pupils. The collected data forms the user profile about the pupils which is then evaluated for determining the best initial GUI for a user and for comparing the results with the results of the Posttest in order to determine, whether the method was successful. The tests can be assigned by using a code consisting of the first letter of the surname, the first letter of the last name and the month of the pupils' birth.

Short Planning: All groups are taught in an identically manner.

Time	Teaching Phase	Planned Teaching Course	Form of Activity, Social Form	Media / Materials
5 min	Welcoming and Access	• Welcoming of the pupils. • Introduction to the Questionnaire.	Teacher-Centered-Teaching	

35 min	Elaboration I	• Answering of the Questionnaire Parts:
		  • A) Personal Questions
		  • B) Questions about Smartphones
		  • C) Questions about Navigation
		  • D) Tasks about Logic
			Individual Work	Questionnaire

15 min	Relaxation Phase	• Video "Die Sendung mit der Maus: Globus"	Individual Exercise	Video, Beamer, Laptop

35 min	Elaboration II	• Answering of the Questionnaire Parts:
		  • E) Tasks about Spatial Thinking
		  • F) Tasks about Perceptual Speed
		  • G) Tasks about the Evaluation of Justice
		  • H) Personal Questions
			Individual Work	Questionnaire

Objectives: <u>Coarse Objective:</u> The pupils deal the first time with spatial justice and get some teaching content brought back to memory

Figure 11.1: Short planning of the first double lesson. (Figure generated with *Libre Office Writer*.)

Material: Questionnaire: The pupils may ask questions while solving the questionnaire, but, to prevent intervention, the task is described in detail on the questionnaire and the pupils can be supported by repeating the task (in the supervising person's own words). The .zip-folder containing the Pretest can be downloaded in German language at the link below:

`http://mathematik.uni-landau.de/download/Platz/Geo-GUI.zip` (retrieved 16/02/2014).

Implementation: One pupil, belonging to the control group, did not participate.

Cluster Analysis: The cluster analysis is used for the assignment of the pupils into the groups of two or three in the second double lesson. Therefore, the personal questions are weighted more than the subject-specific questions, because the pupils in the groups of two should be similar in character. The cluster analysis works similar to the ANNs (Sections 9.3 and 9.5) used within the system for adapting a GUI to a user.
The cluster analyses are methods for class formation, in other words, for classifying statistical units into subsets. These subsets are called clusters. These clusters are not given a priori; they have to be generated. The cluster-generation is done in such a way that objects belonging to a cluster are as similar as possible, while objects from different clusters differ significantly. The clustering is based on an amount of given observable variables, $x_1, ..., x_k$. Let $O := \{o_1, o_2, ..., o_n\}$ be the set of objects that have to be classified and let $C := \{c_1, c_2, ..., c_m\}$ be a set of clusters. A partition of the set of objects O is a cluster set C, which applies:

$$\sum_{i=1}^{m} c_i = O \text{ and the } c_i s \text{ are pairwise disjoint,}$$

that is, each object is located in exactly one cluster.

Definition 11.2.3 *Similarity Measure (cf [Sti96], p.323f)*
Let $O := \{o_1, o_2, ..., o_n\}$ be a set of objects. The mapping

$$\begin{aligned} S :& O \times O \to \mathbb{R} \\ & (o_i, o_j) \mapsto S(o_i, o_j) \end{aligned}$$

is a similarity measure, if:

1. $\forall \; o_i, o_j \in O : S(o_i, o_j) = S(o_j, o_i)$ *and*

2. $\forall \; o_i, o_j \in O : S(o_i, o_j) \leq S(o_i, o_i)$.

Definition 11.2.4 *(Metric) Distance Measure (cf [Sti96], p.324)*

Let $O := \{o_1, o_2, ..., o_n\}$ *be a set of objects. The mapping*

$$\begin{aligned} D :& O \times O \to \mathbb{R} \\ & (o_i, o_j) \mapsto D(o_i, o_j) \end{aligned}$$

is a distance measure, if (O, D) *is a metric space. Then* D *is called metric distance measure.*

Next up, we will take a closer look to the K-MEANS-method. This method is a partitioning, non-hierarchical method. The number of clusters that will be formed is determined a priori. Accordingly, an "optimal" classification of n objects into g clusters, $1 \leq g \leq n$, $n, g \in \mathbb{N}$, can only be made if a quality criterion is defined, for example the minimum distance property. In principle, a global optimal partition can be found by complete enumeration. Therefore, the value of the quality criterion is calculated for all admissible partitions, that is, those for which the quality criterion is defined. Finally, the one with the "best" value is selected. However, this procedure, is usually not practical, because the number of possible partitions for small n and g is already very large. The number of possible partitions of n objects with g clusters is given by:

$$\frac{1}{g!}\sum_{k=0}^{g}(-1)^k \binom{g}{k} (g-k)^n, \; 1 \leq g \leq n.$$

Example 11.2.5: *For* $n = 31$ *and* $g = 3$ *more than* 10^{14} *possible partitions are found.*

Therefore, in practice, heuristic methods that generally lead to locally optimal solutions are used. Often so-called exchange methods are used which run iteratively as follows:

1. Let C_0 be the initial partition.
2. In the partition C_l, $l \geq 0$, each object is checked to see whether the value of the quality criterion is improved, if the object is assigned to another class.
3. The object which causes results in the largest improvement is placed in the appropriate class. The emerging partition is thus denoted by C_{l+1}.
4. Steps 2. and 3. are repeated until no more improvement can be detected with respect to the quality criterion.

The K-MEANS-method requires metric scaled variables and a fixed number of clusters:

1. Starting from an initial partition, which can be formed more or less arbitrarily, for each cluster, the mean vector (centroid) is determined.
2. Then, the Euclidean distances for each object to these centroids are calculated and each object is assigned to that cluster for which this distance is minimal.

3. For clusters, that receive new objects or for those clusters, that lose objects, new cluster centroids will be determined.

4. The steps 2. and 3. are then repeated until no more re-sortings are required. (cf [Sti96], p.323 ff).

When analysing the results of the Pretest two main clusters emerged. Within these clusters individuals with similar characteristics can be found. When considering the results of the subject-specific questions, the groups required for the second double lesson can be determined. Four clusters within the main clusters emerged: very good performance, good performance, average performance, poor performance.

11.2.2 Second Double Lesson

(90 min) (26/09/2012, 10:30am-01:00pm)
The test class was separated into the following groups

- An *adjusted GUI group*, where the GUIs are assigned depending on the results of the Pretest.
- A *GUI group*, where the GUIs are assigned randomly.
- A *control group*, which works without a GUI.

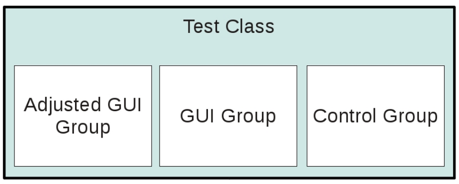


Figure 11.2: Division of the test class for the experimental phase. (Figure generated with *Libre Office Draw.*)

The class consists of 30 pupils, 15 of them were assigned to the *control group*, and 15 were assigned to the intervention group, seven to the *GUI group* and eight to the *adjusted GUI group*. In the Pretest, the pupils were asked about their former experiences with digital devices. The three groups were organised in such a way that pupils with a lot of experience with digital devices were present in equal numbers to pupils with less experience with digital devices. The *control group* was separated into six groups of two pupils and one group of three pupils; the *GUI group* was separated into two groups of two pupils and one group of three pupils; the *adjusted GUI group* was separated into four groups of two pupils; and the groups of two or three consisted of similar characters determined by cluster analysis (see p.212) with a similar result for the subject-specific questions. Thus the groups and the assignment to the four different GUIs did not happen randomly. The cluster analysis works in a similar way as the ANNs described in SECTIONS 9.3 AND 9.5. Two main-clusters emerged. The groups of two or three pupils consisted of pupils of the same cluster. Within the main clusters, an initial GUI was determined by considering the performance of the pupils in the subject-specific questions. Four clusters within the main clusters emerged: very good performance, good performance, average performance, poor performance. The *expert GUI* fits best to pupils with very good performance, the *more advanced GUI* fits best to pupils with good performance, the *advanced GUI* fits best to pupils with average performance and the *novice GUI* fits best to pupils with poor performance. Below, the four mentioned GUIs are described in more detail. In this second double lesson, tasks that prepared for the Posttest (SUBSUBSECTION 11.2.3) were provided. The tasks should be done by the pupils. The *control group* is required to carry out the tasks without using GUIs. The experimental *GUI group*, on the other hand, used a GUI that had not been proposed by the cluster-analysis as best-fitting to the user. This GUI was chosen randomly from the three least appropriate GUIs. The other experimental group, *adjusted GUI group*, used a GUI adjusted to the user based on user information, as described above. Accordingly, resource distribution, path problems and distributional justice were processed by the pupils. For this purpose, some arrangements made at the *Landau Campus* were prepared and an optimal location for a resource was determined by the pupils that included the unevenness of the terrain and thus the time a user needed to reach the resource. First, some fixed supply locations were determined. In FIGURE 11.3 three exemplary supply locations are illustrated. The task was to determine the best location for locating a resource. This location is variable. A location for a resource was first determined individually by each pupil. Then, if injustice was noted, this resource-location was optimised by the pupils. Therefore all possible routes for reaching the resource had to be worked out and the time to cover a route had to be measured. This task may be repeated several times with different amounts and locations for the supply locations. One possible initial situation is illustrated in FIGURE 11.3.

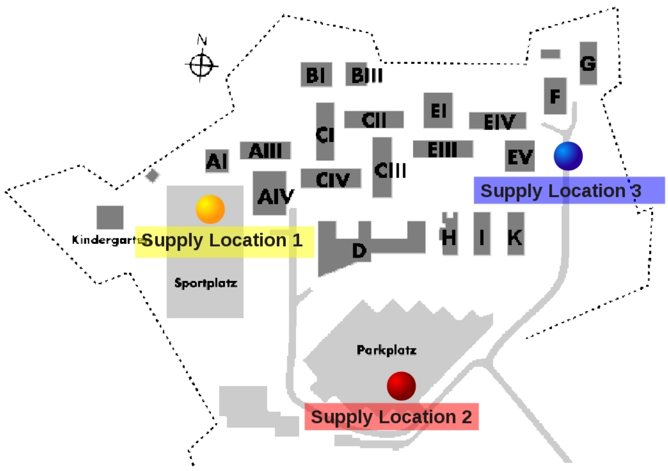


Figure 11.3: Campus map of the *University of Koblenz-Landau, Campus Landau* with a possible location of three supply locations. (Figure generated with *Libre Office Draw*.

The *control group* was given a workbook containing a topographic map of the campus showing the altitude and giving work instructions for working out all possible routes for reaching the resource and measuring the time it takes to cover a route. A smartphone with the *LOCUS free* app for tracking was used. For the *GUI group* and the *adjusted GUI group* four different GUIs were developed for a digital device (PC):

- A *novice GUI* (Figure 11.4) with the option of choosing or moving an object. The background of this GUI illustrates the ground plan of the *University Koblenz-Landau's Landau Campus*. The points "*Gruppe 1*" (blue), "*Gruppe 2*" (red) and "*Gruppe 3*" (yellow) represent the points of the three supply locations. These can be moved. Using a check-box, a point representing the resource can be made visible and moved. Using another check-box, a point can be made visible that represents the calculated location for the resource, depending on the points *Gruppe 1*, *Gruppe 2* and *Gruppe 3*, when only the straight-line distance is considered. Using 23 check-boxes, routes that can be used to cross the *Landau Campus* can be made visible. These are sorted by the cardinal directions, "*NO*" for North East, "*SO*" for South East, "*SW*" for South West and "*NW*" for North West. For each route, the length of the route and an input field for entering the measured time taken to cover the route are attached to the route. In order to distinguish between the routes, they are displayed in different colours.

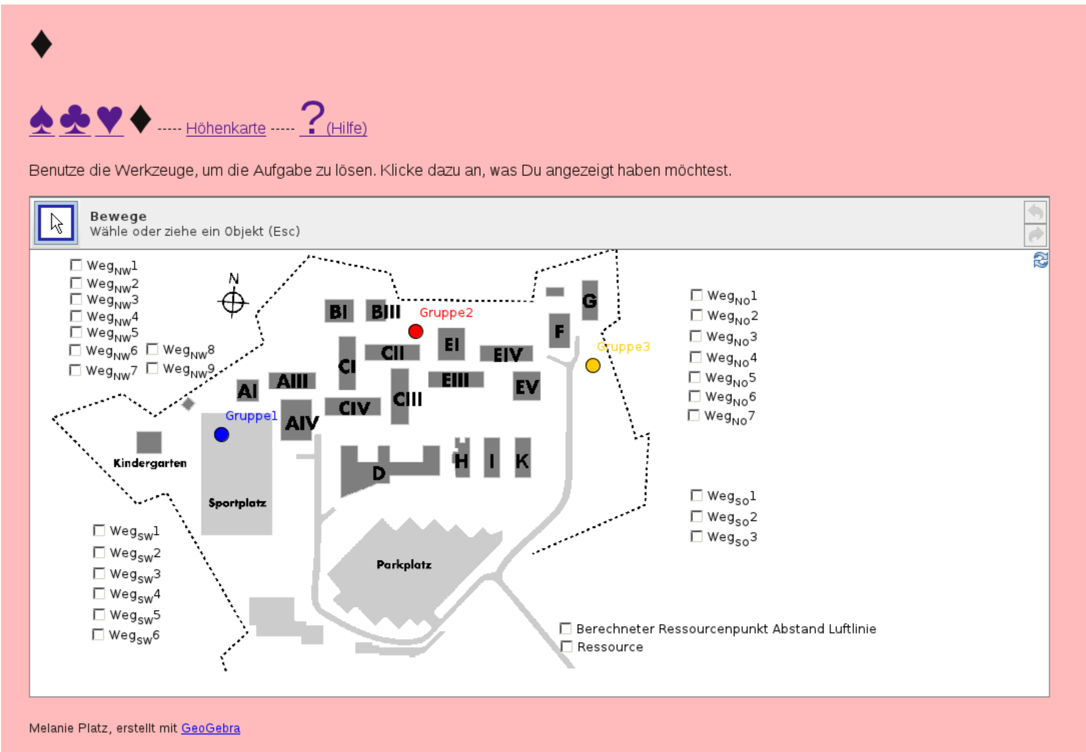


Figure 11.4: *Novice GUI* generated with *GeoGebra* and *JavaScript*.

- An *advanced GUI* (FIGURE 11.5) with the option of choosing or moving an object and measuring an object. In the background, the ground-plan of the *University Koblenz-Landau's Landau Campus* and routes for crossing the campus are illustrated. To distinguish the tracks from each other, they are displayed in different colours. Using a measurement-tool, distances between two objects can be measured, for example route lengths. Using a check-box, input fields for entering the measured time that was taken to cover a route can be displayed. The points *Gruppe 1* (blue), *Gruppe 2* (red) and *Gruppe 3* (yellow) represent the points of the three supply locations and these can be moved. Using a check-box, a point representing the resource can be made visible and moved. Using another check-box, a point representing the calculated location for the resource depending on the points *Gruppe 1*, *Gruppe 2* and *Gruppe 3* can be made visible using only the straight-line distance.

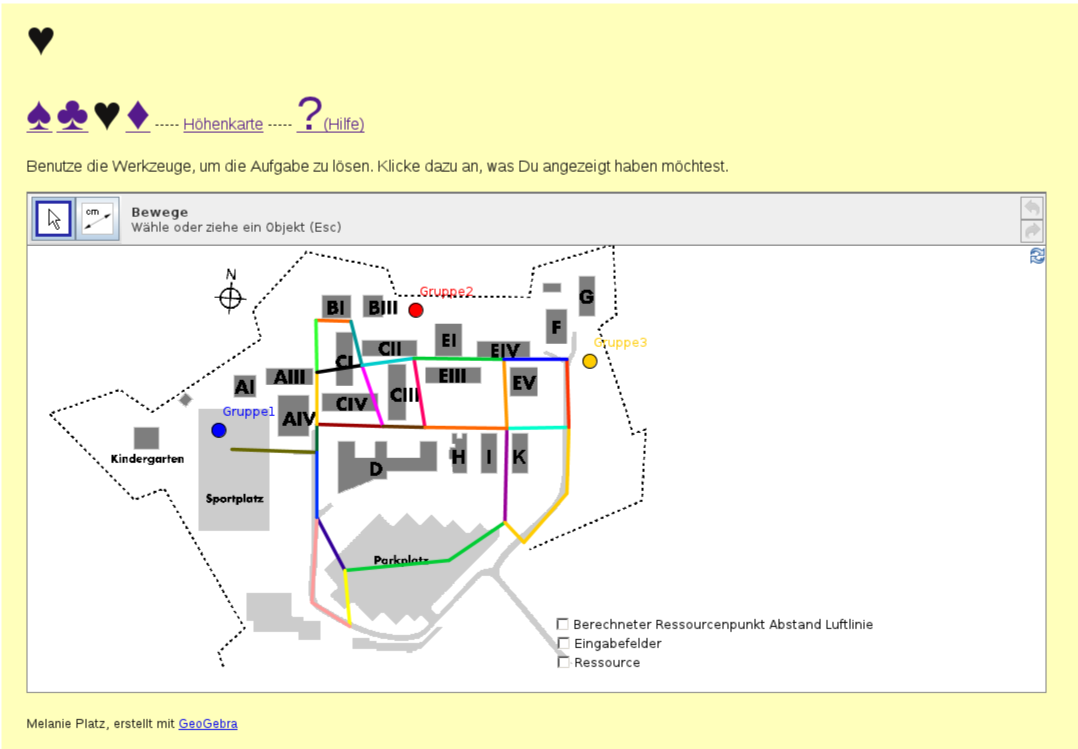


Figure 11.5: *Advanced GUI* generated with *GeoGebra* and *JavaScript*.

- A *more advanced GUI* (FIGURE 11.6) with the option of choosing or moving an object, inserting points, inserting routes, measuring an object and deleting objects. In the background, the ground-plan of the *University Koblenz-Landau's Landau Campus* is illustrated. Using a check-box, a circumcircle sector, that depends on three movable points A, B and C to determine the location for the resource depending on the points A, B and C, if only the straight-line distance is considered, can be displayed. Using another check-box, a monochrome route-plan of routes that can be taken to cross the *Landau Campus* can be illustrated. Yet another check-box displays input fields for entering the measured time that was taken to cover a route. Using another check-box, a point representing the resource can be made visible and then moved.

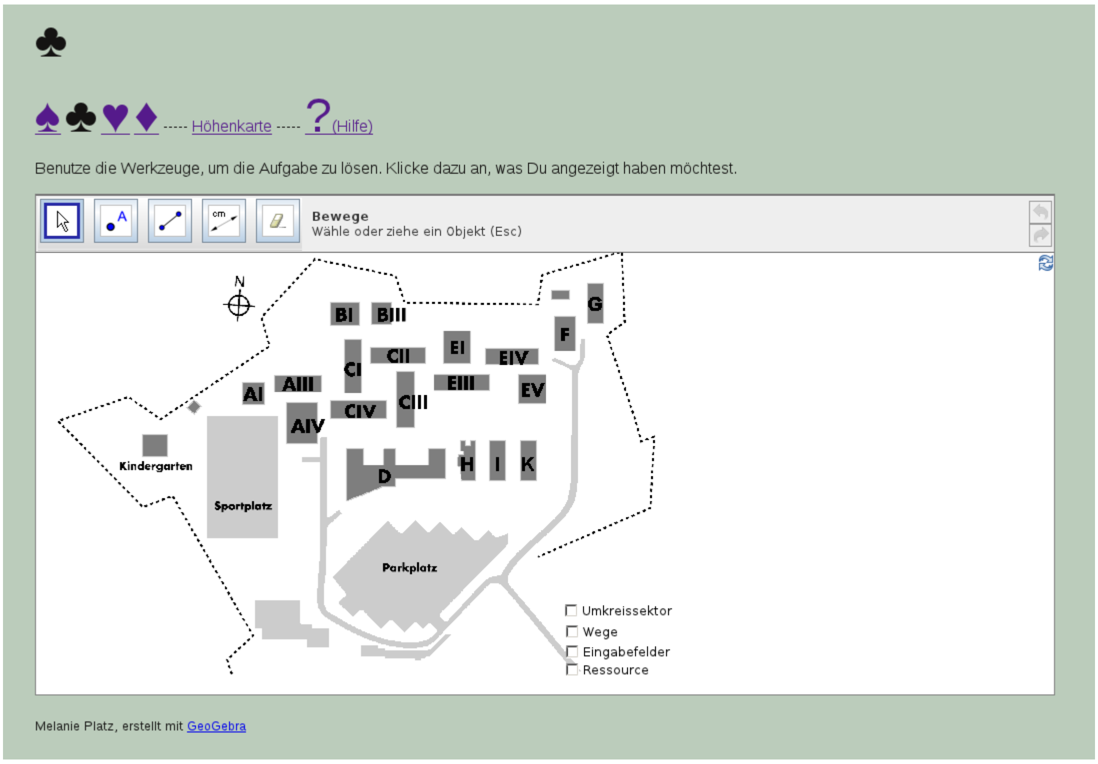


Figure 11.6: *More advanced GUI* generated with *GeoGebra* and *JavaScript*.

- An *expert GUI* (FIGURE 11.7) with the option of choosing or moving an object, inserting points, inserting a circumcircle sector, inserting text-boxes, drawing free lines, moving the drawing sheet, zooming in or out and deleting objects. The background shows the ground-plan of the *University Koblenz-Landau's Landau Campus*.

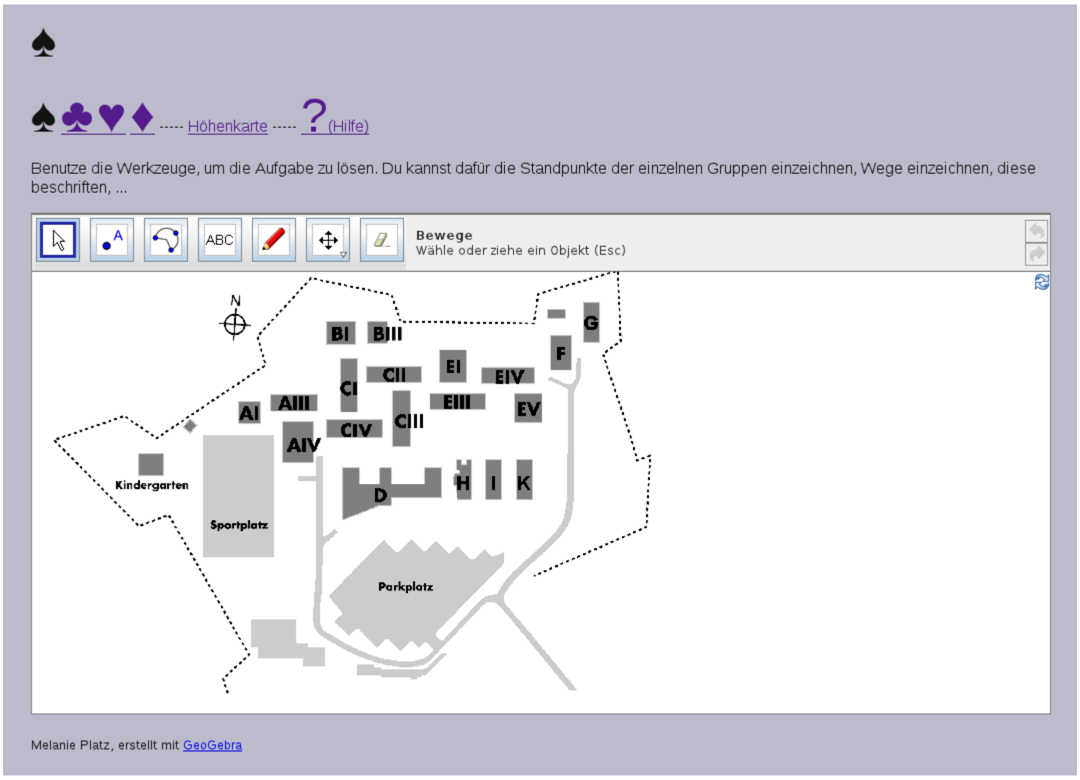


Figure 11.7: *Expert GUI* generated with *GeoGebra* and *JavaScript*.

When the GUI is changed, an alert-window (FIGURE 11.8) opens to remind the pupils to make a note of why they changed their GUI. Additionally, an altitude map (FIGURE 11.9) of the *Landau Campus* and a help-page (FIGURE 11.10) are available.

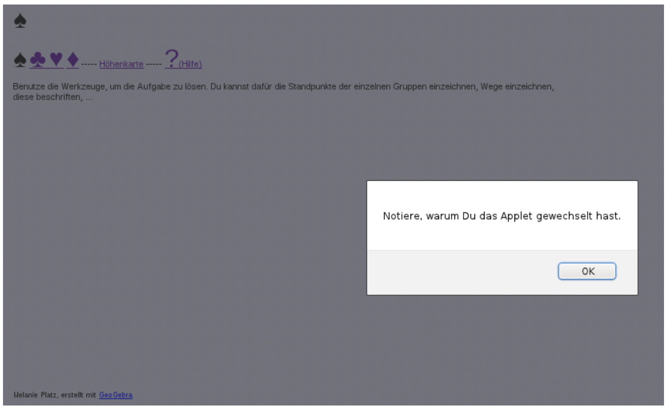


Figure 11.8: Alert-window generated with *JavaScript*.

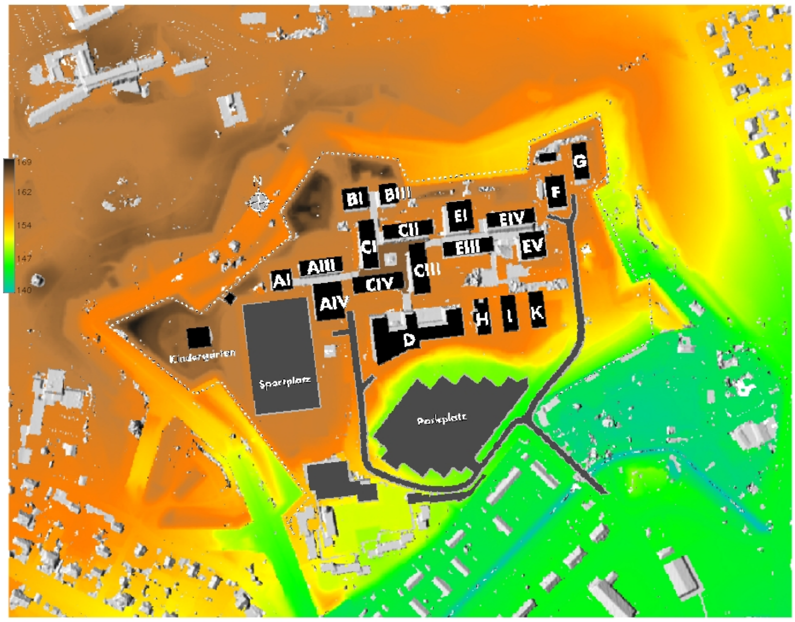


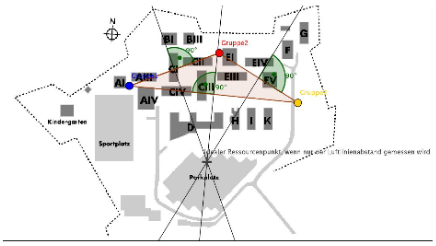
Figure 11.9: Altitude map of the *Campus Landau* provided by *Stefan Jergentz*, generated with *GRASS GIS*.

Hilfe

Durch Anklicken der Felder ♠, ♣, ♥, ♦ kannst Du das Applet wechseln. Begründe dabei immer auf Deinem Aufgabenbogen, warum Du das Applet gewechselt hast, z.B. aus Neugierde oder weil Dir das Applet nicht gefallen hat, ...

Durch Anklicken des Feldes **Höhenkarte** kannst Du Dir eine Höhenkarte des Campus Landau anzeigen lassen. Diese hilft Dir eventuell bei der Messung der Zeitdauer, die man benötigt, um von einem Standpunkt zum anderen zu kommen und somit bei der Bewertung der Gerechtigkeit.

Hinweis 1: Bestimmung des gerechtesten Ressourcenpunktes, wenn nur der Luftlinienabstand beachtet wird
Verbinde die drei Gruppenpunkte zu einem Dreieck. Nun zeichne die drei Mittelsenkrechten ein (im rechten Winkel (90°) zur Dreieckseite genau durch die Mitte der Dreickseite). Der Schnittpunkt der drei Mittelsenkrechten ist der Ideale Ressourcenpunkt, wenn nur der Luftlinienabstand gemessen wird.



Frage: Warum ist diese Methode nicht so gut geeignet, um den gerechtesten Ressourcenpunkt herauszufinden?

Hinweis 2: Diese Karte kann Dir bei der Lösung der Aufgabe eventuell weiterhelfen:

Figure 11.10: Help-page generated with *JavaScript*.

In conclusion, the following GUI elements are included:

- GUI elements with the same GUI element object in each GUI:
  - link to a help-page
  - link to an altitude map of the campus
  - icons for switching to another GUI
  - link to a ground-plan of the *University Koblenz-Landau's Landau Campus*
- GUI elements with different GUI element object in each GUI:
  - (tool for inserting a) representation of the points of the supply-locations
  - (tool for inserting a) representation of the routes that can be taken to cross the *Landau Campus*
  - (tool for inserting) input fields to enter the time taken to cover a route
  - (tool for inserting a) representation of the location of the resource
  - tool for determining the location for the resource depending on the supply locations when only the straight-line distance is considered

The pupils were given an initial GUI, but they could switch to another GUI, if they wanted. If they did so, they were encouraged to record why they preferred another GUI. Therefore the workbook of the *GUI groups* contained a questionnaire to record the approach the pupils used, the thoughts of the pupils while they were working, any (comprehension-) problems encountered with the task or the GUI used and for noting how often and why the pupils changed their GUI. Thus, it was tried to find out

- how injustice is measured by the pupils and how they detected injustice, in other words, which tool they used and
- when and why the pupils switched to/ prefered another GUI.

Short Planning: The groups were for the most part taught identically. Any differences are marked with an A for both the *adjusted GUI group* and the *GUI group* and with a B for the *control group*.

Time	Teaching Phase	Planned Teaching Course	Form of Activity, Social Form	Media / Materials

10 min	Welcoming	• Welcoming of the pupils.
		• Classification of the pupils, assignement in groups of two based on the Pretest:
		  • adjusted GUI-group (GUIs 1-4) (A)
		  • GUI-group (GUIs 1-4) (A)
		  • control group (B)
			Teacher-Centered-Teaching	

5 min	Access	• Presentation of the task: *Placement of a resource on the University Campus with three supply groups.*	Teacher-Centered-Teaching	Laptop, Beamer, Campus Map with starting point
5 min	Elaboration I	• Discussion in the groups of two or three to determine the initial placement point for the resource • Presentation of the proposals of some individual groups • Agreement on one point	Group work Pupil Presentation Class Discussion	Laptop, Beamer, Campus Map with starting point
20 min	Elaboration II	• Work with the GUIs (A)/ Workbooks (B)	Work in groups of two or three	A: PCs, GUIs, B: Workbooks
20 min	Elaboration III	• Evaluation of the selected point by means of field measurements (Tracking with LOCUS)	Group work in the field (Campus Landau of the University Koblenz-Landau)	Smartphones with LOCUS, Workbooks
20 min	Elaboration IV	• Work with the GUIs (A)/ Workbooks (B)	Work in groups of two or three	A: PCs, GUIs, B: Workbooks
10 min	Backup	• Presentation of the group results • Gathering of important results	Pupil presentation Class discussion	Laptop, Beamer, Campus Map with starting points

Objectives: Coarse Objective: The pupils get an idea of spatial justice.
Fine Objectives: The pupils learn something about tools to measure justice.
The pupils improve their spatial orientation.
The pupils learn that you can rate and evaluate different properties of a room (length of a path, time to pass a path)

Figure 11.11: Short planning of the second double lesson. (Figure generated with *Libre Office Writer.*)

Material:

- campus map with supply locations (see FIGURE 11.3)
- *(adjusted) GUI group*
  - workbook *GUI*
    The .zip-folder containing the workbook can be downloaded in German language at the link below:
    `http://mathematik.uni-landau.de/download/Platz/Geo-GUI.zip` (retrieved 16/02/2014).
  - computers with the GUIs
    The GUIs can be accessed at the links below:
    expert GUI:
    `http://mathematik.uni-landau.de/download/Platz/GUI/GUI1.html` (retrieved 16/02/2014)
    more advanced GUI:
    `http://mathematik.uni-landau.de/download/Platz/GUI/GUI2.html` (retrieved 16/02/2014)
    advanced GUI:
    `http://mathematik.uni-landau.de/download/Platz/GUI/GUI3.html` (retrieved 16/02/2014)
    novice GUI:
    `http://mathematik.uni-landau.de/download/Platz/GUI/GUI4.html` (retrieved 16/02/2014)
- *control group*
  - workbook *control group*
    The .zip-folder containing the workbook can be downloaded in German language at the link below:
    `http://mathematik.uni-landau.de/download/Platz/Geo-GUI.zip` (retrieved 16/02/2014).
- smartphones with the *LOCUS free* app

Implementation:

- Every pupil took part.
- The division of the pupils into the groups of two or three worked well, as they cooperated very well in these groups.
- Within the interaction group, the pupils' actions when interacting with the PCs where recorded with the sreencast-software *vokoscreen*.
  - *adGUIexpert*: *expert GUI*
    The group tried out each GUI but finally found a solution after the field work using their initial GUI design: the *expert GUI*. The help option was also used.

- *adGUImoreadvanced*: *more advanced GUI*

  The group did not change their initial GUI and found a solution on their own.

- *adGUIadvanced*: *advanced GUI*

  The group did not change their initial GUI and found a solution after trying each of the options of the *GeoGebra*-applet. They used the altitude map, but not the help option.

- *adGUInovice*: *novice GUI*

  The group switched between the *advanced GUI* and the *novice GUI* and found a solution with the *advanced GUI* after the field work. The altitude map and the help option was used.

- *GUIexpert*: *expert GUI*

  The group tried out each GUI and the help option.

- *GUImoreadvanced*: *more advanced GUI*

  The group did not change their initial GUI, a software bug forced the group to stop before the field work.

- *GUIadvanced*: *advanced GUI*

  The group switched to the *more advanced GUI.*

Results: The cluster analysis, described in more detail on p.212, worked well for the assignment of the pupils into the groups of two or three, in other words, the groups of two or three worked well together. The *adjusted GUI group* was able to solve the task using the GUI the pupils had been assigned. They only changed their GUI, because they were curious. The *GUI group* was not successful in solving the task and changed their GUI several times, because they were unable to interact with the assigned GUI. This shows that the assignment of an initial GUI based on information about the user and the inclusion of measure theory for the evaluation of the information about the user is helpful when it comes to the user. During the teaching unit, the pupils displayed curiousity and were eager to explore the use of the GUIs. This aspect is important for users, who are able to spend time when working with a GUI, when they are not in an emergency situation for example. In this case, the best way for satisfying the user's needs is not necessarily the fastest way. However, in an emergency case, the time taken to deliver a solution plays an important role.

11.2.3 Posttest

(45 min) (27/09/2012, 10:30am-11:15am)

The Posttest is performed within the Posttest *Geo-Math*, which is similar to the Pretest, that is, the Posttest is constructed structurally equivalent with comparable tasks, in same content, is solvable in the same way with the same effort and degree of difficulty. The pupils may ask questions while taking the Posttest, but, to prevent intervention, the task is described in detail on the Posttest and the pupils can be supported by repeating the task (in the supervising person's own words).

Material: Questionnaire: The .zip-folder containing the Posttest can be downloaded in German language at the link below:

`http://mathematik.uni-landau.de/download/Platz/Geo-GUI.zip` (retrieved 16/02/2014).

Implementation: One pupil, belonging to the *control group*, did not participate. This was not the same pupil as the one who did not participate in the Pretest.

Results: In FIGURE 11.12 the results of the self-assessment/ personal opinion questions from Pretest and Posttest are presented.

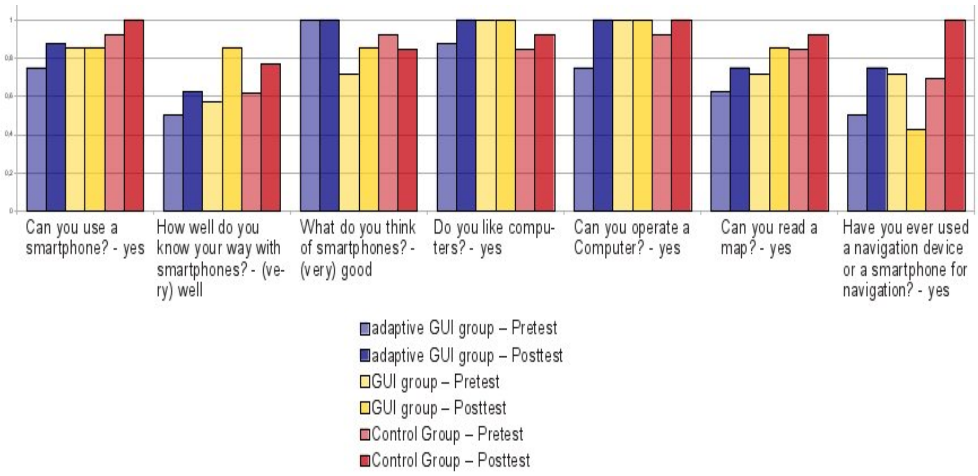


Figure 11.12: Visualisation of the results from Pretest and Posttest: self-assessment/ personal opinion questions. (Figure generated with *Libre Office Calc.*)

11.2.4 Follow-up-test

(45 min) (18/12/2012, 10:30am-11:15am)

Using a Follow-up-test, the sustainability of the knowledge imparted in the second double lesson (SUBSUBSECTION 11.2.2) is evaluated. The Follow-up-test is performed within the Follow-up-test *Geo-Math*, which is created similar to Pretest and Posttest. The pupils may ask questions while taking the Follow-up-test, but, in order to prevent intervention, the task is described in detail on the Follow-up-test form and the pupils can be supported by repeating the task (in the supervising person's own words).

Material: Questionnaire: The .zip-folder containing the Follow-up-test can be downloaded in German language at the link below:

`http://mathematik.uni-landau.de/download/Platz/Geo-GUI.zip` (retrieved 16/02/2014).

Implementation: One pupil, belonging to the intervention group, did not participate. This was not the same pupil as one of the pupils who did not participate in the Pretest and the Posttest.

Results: In FIGURE 11.13 it is apparent, that the performance of the *adjusted GUI group* improved the most in the field of the review of justice, also compared to the *GUI group* and the *control group*. The performance of the *adaptive GUI group* deteriorated noticeably in the field of reasoning. Moreover, in the Follow-up-test the performance of all groups tended to be worse, except in the field

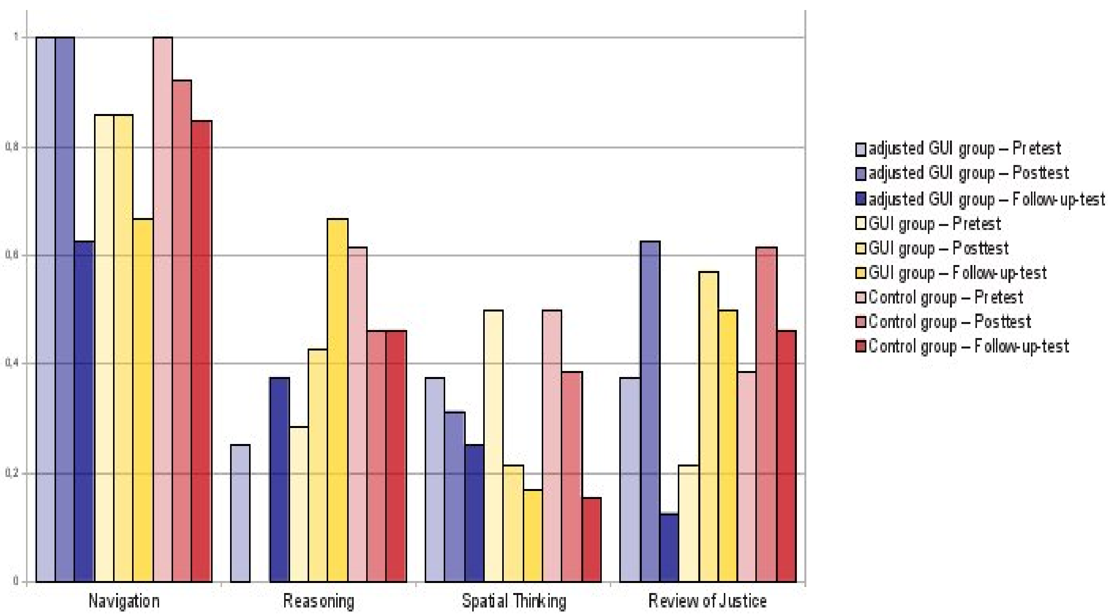


Figure 11.13: Visualisation of the results from Pretest, Posttest and Follow-up-test: questions about navigation, reasoning, spatial thinking, review of justice. (Figure generated with *Libre Office Calc.*)

of reasoning, where the adjusted *GUI group* and the *GUI group* show an improvement. In the field of the review of justice in particular, which was subject of the lesson, an improvement in performance in the Posttest compared to the Pretest and a deterioration in the results of the Follow-up-test compared to the Posttest are visible. This is not a bad result, however, because the GUIs for an application delivering early warning and decision support do not have a tutorial component; in other words, the user is not supposed to learn how to carry out certain actions on her/his own. The GUI is merely intended to give advice and to help the user in hazardous situations by giving the best possible support.

11.3 Empirical Evaluation

11.3.1 Methods

The evaluation was done using multivariate data analysis, that is, ANOVA:

Multivariate Data Analysis. For any empirical research project, the task of data analysis follows the data collection. The results of the data collection can be displayed summarised in the form of a matrix, which is referred to as a data matrix. In the data matrix, the variables are represented in the columns and the statistical units are represented in the rows.

Definition 11.3.1 *Data Matrix (cf [Sti96], p.238)*

$$M_{Data} := \begin{pmatrix} x_{1,1} & ... & x_{1,j} & ... & x_{1,m} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ x_{i,1} & ... & x_{i,j} & ... & x_{i,m} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ x_{n,1} & ... & x_{n,j} & ... & x_{n,m} \end{pmatrix}$$

where $x_{i,j} \in \mathbb{R}$ *denotes the characteristic value of the statistical unit* i *with* $i \in \{1, ..., n\}$ *with the variable* j *with* $j \in \{1, ..., m\}$.

The i-th row of this matrix thus contains the data of the i-th statistical unit for all features or variables. Correspondingly, the j-th column contains the values of the variable x_j for each statistical unit. (cf [Sti96], p.237 f).
A one-factorial ANOVA is carried out to determine, what influence the GUI method and the adjusted GUI method have as teaching methods compared to a teaching method without GUIs and to assess whether it makes sense to determine an initial GUI design for the user based on information about the user. The pupils were combined in groups of two or three and within those groups the pupils were

found to be similar in performance and in their answer structure.

One-Factorial ANOVA. The one-factorial ANOVA is a generalisation of the general testing problem: Two main units are given, which are $N(\mu_1;\sigma^2)$- or $N(\mu_2;\sigma^2)$-distributed with respect to a variable of interest. The hypothesis $H_0 : \mu_1 = \mu_2$ is known to be checked against the alternative hypothesis $H_1 : \mu_1 \neq \mu_2$ by using the t-test. This testing problem can be generalised to more than one main unit. These main units can be interpreted as a factor which is present in two or more stages. For each level there is a sample of observed values. We assume I factor levels with J_i observations per factor level and

$$y_{i,j} \sim N(\mu_i;\sigma^2),\ i = 1,2,...,I,\ j = 1,2,...,J_i,$$

where the $y_{i,j}$ are stochastically independent. $J = \sum_{i=1}^{I} J_i$. Or:

$$y_{i,j} = \mu_i + \varepsilon_{i,j},\ i = 1,2,...,I,\ j = 1,2,...,J_i,\ \varepsilon_{i,j} \sim N(0;\sigma^2),$$

where the $\varepsilon_{i,j}$ are stochastically independent. With the vectors:

$$Y := \begin{pmatrix} y_{1,1} \\ y_{1,2} \\ \vdots \\ y_{1,I} \\ \vdots \\ y_{I,1} \\ y_{I,2} \\ \vdots \\ y_{I,J} \end{pmatrix},\ M := \begin{pmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_I \end{pmatrix},\ E := \begin{pmatrix} \varepsilon_{1,1} \\ \varepsilon_{1,2} \\ \vdots \\ \varepsilon_{1,I} \\ \vdots \\ \varepsilon_{I,1} \\ \varepsilon_{I,2} \\ \vdots \\ \varepsilon_{I,J_i} \end{pmatrix}$$

and the matrix:

$$M_{Design} := \begin{pmatrix} 1 & 0 & ... & 0 \\ 1 & 0 & ... & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 1 & 0 & ... & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & ... & 1 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & ... & 1 \\ 0 & 0 & ... & 1 \end{pmatrix},$$

the columns are linearly independent, results:

$$Y = M_{Design} \cdot E,\ E \sim N(0;\sigma^2 I).$$

This allows the representation of the one-factorial ANOVA model in the form of a regression model. The most common hypothesis test for a one-factor ANOVA is:

$$H_0 : \mu_1 = \mu_2 = ... = \mu_I$$

$$H_1 : \mu_i \neq \mu_j \text{ for at least one pair } i, j.$$

Traditionally, for this test, the following sums of squares are formed:

- Sum of squares between the groups:

$$SS_B := \sum_{i=1}^{I} J_i(\bar{y}_{i,+} - \bar{y}_{+,+})^2,\ \bar{y}_{i,+} = \frac{1}{J_i}\sum_{j=1}^{J_i} y_{i,j},\ \bar{y}_{+,+} = \frac{1}{IJ}\sum_{i=1}^{I}\sum_{j=1}^{J} y_{i,j}$$

- Sum of squares within the groups:

$$SS_W := \sum_{i=1}^{I}(y_{i,j} - \bar{y}_{i,+})^2.$$

The quotient

$$F := \frac{\frac{SS_B}{I-1}}{\frac{SS_W}{J-I}}$$

is $F(I-1; J-I)$-distributed, if H_0 is valid. Large values of SS_B favour the rejection of H_0, small values rather tend to favour the retention of H_0. H_0 can be rejected, if $F > F(I-1; J-I; 1-\alpha)$. (cf [Sti96], p.254 ff).

11.3.2 Empirical Results

In this thesis four variance analyses could be carried out:

1. H_0: The lessons without GUI, but with workbook as static adapted form of representation, the lessons with GUI as interactive adapted form of representation and the lessons with adjusted GUI as interactive adapted form of representation result in the same learning success.

   H_1: The lessons without GUI, but with workbook as static adapted form of representation, the lessons with GUI as interactive adapted form of representation and the lessons with adjusted GUI as interactive adapted form of representation do not result in the same learning success.

2. H_0: The lessons without GUI, but with workbook as static adapted form of representation and the lessons with GUI as interactive adapted form of representation result in the same learning success.

   H_1: The method without GUI, but with workbook as static adapted form of representation and the lessons with GUI as interactive adapted form of representation do not result in the same learning success.

3. H_0: The lessons without GUI, but with workbook as static adapted form of representation and the lessons with adjusted GUI as interactive adapted form of representation result in the same learning success.
   H_1: The method without GUI, but with workbook as static adapted form of representation and the lessons with adjusted GUI as interactive adapted form of representation do not result in the same learning success.

4. H_0: The lessons with GUI as interactive adapted form of representation and the lessons with adjusted GUI as interactive adapted form of representation result in the same learning success.
   H_1: The lessons with GUI as interactive adapted form of representation and the lessons with adjusted GUI as interactive adapted form of representation do not result in the same learning success.

We have three groups: The *control group*, the *GUI group* and the *adjusted GUI group*. We investigate the difference between the results of the Posttest and the results of the Pretest.

- Within the *adjusted GUI group*, we have eight pupils with the following results:

$$x_1 = (1, 2, 0, 3, 2, 0, 4, 0).$$

- Within the *GUI group*, we have seven pupils with the following results:

$$x_2 = (2, 1, 0, 2, 1, 1, 2).$$

- Within the *control group*, we have 13 pupils with the following results:

$$x_3 = (0, 2, 0, -1, 2, 2, 0, -1, 2, 1, 0, -1, -1).$$

The result of the ANOVA is[2] $F = 2.42$ and $F(2; 25; 0.9) = 2.53$.
Thus, H_0 can be accepted, because $F < F(2; 25; 0.9)$, that is, the three teaching methods lead to equal learning success. Above, the level of significance is 0.1. If the level of significance was a larger value, the result would be less reliable. This reliability can be determined by measuring a fuzzy mapping representing the degree of the reliability of a result based on the level of significance. Even if we have a result with lower reliability, that is, larger level of significance, it is still better than having no result. The degree of satisfaction of the significance is given by a fuzzy membership mapping, whereby "0" denotes the fuzzy value "not significant" and "1" denotes the fuzzy value "results are significant with a level of significance α". With respect to the GUI assignment to a user, a necessary decision based on a lower fuzzy value is still better than having decision making via a random experiment on the options of necessary decisions. In our case, the degree of satisfyability of the significance is $\frac{2.42}{2.53} \approx 0.96$.
The other three variance analyses (2., 3. and 4.) delivered worse results, than the variance analysis carried out above.

[2] rounded to two decimal places

11.4 Conclusion

In order to investigate complex dynamical systems and using adaptive GUIs for decision support and to adapt a GUI to South Africa conditions it is important to determine an initial state for the GUI. Using the empirical pilot study described in this chapter, a generic basal concept for the structure of further empirical studies to be performed in South Africa was developed. With this first empirical pilot study it was shown how an initial GUI design can be chosen and what effect this has on the learning success. The result of the ANOVA is, that the teaching methods lead to equal learning success. When comparing the results of the Posttest to the Pretest, the *GUI group* showed the largest improvement. If the teaching part could have been longer, maybe a larger difference might have been measured. This could be done in another empirical study. The opinions about digital devices improved and, according to their self-assessment, the pupils became more used to digital devices (see FIGURE 11.12). Moreover, the cluster analysis worked well for the assignment of the pupils into the groups of two or three. During the GUI teaching unit, the pupils were curious and exploratory in their approach to the GUIs. This aspect is important for users, who are able to spend a lot of time when working with a GUI, when they are not in an emergency situation for example. In this case, the best way for satisfying the user's needs is not necessarily the fastest. However, in an emergency case, the time for delivering a solution plays an important role. The results of the Follow-up-test show, that the lessons were not sustainable, that is, the performance of all groups tended to deteriorate. This is not a bad result, because the GUIs for an application delivering early warning and decision support do not have a tutorial component. This is because the user is not supposed to learn how to carry out certain independent actions more self-dependent. Rather, the GUI is supposed to give advice and help the user in hazardous situations by giving the best possible support. A link to fuzzy logic was emphasised by considering the grade of significance of the empirical results. The degree of satisfaction of the significance is given by a fuzzy membership mapping, whereby "0" denotes the fuzzy value "not significant" and "1" denotes the fuzzy value "results are significant with a level of significance α". With respect to the GUI assignment to a user, a necessary decision based on a lower fuzzy value is still better than having decision making via a random experiment on the options of necessary decisions. The empirical study could be improved by increasing the duration of the teaching unit and the number of testees in the test group. Additionally, further empirical studies should be carried out in the pilot region South Africa.

In the following part, PART V, a conclusion completes the thesis with a summary, a discussion of the results, prospects and closing remarks.

V | Conclusion

The current health delivery situation in South Africa was investigated with the aim of optimising health service delivery. This implied the requirement for the development of an OS-application for digital devices in order to be able to maximise access to health service delivery. In contributing to the optimisation of health service delivery, epidemiological distances are, inter alia, identified as being an issue. However, these distances in the public health domain do not fulfil the symmetry property of the metric, thus the spaces being considered may not be Euclidean spaces. For this reason, topological spaces had to be considered. The target group that was identified for the use of the application is heterogeneous. Consequently, the global determination of a Graphical User Interface (GUI) and its contents were unable to satisfy all the requirements and conditions. Subsequently, in order to achieve adaptivity to the individual conditions of users, Artificial Neural Network (ANN)s were used. The stability of the system can be enabled with ANNs, because of the threshold property. In addition, in order to represent users' statements, which tend to be fuzzy, fuzzy logic was included. Consequently, fuzzy logic operations can be used to control the GUI. Further, measure theory was used for error determination: accordingly, the fuzzy mappings could be measured, as well as the learning processes relating to the ANNs. As a result of the requirement to include measure theory on topological spaces to address our concerns, a generic mathematical structure for a system adapting a GUI to a user was developed. In order to implement the developed system, an Object-Oriented Analysis (OOA) was performed and, thereby a first method for bridging the fields measure theory on topological spaces and Object Orientation (OO) was developed. The scope of this thesis was, inter alia, to conduct an OOA as a precondition for the realisation of Object-Oriented Programming (OOP).

In order to adapt the GUI to South Africa conditions it is important to determine an initial state for the GUI. Using an empirical pilot study carried out with fifth grade pupils from a German school on how to determine an initial GUI and the effect this had on learning success, a generic basal concept for the structure of further empirical studies to be performed in South Africa was developed. Additionally, a link to fuzzy logic was emphasised by considering the grade of significance of the empirical results. The degree of satisfaction of the significance is given by a fuzzy membership mapping, whereby "0" denotes the fuzzy value "not significant" and "1" denotes the fuzzy value "results are significant with a level of significance α". With respect to the GUI assignment to a user, a necessary decision based on a lower fuzzy value is still better than having decision making via a random experiment on the options of necessary decisions.

12 Summary

The development of an OS-application for a digital device that delivers early warning and decision support tailored to different user groups using a GUI adapted to the user is an important milestone for the *ReGLaN-Health and Logistics* project. Consequently, we dealt with the mathematical modelling of a GIST GUI design by applying spatial fuzzy logic and the mathematical visualisation of risk and resource maps for epidemiological issues. This can be done using GIS and adaptive GUI design for an OS application for digital devices in order to supply spatial decision support tailored to different user groups. The aim of this thesis was the development of the mathematical structure for a system for an adaptive GUI tailored to different user groups and for an application which is intended to serve as early warning and decision support system in hazardous situations. For the evaluation and initialisation of the GUI elements, empirical teaching, learning and research dealing with geo-media and GUI elements were undertaken (see Chapter 11).

In this thesis one milestone was the following: The development of an algorithm for an SAGU and, furthermore, the implementation of measure theory on topological spaces in OOA and, thus, in computer science methodologies.

The thesis was structured in the following way:

The thesis started with an introduction and a statement of the objectives in Part I, where the aim of this thesis was also identified as follows: the development of the mathematical structure behind a system for an adaptive GUI tailored to different user groups, in order to provide an OS-application that serves as early warning and decision support system in hazardous situations.

In Part II, the requirements and constraints for the development of this system were analysed. Accordingly, Chapter 1 explains the *ReGLaN-Health and Logistics* project, which is aimed at the optimisation of health care in rural areas of South Africa, and from which this thesis developed. In Chapter 2 of the study we concentrated on South Africa, which is the pilot region for this study, with the focus falling on the South African health system and diseases occurring in South Africa, as well as the current ICT situation in South Africa. In Chapter 3, OS and OC were addressed, and an interim conclusion concluded this part by focusing on risk perception in Chapter 4. In addition, both the socio-economic and the ICT situation in South Africa were investigated in order to identify suitable digital devices and software components for reaching as many potential users as possible.

In Part III, the concepts that were merged for developing an SAGU were described. These included topics such as the GUI for an application delivering early warning and decision support, which was discussed in Chapter 5, OO for creating a prototype of the GUI for an application delivering early warning and decision support, which was discussed in Chapter 6, fuzzy logic and ANNs for implementing adaptivity, which were discussed in Chapter 7 and measure theory on topological spaces for evaluating fuzzy membership mappings on non-Euclidean spaces, which was discussed in Chapter 8. The GUI for an application delivering early warning and decision support has to be adaptive, because we cannot predict every requirement of the GUI. Therefore, we are concerned with adaptivity and the methods for achieving adaptivity in a GUI. Fuzzy logic and ANNs play a key role in adaptivity, as it is dependent on information about the user, for example the geo-location of the user and the actual point in time. Measure theory on topological spaces had to be included, because the distances used in this thesis cannot always be understood in the Euclidean sense, for example epidemiological distances.

In Part IV, the concepts addressed in Part III were applied in order to develop a mathematical structure for an SAGU. Initially, this mathematical structure was presented in Chapter 9. Subsequently, an object-oriented overview of the system was given and maps for visualisation and exemplary GUIs tailored to different user groups were addressed in Chapter 10. The SAGU was implemented in the OOA; therefore its implementation as software is the next step and this can be done in the future by computer scientists. Finally, we conducted a method for an evaluation of certain aspects and properties of the GUIs in Chapter 11. This evaluation was done by means of an empirical pilot study which was carried out with fifth grade pupils from a school in Germany. Subsequently, empirical studies should be carried out in the pilot region of South Africa to obtain more reliable results. The structure of the empirical study carried out in this thesis could be adopted, because it investigated structurally equivalent problems.

In this part, Part V, a conclusion completes the thesis with a summary, the results, challenges, prospects and closing remarks.

13 Results

In Figures 13.1 and 13.2, the links between this dissertation and other fields are presented. The

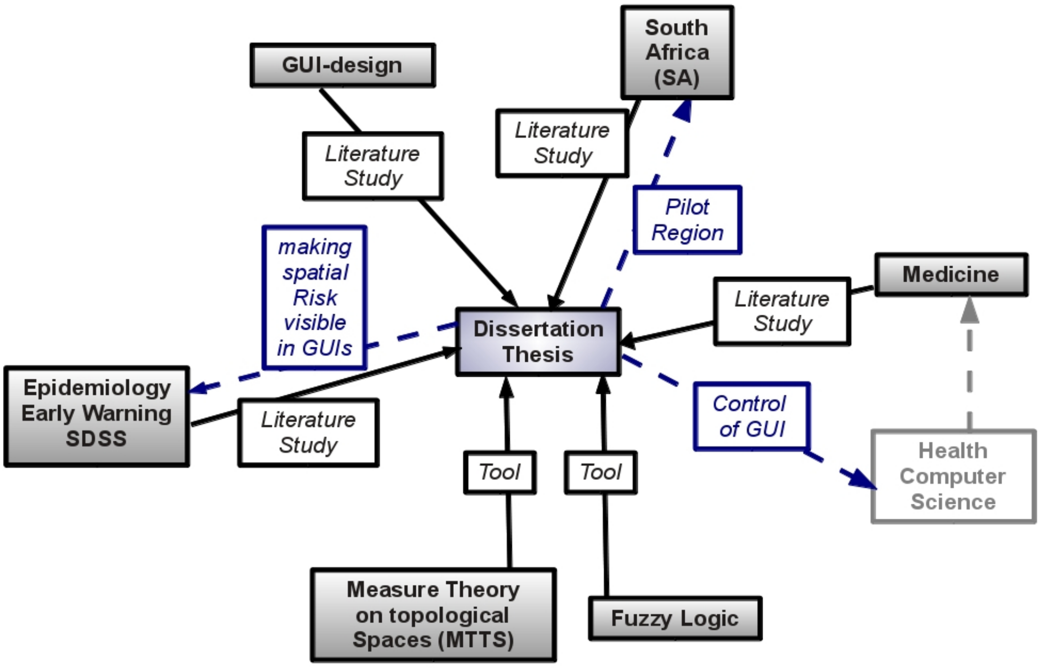


Figure 13.1: Connection between this dissertation thesis and other fields. The black arrows and the black-framed text-boxes with white background show the contents taken from the different fields for the thesis and the blue arrows and the blue-framed text-boxes show, what this thesis delivers back to the different fields. Consistent lines represent in this thesis actually performed contents. Dashed lines represent concepts. Grey-framed text-boxes and arrows are fields and tasks, that where not considered or performed in this thesis. (Figure generated with *Libre Office Draw.*)

GUI for an application delivering early warning and decision support has to be adaptive, because we cannot predict every requirement of every user to the GUI. Consequently, we are concerned with adaptivity and methods to achieve adaptivity in a GUI. Fuzzy logic and ANNs play a key role in adaptivity. Adaptivity is dependent on information about the user, for example the geo-location of the user (GIS tailored) and the actual point in time. In some parts of the SAGU described in Chapter 9, properties of a metric (Definition 8.1.1) are not fulfilled. Therefore we had to consider topological spaces. Thus measure theory on topological spaces had to be included, because the fuzzy mappings operate on these general topological spaces. Moreover, infinite dimensional topological function spaces had to be used, because GUI control works with fuzzy membership mappings, that is, (adaptive) fuzzy logic operations (Chapter 7) determine the GUI structure. Thus, the mathematical fields measure theory on topological spaces (Chapter 8) and fuzzy logic and ANNs (Chapter 7) were used as the basal tools for the development of the mathematical structure behind an SAGU (Chapter 9). A mathematical link between spatial risk and spatial resource availability was used for the adaption processes for GUIs on digital devices. Decisions using non-significant data are supported by the adaption process with dynamic limit functions.

Because of the generic structure of the algorithm, it can be transferred to other spatial decision support that does not operate on Euclidean spaces.

It should be noted that the development of a mathematical structure for the adaptivity of a GUI (Chapter 9) goes beyond the current research. Moreover, measure theory on topological spaces was observed only as it applies to mathematics. In this thesis, measure theory on topological spaces was applied to extra-mathematical topics. Furthermore, an OOA was carried out that included measure theory on topological spaces, which has not been done before. This research developed a preliminary method for bridging the fields of OO and measure theory on topological spaces.

The *ReGLaN-Health and Logistics* project is situated in South Africa, which was the pilot region for the project. The concepts described in this thesis have until now not been tested or implemented in South Africa. However, a literature study was conducted in the field of medicine with respect to both the health situation in South Africa and the diseases occurring in South Africa (Section 2.1). For the development of the GUI, this dissertation considered user groups comprising medical staff, as well as actions to be taken in the case of medical emergencies. The GUI is intended to work in the sense of mHealth, with the aim of improving the health system. The context of this thesis was South Africa, which was the pilot region for the project (Chapter 2). Therefore, a literature study on the country, South Africa, had to be carried out and the GUI prototype has to be adjusted to South African needs, for example the health situation in South Africa (Section 2.1), as well as the ICT (Section 2.2) and economic situation (Chapter 3) and special conditions relating to culture in South Africa (Chapter 5). Both the socio-economic and the ICT situation in South Africa were investigated by means of a literature study to detect suitable digital devices and software components that would reach as many potential users as possible. Cellphones (e.g. smartphones), PCs and digital doorways have, accordingly, been proven to be suitable (Section 2.2), because these devices are able to ascertain the geo-location of the user and are widely used in South Africa. Consequently, the GUI design has to be adjusted to those digital devices. To enable free distribution of the developed software, both OS software and OC were investigated and we identified which OS software and OC could be used directly or modified for our concerns and how the software could be implemented (Chapter 3). The application can therefore be provided free of charge and can be easily edited and modified by local computer scientists in South Africa according to local needs. OS software was used exclusively in this thesis, for example in the development of the UML diagrams. The exclusive use of OS software

was a basic condition for the OOA that was carried out in this thesis. A literature study on South African culture was also done because the visualisation of risk plays an important role in the GUI. For example, the interpretation of both colours and symbols varies for different cultures.
However, it is difficult to ensure the wide use of the application and thus improve the health service delivery for many people in South Africa, because the cultural and social behaviour of people is neither generalisable nor predictable. Nevertheless, by implementing adaptive GUIs in an OS approach and analysing the socio-economic and ICT background, it may be possible to reach a large target group.
A literature study on "good" GUI design was done (Chapter 5) and the findings were applied in Part IV. Basic GUI design guidelines were derived from the literature study and, by considering cultural effects on GUI comprehensibility, design characteristics for users in South Africa, the pilot region, could be derived. Thus, a mathematical structure for a GUI adapted to the individual user was developed, because an average GUI may not necessarily be the best for them. However, an average GUI is used as an initial GUI if no information is available about the individual user, except the geo-location. The initial GUI is not a global GUI, because it is GIST. We refer to the information about users in similar situations, that is, users located at the same geo-location to assign a GUI to the user that worked out fine for users in similar situations. Another literature study for the topics of epidemiology, early warning and SDSS was carried out (Chapter 1). Following this, the concept of making risk visible using risk maps and appropriate GUIs was developed (Chapter 9 and Chapter 10).

In Chapter 6, the tools OOA and UML were taken from computer science in order to make the

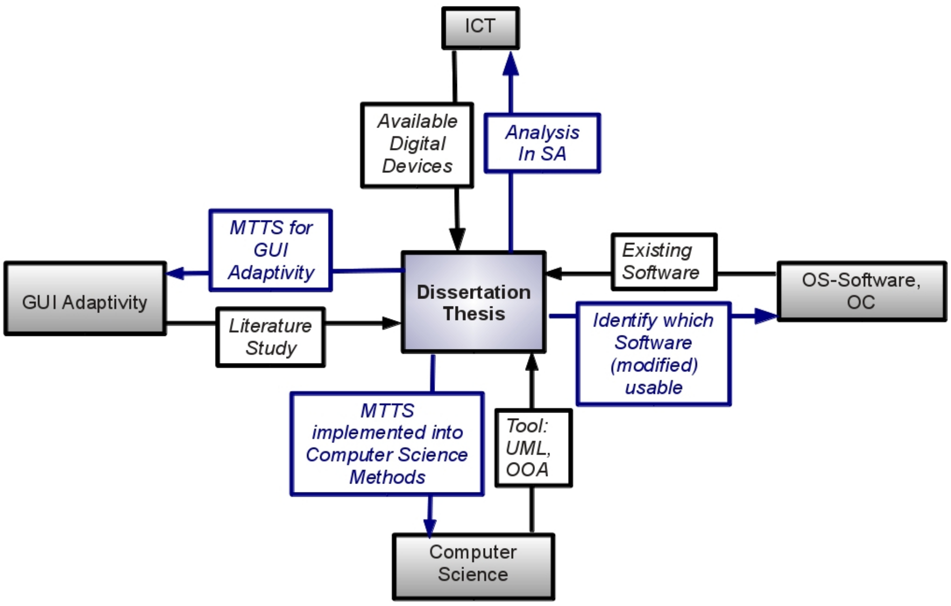


Figure 13.2: Connection between this dissertation thesis and other fields. The black arrows and the black-framed text-boxes with white background show the contents taken from the different fields for the thesis and the blue arrows and the blue-framed text-boxes show, what this thesis delivers back to the different fields. Consistent lines represent in this thesis actually performed contents. Dashed lines represent concepts. (Figure generated with *Libre Office Draw*.)

developed mathematical structure implementable (Chapter 10). As the implementation in computer science will follow the results of this thesis, the implementation in UML within the OOA (Chapter 6) could be concluded from an interface analysis with computer science and implemented within the thesis (Chapter 10). Therefore, measure theory on topological spaces was implemented in the OOA and thus into computer science methodologies. For this purpose, systems of open sets and their convergence had to be made implementable into the OOA.

A literature study on the topic GUI adaptivity was carried out and a mathematical structure for a system that adapts to the user, including measure theory on topological spaces (Chapter 8), was developed (Chapter 9).

In order to investigate complex dynamic systems and the use of adaptive GUIs for decision support and to adapt the GUI to South Africa conditions, an initial state for the GUI was required. Accordingly, a preliminary empirical pilot study was carried out with fifth grade pupils in a school in Germany to show how an initial GUI design could be chosen and the effect this has on learning success (Chapter 11). Although the empirical pilot study was not carried out in South Africa, the method can be transferred to empirical studies in South Africa because a generic framework was tested in the empirical study carried out in this thesis that could be used in empirical studies in South Africa. The empirical pilot study carried out in this thesis tested various spatial aspects. Further empirical studies could consider problems with similar structures.

The results of this thesis can be illustrated by means of an innovation life cycle (Figure 13.3). Thus the concept was created, an OOA has been done with the inclusion of measure theory on topological spaces, which will lead to the development of a prototype in the future. However, the deployment has not yet taken place and only an initial idea for the evaluation was described with an empirical pilot study, where it was shown how an preliminary GUI design could be chosen. Up to now, no prototype has been developed and no implementation has been done in South Africa, although this should happen in the future. Nevertheless, basic modules to enable further progress have been developed in this thesis. A next step could entail the automating of processes for implementation into computer science and, thus subsequently, the development of a prototype.

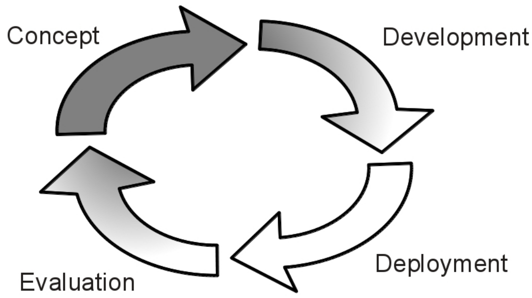


Figure 13.3: The results of this thesis illustrated via an innovation life cycle. Grey filled means achieved step in this thesis, colour gradient filled means partly achieved step in this thesis and white filled means not achieved step in this thesis. (Figure generated with *Libre Office*. Compare [CHC12], p.19.)

14 Challenges

The following limitations have to be acknowledged in the use of ANNs: If we want to minimise an error, it cannot be guaranteed that the ANN will find the global minimum of the error mapping. Additionally, the learning speed of an ANN may be too slow if, for example, a user's needs change too quickly and the ANN cannot sustain the adaption process. If this happens, the GUI changes to manual mode, whereby the user can adjust the GUI to suit her/his needs herself/himself.

The tools OOA and UML (Chapter 6) were taken from computer science in order to make the mathematical structure implementable (Chapter 10). Further implementation in computer science could not be done in this thesis and the OOP has to be done in the future by computer scientists. Therefore, a complete development cycle could not be performed.

The application is intended to reach a potentially large target group. However, it is difficult to assure wide use of the application and thus improve health service delivery for many people in South Africa, because the cultural and social behaviour of people is neither generalisable nor predictable. Nevertheless, through the implementation of adaptive GUIs in an OS approach and the analysis of the socio-economic and ICT background, a potentially large target group may be reached.

For financial reasons, the empirical pilot study in this thesis was not carried out in South Africa, but the method used in the study of this thesis could be transferred to empirical studies in South Africa. The empirical pilot study carried out in Germany tested spatial issues. Therefore, in further empirical studies problems with similar structures will be considered. The limitations experienced with respect to the empirical study carried out in this thesis include the small target group (one school class consisting of 30 pupils) and the fact that the ANOVA led to a non-significant result. Nevertheless, the concept is generic for the determination of an initial GUI for a user, because even without a significant result it is better to assign the most suitable GUI to the user on the basis of the information available about the user, rather than assign an arbitrary GUI to the user.

15 Prospects

Although the implementation in computer science could not be done in this thesis, the editability of a GUI was enabled by such OS standard technology. The implementation on mobile devices and a solution for offline usage would be the next step. In the future, a prototype will be developed. For this purpose, the processes have to be automated for implementation in computer science. Once a prototype for one type of digital device (e.g. smartphones) has been developed, it can be tested in South Africa using empirical research and, subsequently, improved in line with the results of the empirical studies. Afterwards, the application could be made available to the pilot region, South Africa. Eventually, the application could be modified for other digital devices (e.g. cellphones, PCs and digital doorways). If the application works well in South Africa, it could be adapted to and made available for other countries.

The stability of the SAGU is achieved by including ANNs. Accordingly, in the beginning, that is, when little information about the user is available and the ANNs are still untrained, a GUI change is proposed to the user very quickly and, then, a return to the initial GUI is proposed. The threshold property of the ANNs provides a proposal for a GUI change only with more training of the ANNs, if it is later found that the user may have problems understanding the current GUI and that another GUI would suit the user's needs better. A method for training the ANNs should thus be developed in the future.

Some ideas for improving the GUI are the following (examples of GUI screenshots can be found in CHAPTER 10): When developing the various GUI designs, some of the GUI guidelines described in CHAPTER 5 will be implemented, for example a state feedback via a time bar if a long period of time elapses between action feedback and objective feedback. Telemedicine (SUBSECTION 5.5.1) will be used, for example, to support a person in an accident situation where somebody has to intervene to help an injured person and no medical doctor is physically available. Risk perception (CHAPTER 4) can be used to replace scales for the better comprehension of risk. AR (SUBSECTION 5.5.2) can be used for displaying an invisible risk (for example an epidemiological risk) or better visualisation of complex tasks (e.g. medical treatment carried out by a lay person - AR can be implemented in telemedicine).

The *Project Reality View* (see `http://wiki.navit-project.org/index.php/Augmented_Reality` (retrieved 16/02/2014)), a *Navit* extension based on AR, will be developed in cooperation with the

Geo-information Group of Potsdam University based on the *Android* platform. The *Project Reality View* could easily be modified for the application for early warning and decision support to improve the comprehensibility of the navigation. Furthermore, *Navit* could be equipped with an adaptive GUI design based on the approach developed in this thesis. AR could also be implemented for the identification of species (p.49), namely, vectors. For example this approach could be adapted for the identification of anopheles mosquitoes. Moreover, crowd sourcing could subsequently be used for the collection of data on the occurrence of anopheles mosquitoes in certain areas. In other words, when somebody identifies an anopheles mosquito using our application, the user's location delivered via GPS will be stored in a database. Based on this data, actual risk maps could be generated. One difficulty that has been identified, however, is the photo recording quality of cellphones, because very high quality is required. Software for the determination of several GUIs for different digital devices, for example smartphones, other cellphones with smaller displays, cellphones displaying only text information, and PCs will be developed. Some other requirements for the digital devices and the GUI are described on p.197. Certain GUI elements will be evaluated by further empirical studies carried out in South Africa with regard to adaptivity, comprehensibility, benefit for the user and conceiving the spatial sense.

16 Closing Remarks

This thesis arises from the *ReGLaN-Health and Logistics* project, a vibrant project in which members of different research fields collaborate with the aim of improving health service delivery (see p.8). The intention is to do this by, inter alia, creating an application for digital devices to deliver decision support. This project has been developing and growing for a number of years, with each member contributing to the success of the project through her/his research. Accordingly, this thesis contributes to the project by proposing a mathematical structure for a system for an adaptive GUI application, with a prototype of the application being developed in the next few years. The next step in this development structure should be its implementation as a generic IT framework. By including many researchers from different fields, who work together and look at the application from different angles, we may be able to overcome disciplinary boundaries.

Additionally, the concepts of OS and OC are essential to the project and, thus, the application will be made freely available to the user. Moreover, the application can be easily edited and modified by local computer scientists in South Africa in order to adapt the software to their needs.

Hopefully, the application will be widely used and will be of great value to the population and, thus, ultimately improve the health system.

Backmatter

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Melanie Platz